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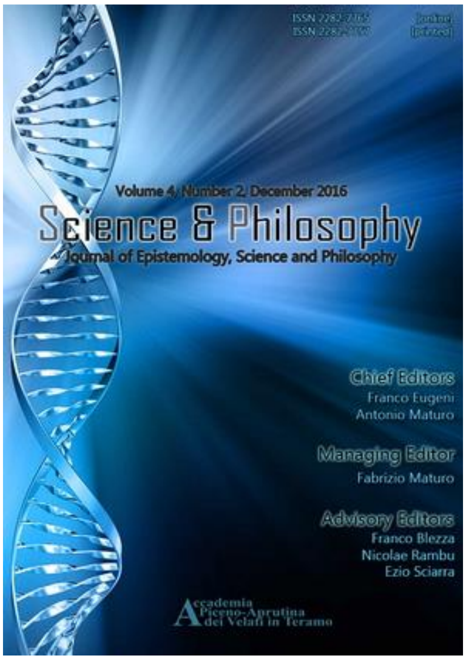

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Updating Statistical Measures of Causal Strength

H. D. Vinod *

Abstract

We address Northcott's (2005) criticism of Pearson's correlation coefficient 'r' in measuring causal strength by replacing Pearson's linear regressions by nonparametric nonlinear kernel regressions. Although new proof shows that Suppes' intuitive causality condition is neither necessary nor sufficient, we resurrect Suppes' probabilistic causality theory by using nonlinear tools. We use asymmetric generalized partial correlation coefficients from Vinod (2014) as our third criterion (denoted as Cr3) in addition to two more criteria (denoted Cr1 and Cr2). We aggregate the three criteria into one unanimity index, $UI \in [-100, 100]$, quantifying causal strengths associated with causal paths: $X_i \to X_j$, $X_j \to X_i$, and $X_i \leftrightarrow X_j$.
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1 Introduction

Scientists and philosophers have debated the measurement of causal directions and strengths for a very long time. In pharmaceutical research, the Food and Drug Administration (FDA) has favored randomized controlled trials as the gold standard for assessing whether a drug is safe and effective. Many social scientists and philosophers have noted that controlled trials are often impractical, costly, time-consuming, and ethically unsuitable. Cartwright [2007] goes beyond the narrow scope of controlled experiments to argue that ultimately their basis remains deductive, implying that when assumptions fail to hold precisely, the causal strength measurements from controlled trials also become suspect.

Practitioners have long used Pearson's correlation coefficient 'r' developed in the 1890s as an indicator of causal strength. Rodgers and Nicewander [1988] list 13 interpretations of r including: a type of mean, a type of variance, the ratio of two means, the ratio of two variances, the slope of a line, the cosine of an angle, the tangent to an ellipse, and so forth. Building upon Sober [1988], and focusing on only two interpretations of r, Northcott [2005] highlights its limitations for measuring causal strengths. This paper refers to a newer generalized correlation coefficient r^* from Vinod [2014] and Vinod [2017b] implemented in a free software package Vinod [2017a] to overcome those limitations and lead to a newer practical measure of causal strength.

We limit the scope here by considering only those noisy causal strengths which can be computed in terms r or r^*. Hence, this paper bypasses the considerable literature dealing with Directed Acyclic Graphs (DAGs) or Pearl's 'do' operator, Pearl [2010]. We shall see that a study of r^* can include "variation in the features under study" representing counter-factuals described by Cartwright [2003]. Salmon [1977] [p. 151] suggests replacing the probabilities of *events* by causally connected *processes* defined as "spatio-temporally continuous entities" having their own physical status. Hence, our exploratory assessment of causal relations in this paper is assumed to be describable by regression equations between passively observed data generating processes (DGPs)–not between events. Our scope also excludes deterministic causal relations expressed as functional relations without random components. Thus, for example, Boyle's law (pressure *volume = a constant) where all component variables (pressure and volume) can be independently controlled in a laboratory, is beyond our scope.

Northcott distinguishes between absolute causal effectiveness CE_{abs}, what he calls "Gallalean idealization" isolating the contribution of a particular cause on the one hand, and CE_{rel}, or relative causal effectiveness representing the proportion of total noise explained by the cause. He argues that we are rarely interested in the relative concept, and that Pearson's r cannot measure CE_{abs}. One aim of this paper is to show how a generalized r, and two additional criteria together can indeed quantify CE_{abs}.

Consider a set of p random variables $V = (X_1, \dots X_p)$, with subscripts from the index set, $V_I = \{1, \dots p\}$, and their joint density, $f(V)$. Unlike Cartwright, it is convenient here to let the same symbol X_i stand for observable proxies, as well as, the true underlying (cause or effect) variables. The conditional density is, by definition, the joint density divided by the marginal density, $f(X_i)$. The latter is the Radon-Nikodym derivative of the joint density, either with respect to the Lebesgue or the counting measure. The idea that X_i causes X_j is conveniently denoted by the causal path $X_i \to X_j$. For concreteness, the reader can think of X_i as a treatment, X_j as an outcome, and X_k as a set of background conditions often called control variables (e.g., age, gender, ethnicity), which affect the outcome.

Assume we have T observations denoted by $V_t = (X_{1t}, \dots X_{pt}), t = 1, 2, \dots T$. Let us denote by $LjRik$ a model having X_j on the left-hand side (LHS), and X_i plus a set of variable(s) combined into generic X_k on the right-hand side (RHS).

$$LjRik: \quad X_{jt} = f(X_{it}, X_{kt}) + \epsilon_{j|ik}, \tag{1}$$

where we use a generic f to denote a possibly nonlinear function, and ϵ denotes unobserved shocks or errors. According to one of 13 interpretations mentioned above, Pearson's r is a signed square root of the coefficient of determination R^2_{LjRik} of regression with linear f in (1).

Northcott [2005] lists the following two ambiguities associated with using Pearson's r.

(i) **Variable level versus its variance and covariance**: Northcott uses the example of X_j as the level of stock market price, and X_i as some variable affecting the stock price to argue that R^2 is focused on the variance of the stock price, which is of interest only to hedge funds focused on volatility, ignoring the stock price levels which are of interest to most investors. Our second criterion (Cr2) will explicitly consider absolute values of regression residuals $|\hat{\epsilon}_{j|ik}|$, obviously affected by levels of (X_i, X_j, X_k) even if we standardize the data variables to have zero mean and unit standard deviation.

(ii) r **cannot handle counter-factuals**. Northcott himself suggests a solution to the second problem by comparing (1) with an auxiliary regression representing a "baseline counter-factual" using the complete absence of X_i from the right-hand side as in the model:

$$LjRk: \quad X_{jt} = f(X_{kt}) + \epsilon_{j|k}. \tag{2}$$

Denote by R^2_{LjRk} the coefficient of determination of regression (2). Now Northcott implicitly suggests a simple extension of Pearson's r for computing the causal strength of $X_i \to X_j$ by computing

$$(X_i \to X_j)_{abs} = R^2_{LjRik} - R^2_{LjRk}. \tag{3}$$

When two more (original and auxiliary) regression equations are analogously defined by flipping X_i and X_j, in equations (1) and (2), we can define the strength of the causal path in the opposite direction by:

$$(X_j \to X_i)_{abs} = R^2_{LiRjk} - R^2_{LiRk}. \tag{4}$$

The strengths of the two paths from (3), and (4) will not, in general, be the same. Thus, even though the matrix of Pearson's r coefficients is symmetric, we have

$$(X_j \to X_i)_{abs} \neq (X_i \to X_j)_{abs}, \quad a.e., \tag{5}$$

where $a.e.$ denotes "almost everywhere" in a relevant measure space, in the sense that any violations of (5) are a 'set of measure zero' or extremely rare. Remark 1 in the sequel exploits the difference in the absolute magnitudes of the two sides of (5) to help assess the causal direction.

Vinod [2014] proves that a matrix of generalized correlation coefficients r^* is asymmetric. However, we cannot entirely rely on (4) or (5) as the sole criterion, because Northcott's item (i) listed above regarding variable levels remains unaddressed.

Mandel [2017] Table 4 summarizes ten definitions and concepts of causality from the literature, of which the 5-th is Suppes' probabilistic causality discussed in Hitchcock [2018]. It defines: X_i probabilistically causes X_j, if the information that X_i occurred increases the likelihood of X_j occurrence. The intuition behind "probabilistic theory of causation," Suppes [1970], is that if the causal path ($X_i \to X_j$) holds, we should have:

$$P(X_j|X_i) > P(X_j) \quad a.e., \tag{6}$$

which is 'probabilistic,' because it holds ($a.e.$), meaning that it may be violated on rare occasions. In modern probability (measure) theory, the notion of rare violations is described by the expression ($a.e.$). We require the number of violations of the inequality (6) comprise 'a set of measure zero.'

Eells [1986] questions the intuition behind the inequality (6) by showing that a genuine cause need not raise the probability of a genuine effect when interacting with a third factor might be present. Some philosophers including Salmon [1977] have long criticized Suppes' theory with examples showing its logical failures. It is convenient to refer to those criticisms collectively as "old proofs," allowing us to state that the following Lemma provides a formal new proof.

Lemma 1: Suppes' condition (6) is neither necessary nor sufficient for causality

Proof:

Let X_k denote an additional omitted cause, which might be a confounder. By definition, 'confounder' X_k is a plausible underlying cause behind the apparent cause. Now, it is possible to construct counterexamples where the true causal paths are: ($X_k \to X_i$), and ($X_k \to X_j$). For example, let X_i denote the event of an atmospheric barometer falling sharply, and let X_j denote a weather storm event. These X_i, X_j events satisfy eq. (6). However, the barometer gauge itself does not cause the storm! The true cause X_k 'falling atmospheric pressure' is hidden from (6). Since barometer reading X_i is not necessary for the storm event X_j, this is a counterexample. Thus the "necessity" of (6) is rejected.

We reject sufficiency by using the definition of conditional probability as follows. Since conditional probability equals joint divided by marginal, we can rewrite (6) as

$$\frac{P(X_i \cap X_j)}{P(X_i)} > P(X_j),$$

or upon multiplying both sides by $P(X_i) > 0$ as:

$$P(X_i \cap X_j) > P(X_i) * P(X_j).$$

The inequality's sense remains intact if we divide both sides by a positive quantity, $P(X_j) > 0$, to yield the inequality:

$$\frac{P(X_i \cap X_j)}{P(X_j)} > P(X_i).$$

Thus we must always have

$$P(X_i|X_j) > P(X_i) \quad a.e. \tag{7}$$

This puzzling implication proves that Suppes' test satisfies conditions for $X_j \to X_i$ as well as $X_i \to X_j$ at all times. A result finding bidirectional causality $X_i \leftrightarrow X_j$ all the time means that the condition (6) is logically flawed, insufficient to distinguish between $X_i \leftrightarrow X_j$, and $X_i \to X_j$. **QED**.

Some philosophers and economists (e.g., Clive Granger) have suggested that the path $X_i \to X_j$ should further require that X_i must occur chronologically before X_j occurs, to help achieve a desirable asymmetry property. However, this is needlessly restrictive and inapplicable for human agents (who read newspapers) acting strategically at time t in anticipation of future events at time $t+1, t+2, \ldots$. Sayre [1977] also argues that temporal directionality is not needed.

Remark 1: Asymmetry via flipped models

Logically consistent probabilistic causality theory must retain robust asymmetry even when our causality testing condition(s) are stressed by flipping the cause and effect (X_i and X_j). Since equations (6) and (7) suggesting opposite causal directions are proved to coexist, we need to go beyond the inequality signs, and consider the relative magnitudes of the differences: $(P(X_j|X_i) - P(X_j))$, and $(P(X_i|X_j) - P(X_i))$, in order to generalize Suppes' non-deterministic theory.

Remark 2: Confounders and controls distinguished

The causal path $X_i \to X_j$ assessment is often affected by two types of often present related events X_k. It is convenient to distinguish between two types of X_k: (i) 'confounder' and (ii) 'control' variables, even though the two may be synonymous for many readers. Recall that, by definition, 'confounder' X_k is a plausible underlying cause behind the apparent cause X_i for the outcome X_j. For example, the true cause of weather events X_j is 'atmospheric pressure' X_k and not 'barometer reading' as X_i. Second, we define X_k as a 'control' event if both (X_i, X_k) may be causing X_j, but we are interested in knowing if X_i causes X_j over and above the effect of X_k. For example, let X_j be health outcome, X_i be some medicine, then X_k, the patient's age, is commonly used as a control. A confounder can be treated as a control, but the converse may not hold true.

This paper develops a practical probabilistic theory of causality. Our Theorem 1 in the sequel proves that the revised theory does not suffer from the logical problems with Suppes' theory. When we try to develop computational methods for implementing the revised theory, we find that there are at least three coequal empirical criteria, denoted by Cr1, Cr2, and Cr3, which quantify the support for the causal path $X_i \to X_j$. It is not possible to prove why one criterion should dominate others. Hence, let us use the familiar 'preponderance of evidence' standard by computing a weighted sum of the three criteria denoted by $ui \in [-100, 100]$, and call it a sample unanimity index. Given a 5% threshold, say, (or $\tau = 5$), the index allows us to propose the following decision rules:

Rule 1: If $(ui < -\tau)$ the causal path is: $X_i \to X_j$.

Rule 2: If $(ui > \tau)$ the causal path is: $X_j \to X_i$.

Rule 3: If $(|ui| \leq \tau)$ we obtain bi-directional causality: $X_i \leftrightarrow X_j$, that is, the variables are jointly dependent.

Vinod [2017b] reports simulations showing good performance of these rules from a journal specializing in simulations.

Complete computational details for using these decision rules on any given data set are a part of an open source and a free software package called 'generalCorr', Vinod [2017a], in the computer language called R. It is readily available in an open forum for further criticism and development. The package comes with three vignettes that provide technical details about the algorithms used along with examples and citations to additional relevant papers, including Zheng et al. [2012].

A referee has pointed out that underlying ideas were partially anticipated by Yule in the late 1890s.

An outline of the remaining paper is as follows. Section 2 uses these Remarks 1 and 2 while avoiding logical problems with Suppes' theory to propose a revised version comprising our Theorem 1. We must translate the necessary and sufficient (iff) conditions for revised probabilistic causality into decision rules using the available data in the form of DGPs (X_i, X_j, X_k). This requires some sophisticated statistical tools. The intuition behind these tools is discussed in Section 3 without technical details. Section 4 briefly reviews kernel regressions for fitting nonlinear nonparametric functions. The residuals of these regressions are used to quantify Theorem 1 in Section 5. Section 4 specifies our three criteria Cr1 to Cr3. Section 6 develops a quantification of our Cr1 to Cr3, leading to a sample unanimity index $ui \in [-100, 100]$, summarizing the three criteria into a single number. Section 7 contains our final remarks.

2 Revised Probabilistic Theory of Causality Among DGPs

Following Remark 2 we define the set of variables X_k as containing both confounder and control variables. Since there are situations where X_k variables are completely out of the picture, we need two versions of the following result to accommodate both situations.

Theorem 1: Revised probabilistic Causality

(Version a) Assuming data on X_k are available, the causal path $X_i \to X_j$ holds if and only if (iff)

$$(P(X_j|X_i, X_k) - P(X_j|X_k)) > (P(X_i|X_j, X_k) - P(X_i|X_k)), \quad a.e. \tag{8}$$

(Version b) Assuming data on X_k are available, the causal path $X_i \to X_j$ holds if and only if (iff)

$$(P(X_j|X_i, X_k) - P(X_j)) > (P(X_i|X_j, X_k) - P(X_i)), \quad a.e. \tag{9}$$

Proof: Our proof removes the obstacles to proving 'necessity' described in Lemma 1 above by explicitly including X_k variables, which belong in the set of conditional variables in both versions of Theorem 1. Logical problems with Suppes' condition arise from the simultaneous existence of equations (6) and (7). We remove it by using flipped models (Remark 1) to impose asymmetry and focusing on relative sizes of inequalities of flipped models.

The iff condition from Theorem 1(Version a) becomes

$$(f(X_j|X_i, X_k) - f(X_j|X_k)) > (f(X_i|X_j, X_k) - f(X_i|X_k)) \quad a.e. \tag{10}$$

The slightly simpler iff condition from Theorem 1(Version b) becomes

$$(f(X_j|X_i, X_k) - f(X_j)) > (f(X_i|X_j, X_k) - f(X_i)) \quad a.e. \tag{11}$$

Since we eschew consideration of 'events' and focus on probabilities of DGPs, we can use widely accepted multiple regression to remove the effect of X_k, not readily available if we were to study probabilities of events. Besides DGPs, a further novelty here is to use nonlinear nonparametric kernel regressions (instead of linear regressions). Its advantages explained later include greater realism and superior statistical fits.

3 The Intuition behind Empirical evaluation of Theorem 1

The iff conditions established in Theorem 1 involve various probability density functions $f(.)$. Now consider the construction of these densities from data in the form of $t = 1, \ldots, T$ observations on DGPs for (X_i, X_j, X_k). We describe the intuition behind modern statistical methods used here, quantifying equations (10) and (11) starting with the simplest.

(a) **Marginal Densities:** Elementary statistics teaches us how to classify the data series X_i into a few class intervals and draw histograms. More advanced statistics papers describe smoothing of histograms into empirical approximations to marginal density functions $f(X_i)$, separately for each variable.

(b) **Joint Densities:** When we study two or more variables simultaneously, placing them along two or more axes, we need simultaneous class intervals for joint variation arising from multi-way histograms based on a joint binning of the data. Instead of histogram smoothing, quantification of joint densities of two or more variables is handled by using kernel weighting in higher dimensional spaces using modern computer-intensive methods.

(c) **Conditional Densities:** One defines conditional densities as 'joint density' divided by 'marginal density,' similar to conditional probabilities.

(d) **Standardization:** Recall that a typical evaluation of iff conditions requires quantification of the difference between two (conditional) densities. For example, the left-hand side of (10) is: $(f(X_j|X_i, X_k) - f(X_j|X_k))$. Since units of measurement of variables in different DGPs are likely to be distinct, it is intuitively obvious that one cannot simply subtract one density from another. We must first standardize all variables to have zero means and unit standard deviations, technically known as requiring densities to have numerically comparable supports.

(e) **Stochastic Dominance:** Financial economists need tools to choose between two or more risky assets (e.g., buying Facebook or Amazon stocks) using the probability distributions of their expected future returns, extrapolated from data on past stock returns. When comparing two or more densities, we need to examine their distinct features exemplified by local mean, variance, skewness, and kurtosis. There are well-developed methods in Finance based on the concept of stochastic dominance of four orders to compare the four moments of the density, respectively. Empirical versions of stochastic dominance methods yield four sets of numbers comparing four orders of integrals of two 'empirical cumulative density functions' being compared. One can compute such four sets of numbers when comparing any two densities.

(f) **Conditional Expectation Functions & Kernel regressions:** Since joint densities $f(X_i, X_j, X_k)$ have three or more dimensions, they are difficult to quantify into one set of T numbers. Typical conditional density functions needed in our iff conditions such as $f(X_j|X_i, X_k)$, obtained from ratios of joint and marginal densities, are also multi-dimensional and difficult to quantify. Hence, we use T estimates of fitted values of kernel regression models associated with conditional *expectation* functions $Ef(X_j|X_i, X_k)$ evaluated at each $(t = 1, \ldots T)$.

In traditional linear regressions, conditional expectation functions contain regression coefficients that remain constant for all t and yield fitted values. Moreover, flipped linear regressions, $X_j = a + bX_i$ and $X_i = \tilde{a} + \tilde{b}X_j$, have identical R^2 values. Hence, we cannot assess causal directions from measures of goodness of fit of the flipped linear regressions. Therefore, we must use more sophisticated nonlinear kernel regressions described in the next section.

4 Kernel Regression Review

The linearity of the regression model is often a matter of convenience rather than an evidence-based choice. Back in 1784, the German philosopher Kant said: "Out of the crooked timber of humanity no straight thing was ever made." Since social sciences and medicine deal with human agents, evidence supporting linearity is often missing.

The main reason for using nonparametric nonlinear kernel regression in applied work is to avoid misspecification of the functional form. The best-fitting kernel regression line is often jagged, which does not have any polynomial or sinusoidal form. However, it provides a superior fit (higher R^2) by not assuming a functional form.

A disadvantage used to be its computational difficulty, which has recently disappeared. The remaining disadvantages are that kernel regressions fail to provide partial derivatives and that out-of-sample forecasts can be poor. Fortunately, partial derivatives and out-of-sample forecasts are irrelevant for determining causal structures.

Let us replace the generic function f from (1) by the population conditional expectation function $G_1(X)$ when X_{kt} variables are omitted for ease of exposition without loss of generality. It will be estimated by nonlinear and nonparametric kernel regression. The sample estimate of G_1 is:

$$g_1(X) = \frac{\sum_{t=1}^{T} X_{jt} K(\frac{X_{it}-X}{h})}{\sum_{t=1}^{T} K(\frac{X_{it}-X}{h})}, \tag{12}$$

where $K(.)$ is the well known Gaussian kernel function, and h is the bandwidth chosen by leave-one-out cross-validation. The exposition becomes complicated when several regressors are involved, each needing a separate bandwidth of its own as described in Li and Racine [2007]. It is well known that kernel regression fits are superior to OLS.

Assuming that g_1 in eq. (12) belongs to $\mathscr{B}$, a class of Borel measurable functions having a finite second moment, then g_1 is an optimal predictor of X_j given X_i, in the sense that it minimizes the mean squared error (MSE) in the class of Borel measurable functions, [Li and Racine, 2007, Theorem 2.1]. Also note that kernel regression estimates are proved to be consistent and asymptotically normal (CAN) under certain assumptions, partly considered in the next subsection.

4.1 Kernel regression consistency

The sample kernel regression estimate g_1 of the population conditional expectation function G_1 is consistent provided true unknown errors in eq. (1) are orthogonal to the regressors. That is, the following probability limit should be zero, or:

$$\text{plim}_{T\to\infty}(\epsilon_{j|ik} X_{it})/T = 0. \tag{13}$$

We note in passing that nonlinear nonparametric kernel regressions prevent inconsistency induced in linear regressions by hidden nonlinearities. Assume that we approximate the hidden nonlinearity by a high order polynomial. Now a researcher using a linear model is implicitly letting high order polynomial terms merge into the regression error. Since the merged error will be correlated with the regressor due to misspecification, it will induce inconsistency.

4.2 Implicit Counter-factuals in Cross-validation:

Counter-factuals are defined as "what has not happened but could happen" in the avaliable data. Since experimental manipulation is often not an option, especially in social sciences, many authors use virtual manipulation involving counter-factuals, implicit in cross-validation described next.

Considering $\{X_{it}, X_{jt}, X_{kt}\}$ data, when we pretend that t-th observation is absent, even though it is present, we have a counter-factual. Now leave-one-out cross-validation used to determine bandwidth h appearing in (12) of kernel regressions minimizes a weighted error sum of squares

$$min_h \quad \frac{1}{T}\Sigma_t[Y_t - \hat{g}_{1,-tL}]^2 W(X_t), \tag{14}$$

where $W(.)$ is a weight function, subscript $(-t)$ denotes omitting t-th observation, and where the subscript (L) refers to a local linear fit. We employ cross-validation as a counter-factual in our determination of (g_1) conditional expectation functions, which will eventually determine our causal direction and its strength.

5 Quantification of Theorem 1 from residuals

Recall that numerical quantification of the causal path $(X_i \rightarrow X_j)$ between standardized DGPs from Theorem 1(b), requires that we evaluate the four-part inequality: $[(P1 - P2) > (P3 - P4)]$, where the four parts P1 to P4 are readily seen from (10) to be:

$$(f(X_j|X_i) - f(X_j)) > (f(X_i|X_j) - f(X_i)).$$

Quantification of the first part, P1:

In randomized controlled experiments, the conditioning variables are randomly assigned to experimental units $(X_{it}, t = 1, \ldots, T)$, and the researcher records corresponding values of X_{jt}. Whether experimental or passively observed, one can construct T_i class intervals and record the probabilities (relative frequencies) of various values of X_j in each class interval (group). Midpoints of class intervals and associated relative frequencies as probabilities yield T_i numbers representing the conditional density P1 $=f(X_j|X_i)$. The range of observed values of X_i represents the 'support' of P1 density. If each class interval is to have at least five observations on an average for a reasonable estimate of conditional density, T will be five times larger than T_i. In general, T_i class intervals will be far fewer than T, expressed by $T_i << T$. We like to avoid cumbersome construction of class intervals, which always involves loss of information since they are not 'sufficient' statistics.

Paragraph (f) of Section 3 notes that conditional expectation functions from kernel regression of X_j on X_i can yield consistent estimates of fitted values of $\hat{X}_{jt|i}$ containing T numbers. We have noted before that fitted values obtained by using bandwidths from leave-one-out cross-validation perform a form of counterfactual, relevant for conditional density estimation.

Next, consider the **quantification of P2**, which is $f(X_j)$. Note that data on X_{jt} can readily construct an empirical cumulative distribution function (ECDF) whose Radon-Nikodym derivative is X_{jt}, providing T numbers which represent the density P2.

Regression residuals are defined as 'observed minus fitted' values: $e_{jt|i} = X_{jt} - \hat{X}_{jt|i}$. Using quantified P1 and P2 described above, **quantification of P1–P2** is available from negative residuals or $-e_{jt|i}$.

Consider **quantification of P3–P4**. Note that this is completely analogous to P1–P2, when we simply flip i and j. Hence, it is easy to verify that P3–P4 is quantified by the negative of the residuals of a flipped kernel regression or $-e_{it|j}$.

Finally, we can assess whether the inequality $P1 - P2 > P3 - P4$ in Theorem 1(b) holds by computing the following inequality, where we replace negative residuals by positive ones, change the sense of the inequality from (>) to (<) and also insert absolute value symbols to yield:

$$|e_{j|i}| = |X_j - E[\hat{f}(X_j|X_i)]| \; < \; |X_i - E[\hat{f}(X_i|X_j)]| = |e_{i|j}|, \quad a.e. \tag{15}$$

It is easy to verify that the densities in Theorem 1(a) are similar to the P1 to P4 discussed above, except that we have to condition all densities on control variables X_{kt}. Thus causal path $X_i \to X_j$ after removing the effects of confounder or control variable(s) X_k can be approximately assessed by

$$|e_{j|ik}| = |X_j - E[\hat{f}(X_j|X_i, X_k)]| \; < \; |X_i - E[\hat{f}(X_i|X_j, X_k)]| = |e_{i|jk}|, \quad a.e., \tag{16}$$

An intuitive interpretation of this inequality is that the causal path $X_i \to X_j$ requires kernel regression LjRik with causal variable X_i on the RHS should have a superior fit compared to the flipped regression LiRjk.

The inequalities (15) and (16) are considered fuzzy since they hold with some exceptions, almost everywhere but not everywhere. A concrete example is illustrated in Vinod [2017a] using European data. It has the crime rate as X_i, police deployment rate as X_j. The causal path from high crime to high police deployment, $X_i \to X_j$, requires that regression residuals for the model with X_i on RHS are "smaller" in some fuzzy sense than the flipped model with X_j on RHS.

6 Criteria Cr1 to Cr3 using residuals

The discussion so far has quantified Theorem 1 as amounting to certain fuzzy inequalities among T numbers obtained from residuals of kernel regressions with flipped i and j in equations (15) and (16). Our next task is to develop three criteria (Cr1 to Cr3) using these residuals to quantify the strength of the causal path, which will eventually yield our unanimity index ui.

Our first criterion Cr1 described next evaluates finite sample implications of consistency of conditional expectation functions in kernel regressions described in Section 4.1. We plug observable residuals into the consistency condition (13), yielding two sets of probability limit ('plim') expressions. We simply compute absolute values of T multiplicative expressions: $|e_{j|ik}X_{it}|$ and $|e_{i|jk}X_{jt}|$. Our Cr1 exploits the theoretical result that closeness to zero of these expressions implies faster convergence.

6.0.1 First criterion Cr1 for $X_i \to X_j$

Long ago, Koopmans [1950] formulated the consistency requirement of eq. (13) as exogeneity of X_i and went on to require that each right-hand side (RHS) variable should be exogenous in the sense that it should "approximately cause" the LHS variable. Being the oldest, we let this criterion based on comparing residuals provide our first criterion.

Since Kernel regressions are consistent, the conditional expectation functions are also consistent. Since speeds of convergence can differ, one should prefer the conditioning with a faster convergence rate. The obvious finite sample indicators of speeds of convergence are available from eq. (13) when we replace the true unknown errors by residuals. If the conditional expectation function when X_i on RHS converges faster to its true value than its converse, the T values implied by the 'plim' expression of the LjRi model should be closer to zero than the similar 'plim' expression of the flipped LiRj model.

Hence, the condition for the causal path $X_i \to X_j$ based on the faster convergence rate of the LjRik model, than that of the flipped LiRjk model, is the inequality:

$$\text{Cr1}: \quad |e_{j|ik}X_{it}| < |e_{i|jk}X_{jt}|, \quad a.e., \tag{17}$$

where the residuals are comparable in numerical magnitudes because we have standardized all variables.

Working with residuals overcomes a criticism of 'r' defined in terms of variances and covariance by Northcott [2005] mentioned before, that 'r' ignores data 'levels.' Residuals are obviously sensitive to levels.

Note that we have T numbers for each side of the inequality, and we want to compare whether one set is "larger" than the other, analogous to the investor's problem in choosing one distribution of two risky investment options. An old solution to the problem uses stochastic dominance methods for four orders approximating local mean, variance, skewness and kurtosis of the two distributions, yielding four sets of numbers for a thorough and sophisticated comparison of two investment options. Here, we simply apply the tools from Finance to our problem.

6.0.2 Second criterion Cr2 for $X_i \to X_j$

Our second criterion simply restates the fuzzy inequality (16) quantifying our Theorem 1. Recall that it checks whether independent changes in X_i lead to (dependent) changes in X_j, leading to LjRi model providing a better fit than LiRj. Hence we require:

$$\text{Cr2}: \quad |e_{j|ik}| < |e_{i|jk}|, \quad a.e. \tag{18}$$

Note that Cr2 is similar to Cr1, requiring a comparison of two sets of T numbers. Hence we quantify Cr2 by using stochastic dominance of four orders.

6.0.3 Third criterion Cr3 for $X_i \to X_j$

Following Vinod [2014] an aggregate manifestation of the 'a.e.' inequality (16) involving residuals: $e_{j|ik}, e_{i|jk}$, can be stated in terms of a higher coefficient of determination R^2 for one of the two flipped models. The effect of X_k variable(s), if any, on X_i, X_j is netted out in these computations to yield our third criterion:

$$\text{Cr3}: \quad R^2_{j|i,k} = 1 - \frac{\Sigma(e_{j|ik})^2}{(\text{TSS})} > 1 - \frac{\Sigma(e_{i|jk})^2}{(\text{TSS})} = R^2_{i|j,k}, \tag{19}$$

where TSS denotes the total sum of squares, which is $(T-1)$ for standardized data, and where we denote the conditioning in the two models by subscripts to R^2.

An equivalent requirement using generalized partial correlation coefficients from Vinod [2017a] for $X_i \to X_j$ is:

$$|r^*_{(j|i;\,k)}| > |r^*_{(i|j;\,k)}|. \tag{20}$$

Two advantages of Cr3 are that it involves a simple comparison to two summary statistics that can be computed without having to standardize the data.

R package 'generalCorr' reports the generalized partial correlation coefficients in eq. (20), if desired. The R function `pacorMany` provides partial correlation coefficients of the first column paired with all others after removing the effect of a specified list of control variables. Recall that Theorem 1(a) eq. (10) considers netting out of the confounders X_k from both causal X_i and outcome X_j variables, exactly as it is implemented in computing (20).

Since quantification of Cr1 and Cr2 yields four numbers for four orders of stochastic dominance, we need a weighting scheme to get one summary number each denoted by $N_{cri}, i = 1, 2$. Cr3 yields only one sign leading to N_{cr3}. Our unanimity index transforms a weighted sum of N_{cri} into $ui \in [-100, 100]$ needed for the decision rules Rule 1 to Rule 3. Vinod [2017b] reports successful simulations using the rules and tools for bootstrap-based statistical inference to study sampling variability of ui.

7 Summary and Final Remarks

This paper supports Northcott [2005] view that the Pearson correlation coefficient 'r' is unsuitable for measuring causal strength or absolute causal effectiveness (CE_{abs}). Instead of 'r' we suggest recently developed generalized partial correlation coefficients $r^*_{i,jk} \neq r^*_{j,ik}$ based on nonparametric kernel regressions, Vinod [2014]. Partial correlations help quantify our third criterion Cr3. Since Cr3 is sensitive to variances and covariances but not data 'levels,' we overcome Northcott's criticism of 'r' by not relying on Cr3 alone. We use additional coequal criteria, Cr1 and Cr2, based on certain residuals sensitive to data levels.

Our Lemma 1 in Section 1 provides a new proof showing that the condition $P(X_j|X_i) > P(X_j)$ proposed in Suppes' "probabilistic causality theory" is logically faulty for the causal path $X_i \to X_j$, because it always coexists with the opposite path $X_j \to X_i$.

Our Theorem 1 incorporates additional variables X_k if confounding variables vitiating the causal paths are present and goes on to update Suppes' theory by providing new iff conditions in the form of inequalities involving conditional and marginal densities. Direct quantification of inequalities among conditional densities using limited, nonexperimental, and passively observed data (without manipulation) is obviously difficult. Hence, we suggest a deeper study of two nonparametric kernel regressions, LjRik: $X_{jt} = f(X_{it}, X_{kt}) + \epsilon_{j|ik}$, and a flipped regression where i and j are interchanged, LiRjk.

We employ three coequal empirical criteria Cr1 to Cr3 to quantify support for the causal path $X_i \to X_j$ using the absolute values of residuals of two regressions LjRik and LiRjk. Sophisticated comparisons need to look at all locally defined mean, variance, skewness, and kurtosis properties of empirical cumulative distribution functions implied by absolute values of residuals. We employ readily available tools from Financial Economics to quantify our criteria and develop a unanimity index ui used to provide decision rules for choosing between three causal paths $X_i \to X_j$, $X_{,} \to X_i$, and $X_i \leftrightarrow X_j$.

Our methods are completely transparent (open source, free) described in the R package 'generalCorr' and its three vignettes, providing examples, simulations, and all details. One R command, `causeSummary` is worthy of further attention as an approximate implementation of Theorem 1. The 'generalCorr' decision rules are recently used in Lister and Garcia [2018] to conclude that global warming causes anthropod deaths, in Allen and Hooper [2018] to explore causes of volatility in stock prices, in Mlynczak and Krysztofiak [2019] to study causal links between sports and cardio-respiratory issues faced by elite athletes, and in Fousekis [2020] to study US commodity futures markets.

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The history and the meaning of Einstein's Principle of Equivalence

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Abstract

We review from a historical and a didactic point of view the Equivalence Principle, which was considered by Einstein as the corner stone of his new theory of Gravitation: the General Relativity. Before and after the enormous success of his theory, this principle was the subject of studies and discussions. Still today, after more than one century, the debate about its interpretation, application and generalization is very fertile. Einstein soon understood the revolutionary significance of his idea and defined it as "the happiest thought of my life".

Keywords: History of Physics, Didactics of physics, Theory of Gravitation.†

1 Introduction

General Relativity is the theory that explains the motion of the bodies in the gravitational field in a geometrical way as caused by the curvature of space-time. All the theoretical scheme is based on two fundamental postulates: the Equivalence Principle and the Principle of General Covariance. The first postulate is the argument of this review (that updates an invited lecture on this subject held at a meeting in 2005 [1]) because it represents one of the best ideas that human mind has ever produced, while the second principle is more mathematical but not less important, being used by Albert Einstein to write the field equations in the final form in November 1915 after ten years of hard work and failed attempts.

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Since 1905, the annus mirabilis when Albert Einstein issued five fundamental papers for the History of Physics [2], he devoted all his efforts to try to make Newton's Gravitational Force consistent with his own Special Relativity [3]: in particular, he wanted to generalize Galileo's Principle of Relativity (which was valid for observers in straight line motion at constant velocity), even to accelerated observers. Galileo described his principle through the famous "mental experiment" of the ship[‡]. He suggested that we should perform a lot of experiments in a cabin below decks on a large ship. If we repeat the same experiment when the ship is standing still and when the ship is moving with constant velocity, we will always obtain the same result. From Galileo's reasoning we can conclude that *"there is no way to distinguish between what is experimented by an observer in the still ship and what is found by an observer in the ship in straight line motion at constant velocity".*

Einstein, in a lecture held in Kyoto [5], remembered when in 1907 [6] he had had the great idea to generalize this principle: "I was sitting in a chair in the Patent Office at Bern when all of a sudden a thought occurred to me:<<If a person falls freely, he will not feel his own weight>>" that is a freely falling observer feels no gravitational field. In this paper, we will try first to understand the meaning of this observation using the Galileo's method of the mental experiments, second to deduce the surprising consequences of this result on the role of space and time, on the vision of Gravitational theory and on the evolution of the entire Universe; finally, we will briefly discuss the limits and the violations of the equivalence principle.

2 The spaceship and the elevator

In order to explain his idea, Einstein resorted to a mental experiment which was very similar to Galileo's one. We can describe it in a simple way, using two observers (we will call them James and Tony). We will place one (James) in a lift still on the Earth and the other (Tony) in a spaceship far from stars and planets, so that it cannot undergo any gravitational influence. As in Galileo's ship example,

[‡]The Principle of Relativity in the words of Galileo [4]:"Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though doubtless when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still".

the two observers are not allowed to look out; moreover, it is supposed that the vacuum has been created both in the lift and in the spaceship.

James makes some experiments and realises that inside the lift the objects fall towards the floor with an acceleration $g=9.8m/s^2$. On the other side, Tony, in the spaceship far from any sort of matter and hence not subject to gravity, bounces inside the cabin together with all the objects which he has brought with himself and which are hanging in space.

In these conditions, is it possible to simulate gravity inside the spaceship, so that Tony can think he is on the Earth? Yes, it is. If we light the rockets of the spaceship with an acceleration $g=9.8\ m/s^2$, Tony will keep his feet anchored to the floor and will see the objects falling down just with the same acceleration as if he were on the Earth, even if, actually, they are not the objects that are falling on the floor, but it is the floor that is moving towards the objects. Whatever experiment Tony makes inside his accelerated spaceship, he will obtain the same results observed by James in the lift. So Einstein has reached his aim: he has found a principle which is similar to Galileo's one and is valid for accelerated observers.

The Equivalence Principle states that "*there is no way to distinguish between the effects observed in a constant gravitational field (lift) and the ones observed in a reference frame moving with uniform acceleration (spaceship)".*

In a similar way, we can simulate the absence of gravity, as Einstein had guessed in Bern Patent Office: we have only to cut the lift cables and to let it go into free fall, so that the situation becomes equivalent to the spaceship travelling with the engine switched off. Of course this is only locally true, i.e. if we limit ourselves to a small area of space where the acceleration of gravity (which depends on the distance from the centre of the Earth) is really constant and where we can neglect the tidal effects which exist in a real gravitational field but do not exist in a field simulated by an accelerated frame.

3 Galileo's experiment of the fall of heavy bodies

The validity of the equivalence principle is based on the experiment of Galileo who, after dropping different objects from the Tower of Pisa, discovered that, if the air friction can be neglected, the bodies reach the floor simultaneously. In the First Dialogue of his famous opera of 1638 [7] Galileo asserts that "...if the resistance of the medium was abolished, all the bodies of different materials would go down with the same speed". It means that all massive objects in free fall undergo the same acceleration in a gravitational field, regardless of their mass and composition. This experiment, which brought with it the equivalence between inertial mass and gravitational mass, was not given the due importance. Einstein noticed this "neglected clue […] shunned by everyone for three hundred years" [8] and made it the basis of his General Relativity. Let us try to understand why it is so important. We can imagine for a moment that Galileo was wrong, and gravity does not attract all the objects democratically with the same acceleration. Then the two observers, James and Tony, would not see the same results and the equivalence principle would be wrong. Actually, James in the lift would drop a hammer and a feather and would observe that the hammer is the first to reach the floor. On the contrary, in the spaceship they are not the objects which fall down, but it is the floor which moves towards the objects, so Tony cannot see the hammer falling before the feather, since the floor goes towards all the objects in the same way. To conclude, there is equivalence between James's results and Tony's ones only if Galileo was not wrong, otherwise the experiment of the fall of heavy bodies will give different results for the two observers. For the same reason, the equivalence principle would not work if we were making our experiments with an electric field rather than with gravity. Let us imagine to repeat Galileo's experiment with a charged hammer and feather that are placed in a constant electric field *E*. The force acting on the bodies is:

$$\vec{F} = q\vec{E} = m\vec{a} \Rightarrow \vec{a} = \frac{q}{m}\vec{E}$$

The term *q/m* will depend on the falling object; every object will have its own charge and its own mass and will be attracted towards the floor with an acceleration different from the others, so the hammer and the feather will not reach the floor simultaneously because the charge to mass ratios of the hammer and the feather are different:

$$\frac{q_h}{m_h} \neq \frac{q_f}{m_f}.$$

There is an interesting and noteworthy case where the Equivalence Principle is valid also for electromagnetism, because locally all the bodies have the same charge to mass ratio. This really happens inside the atom. If we consider only Coulomb's attractive Force exerted by the nucleus on the electrons, these latter have all the same charge and the same mass.

On the contrary, the Equivalence Principle can always be applied with the gravitational field: for example, near the surface of our planet, the acceleration

$$g = \frac{GM_T}{R_T^2} = 9{,}8 \text{ m/s}^2$$

does not depend on the masses of the falling objects, but on the mass and the radius of the Earth which attracts them. If all the bodies undergoing the influence of gravity fall from the same height, they will reach the floor of the lift in the same moment, as well as the spaceship floor, going upward, reaches all the bodies which are at the same height in the spaceship at the same moment. Now Einstein could start formulating his theory based on the Equivalence Principle, if he had not another objection to overcome.

4 The experiment with a beam of photons

If we think it over, there is actually a case where the two observers could obtain different results and Einstein's principle would not be valid. Let us resort to a mental experiment again. We will take a small laser pointer and fix it in the lift at 20 cm from the floor. We will switch it on and the photons will be projected on the opposite wall of the lift, where we will see a red point. The photons will be propagated in straight line as they have a zero mass and the gravity force does not influence them. We will take a ruler and, measuring the distance from the point to the floor, we would expect to find the value of 20 cm. However, let us see what happens in the spaceship when we repeat the same experiment. We will switch on the laser pointer and, as the rockets are working, in the time taken by the photons to reach the opposite wall, the floor will have moved a little upward. As a consequence, we will not expect the value of 20 cm, but we will obtain a slightly lower value (19,99…cm). Actually, we might not be able to measure the difference even by a precision instrument (it is of the order of 10^{-15}cm), but without any doubt the value is lower than 20 cm. So, Einstein's equivalence principle is violated, since we have found at least one experiment where the result obtained by Tony is different from the one obtained by James. Now we have a way to distinguish if we are in a real gravitational field or in a system with a constant acceleration.

On the ground of this argument, Einstein should renounce his principle, but he thinks that the principle is right, while the previous results are wrong. In particular, Einstein maintains that if the measure on the spaceship is 19,99….cm, then also the measure in the lift must necessarily be 19,99…cm. The course of the photons must slightly bend downward, i.e. also the photons must be influenced by the Gravitational Force.

By 1704 Isaac Newton had already wondered about the "weight" of light: " Do not bodies act upon light at a distance, and by their action bend its rays; and is not this action the strongest at the least distance?"

It is possible to answer the first question of Newton's "Optics", imagining that the photons themselves have a gravitational mass E/c^2 (hence they undergo the Newtonian acceleration of gravity) and calculating that, on its way towards the earth, a beam of light coming from a far away star, is deflected 0,87 arc seconds when it is skimming the Sun.

A similar approach was used by Einstein [9] in 1911 (he found 0.83 arcsec), but straightforward he realized that there may be an alternative explanation without any need to invoke a gravitational force.

From Fermat principle a beam of light always follows the path of minimal time to go from a starting point A to a final point B. The consequence is that the light beam travels along a straight line that is the shortest path between the points A and B. But in the example of the lift, the path of the light beam is curved. So "if a light beam can follow a curved path, then this curved path must be the shortest distance between two points – which suggests that space itself is curved [10]"

According to this revolutionary interpretation of gravity, the bodies in the lift fall downward and the photons curve, not because there is a force attracting them, but because they live in a "curved space-time". On the basis of this theory, the Sun curves the space around itself with its mass and the planets turn round it because they cannot do otherwise as they have to move in a curved space. The consequence of the curvature is that the distance between two points in space-time is not ruled any more by Pythagoras theorem in four dimensions

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

but it must be generalized finding some suitable functions of coordinates, for example f(x), g(x) etc in the formula:

$$ds^2 = f(x,y,z,t)dt^2 - g(x,y,z,t)dx^2 - h(x,y,z,t)dy^2 - n(x,y,z,t)dz^2$$

solving the new field equations.

Einstein took four years before writing, in November 1915 [11], the new equations which describe the gravitational field no longer as a force but as a curvature of space-time. The completely new feature of the theory of General Relativity is well summed up by Hawking's words: "Einstein made the revolutionary suggestion that gravity is not a force like other forces but is a consequence of the fact that space-time is not flat, as it has been previously assumed but it is curved or "warped" by the distribution of mass and energy in it. Bodies like the Earth are not made to move on curved orbits by a force called gravity; instead, they follow the nearest thing to a straight path in a curved space, which is called a geodesic [12]"

The physicists have to make an extraordinary conceptual jump, as Silvio Bergia comments on [13]: "It consists in supposing simultaneously that gravitation is not a force, that the motions it determines are free, in a way to be explored; and that a free motion must not necessarily be straight and uniform".

And now two simple observations. Even if Einstein's new idea seems to come from a consequence of Fermat principle, the curious thing is that the final result that he reaches corresponds exactly to the opposite of that principle. The curve of the shortest distance in space (the geodesic) corresponds in space-time to the one of the longest time.

The geodesic motion in a curved space-time is the analogue of the motion at uniform velocity along a straight-line in flat space and both of them occur in the absence of external forces. Even if Galileo certainly did not imagine a curved space-time, he was right in considering the motion of planets as a natural "free" motion (his idea of a "circular inertia").

In November 1915 Einstein [14] repeated the calculation of deflection of light beams using his new theory and obtained the double of the value found in 1911 by Newton's gravitation. So, the theory of gravitation reaches a crossroad with three possible ways out:

1) The photons have no rest mass, the gravitational force does not influence them and therefore they are not deflected when they go by the sun. The Equivalence Principle is not valid.

2) The photon's energy can be considered as a measure of its mass and massive particles travelling at almost the speed of light near the Sun will form a deflected beam, according to Newton's theory of about 0,87 arc seconds.

3) The gravitational force does not exist, but the photons, following the shortest way in the space-time curved by the mass of the Sun, form a deflected beam of 1.74 arc seconds, according with what had been predicted by Einstein's General Relativity based on the Equivalence Principle.

To settle the question it was necessary to wait until 1919, when there was the first solar eclipse, during which Eddington was the first to measure an apparent shifting of the position of the stars near the Sun finding a value of the deflection of the light beams close to 1,74 arc seconds. All the following experiments will always confirm Einstein's General Relativity, allowing also to explain the perihelion precession of Mercury. It will predict the existence of gravitational waves [15], permit the birth of Scientific Cosmology [16] and to devise a series of completely new phenomena among which the gravitational lenses and the black holes are the most extraordinary ones.

5 Discussion

After more than a century the Equivalence principle is still a subject of debate and a source of open questions. The historical evolution of the principle has also been studied in a more technical way [17] with respect to the didactic approach followed in this paper, together with the discussion about the critical points. Just to give some examples, we want to list the problems faced by our research group.

- Is the Einstein principle always and everywhere valid?

Of course, as each principle in physics, it has limits of applicability both in classical domain and in quantum regime. As we have underlined at the end of section 2, the principle works if we limit ourselves to a small area of space where the acceleration of gravity (which depends on the distance from the centre of the Earth) is really constant and where we can neglect the tidal effects which exist in a real gravitational field but do not exist in a field simulated by an accelerated frame. In the case of a real elevator, we can take for example two small balls (test particles) and put them on the top of the elevator at the maximal distance from each other and leave them fall in the Earth gravitational field until they reach the floor of the elevator. We have explicitly studied [17] this case considering a small elevator with a square floor surface whose side is 80 cm and height $h = 220$cm. We have calculated that a measure on the floor of the contraction of the distance between the two balls due to the tidal forces of the Earth field is about 2.76 X 10^{-5}cm hence very small.

- Instead of placing the elevator in a central field, is it possible to create a real uniform gravitational field?

The idea is the following: a massive body and the floor of the enclosure become closer and closer either because the body falls towards the floor attracted by an external mass (in the elevator on Earth), or because the floor moves towards the body accelerated by an external force (in the rocket in the space far from any massive object). In principle, a third way can exist: the space between the floor and the body can disappear if it undergoes a suitable contraction, that is the contrary of the cosmological expansion. So it is worthy to analyze this theoretical possibility suggesting a geometry that can simulate the effects of a real uniform gravitational field [18].

- Finally, is the equivalence principle valid in quantum mechanics?

The answer is that in Quantum Mechanics inertial and gravitational masses are not equivalent and this is confirmed by the experiment. For example, we have showed [19] the deep conflict between quantum theory and gravity analyzing a quantum bouncing ball system. In particular, we have emphasized graphically that the behavior of quantum particles in gravitational fields is mass dependent. Hence the Principle of Equivalence is not valid in the quantum regime and the problem of unification of Quantum Mechanics and General Relativity remains open as the one of the radiation emitted by a charged body in the Einstein's lift [20].

These are only some of the possible questions. So many years of studies and experiments have not exhausted all the beauty, the astonishing consequences and the mysteries of a principle and a theory born on a day in 1907 from the idea of a man "sitting in a chair in the Patent Office at Bern".

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The logical perception of the Origin of Matter and the outside of the Universe

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Abstract

This paper provides new perception of how matter exists out of total void. A void without description. Due to the existence of the void we conclude that the void has complementary. The possibility that the void complementary is substance e.g. the universe is dismissed and the conclusion is the void complementary represents the total information about all potential scenarios, i.e. stories. Yet, stories are realized within universes and the question remains how the universe exists if the void complementary does not incorporate substance? The conclusion is that matter is a logical state within the void complementary, wherein the universe has complementary such that the total matter of the universe and its complementary collapses timelessly into zero matter i.e. null matter. The complementary universe cannot be void and therefore it carries negative matter. The universe is identified as the inner universe with positive matter having mass ≥ 0 (wherein mass $= 0$ is recognized as light) and its complementary negative matter with mass < 0. The complementary universe is everything which is not the inner universe incorporating the outside universe. The complementary universe carries particles that travel at speed higher than speed of light up to infinity and it has an open structure up to infinity. Particles of the inner universe attract each other and are bounded by speed of light resulting in universal close structure. The question: "into what the universe is expanding?" is now being clarified. The universe is expanding into its complementary universe. Furthermore, we concluded that the general relativity theory is not complete in the sense that the theory does not considered forces imposed on the inner universe from outside by its complementary universe.

Keywords: Void, Complementary, Void Complementary, Universe, Universe Complementary, Metaphysics, Logic, Brain, Neural Network, Synchronization, Synchronized Groups, Order, Disorder, Universe expansion, Linear Schrödinger Representation, Frequency, Phase, Hologram Surfaces, Virtual, Reality[2].

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1. Introduction

The question what is the origin of matter was, and still is, one of the most fundamental issues mankind has been dealing with since it was capable of asking such questions. Theories that matter emerge as a sudden change of a quantum void such as of Casimir effect, quantum phenomenon [1], quantum creation [2], quantum geometry [3], quantum topology [4] does not comply with the basic assumption of total void, a void without description. The basic question being asked in this paper is the existence of matter out of total void.

The idea that matter existence out of void is impossible as discussed in [5], "since it is metaphysical impossible that the universe came into being spontaneously out of nothing" is not entirely correct as shall be explained in this article.

The question if the universe is expanding, to what it is expanding into and if the universe is shrinking, from what it is shrinking or even doing both? This question is connected to the question of how matter exists out of the void and it is considered in the article pursuant the question of how matter exists out of total void.

Understanding that the universe has a complementary universe with real physics and forces put in mind the question: "Is Einstein's General Relative theory complete? in the sense that it contains all potential variables that may influence the local geometry?" As it seems, there are additional external forces imposed onto the universe (inner universe) from its complementary universe, i.e. the outside universe may influence universal structure and topology.

Additional questions are considered as part of generalize concept of the relationship between physics and metaphysics including short answers related to dynamic, coordinate system, the location of universes and short description of the brain neural synchronization that enables to store and retrieve virtual information, are considered.

This article is part of a serious of articles exploring the metaphysical space. Following articles, we shall discuss what is the pure consciousness and the possibility of the existence of consciousness in a space lack of physics, i.e. the metaphysical space.

2. The Void and its Complementary

2.1 **Axiom:** *VOID is the only natural state. That is to say, the only thing that exists is void.*

The paradox of saying that the only existing thing is the void, means that something does exist even though it is void. The existence of the void is the paradox of existence, due to its existence there has to be a complementary to the void.

2.2 ***Theorem 1:***

The void has complementary.

2.2.1 **Proof:**

Due to the existence of the void there is a complementary to the void.

2.2.1.1 ***What would be the void complementary?***

Void complementary means any scenario which is not void. Let's consider three possibilities for the void complementary:

a) The void complementary is substance, e.g. the universe or just a single atom, as it does not really matter if substance is single atom or the entire universe.
b) The void complementary is also void!
c) The third possibility is the one the paper is focused.

If the void complementary is substance, it means that the substance, e.g. even single atom, is complete. That is to say, single atom is the entire universe. This follows, since the complementary of the substance e.g. the complementary of the universe, is void, therefore, the substance is complete and incorporates all possible sort of matter, i.e. positive matter as well as negative matter. Positive matter means matter with mass ≥ 0 and negative matter means matter with negative mass < 0. Therefore, we define sort of matter:

- **Definition 1:** Positive matter (positive substance):
  Positive matter means substance with positive mass ≥ 0. (mass = 0 is related to light)

- **Definition 2:** Negative matter (negative substance):
  Negative matter means substance with negative mass < 0.

Furthermore, if the positive substance is complete it means the positive universe (i.e. the one that we are living within i.e. having mass ≥ 0) and its physics would be complete universe representing complete physics as well. What does it mean complete physics? It means physics that would reflect any possible scenario due to its completeness. For example, particles that may travel at speed reaching infinity, while complete universe means that the universe would represent any possible matter and any possible volume and any possible shape and so on. Theorem 2 below is, therefore, to argue that the void complementary is not substance.

2.3 *Theorem 2:*

The void complementary cannot be substance (i.e. single Atom or the Universe).

2.3.1 **Proof:**

If the substance is of positive nature, i.e. having positive mass, then it does not contain negative particles having negative mass and therefore it is not complete. If the substance is of negative nature, reflected by negative mass, then it does not contain positive particles having positive mass, therefore, it is not complete.

If the substance is of both natures i.e. positive and negative, then either the sum of all particles would reflect zero matter i.e. positive matter merged with negative matter resulting in null matter, then in such case the void complementary is not substance. This contradicts the assumption that the void complementary is substance.

If the sum of all particles is positive mass (including the case where the total matter tends to plus infinity) or of negative mass (including the case where the total matter tends to minus infinity) it would reflect incomplete substance, i.e. either more positive substance or more negative substance. Therefore, the void complementary is not substance.

Furthermore, complete substance would reflect complete physics i.e. physics that reflects all possible physical scenarios. According the Einstein General relative theorem, velocity of the positive substance is bounded. This means that the velocity of any particle, including particles of light, is bounded by certain velocity. According to Einstein theory such boundary would be the speed of light. We cannot find in our universe particles who may travel at speed above speed of light reaching infinity, therefore, physics is not complete as well.

Additionally, if the positive universe is expanding, it means the universe does not incorporate the entire potential volume, which in turn means the positive universe evolves. If the positive universe is shrinking, then some space of the universe is dropped down. In both cases the outcome is incompleteness of the universe due to the change in the universe potential volume. In other words, the universe is evolving no matter in what direction expanding, shrinking or doing both.

These arguments contradict the completeness of the universe or its physics. Therefore, the void complementary cannot be substance. Furthermore, the above argument suggests that the universe has a complementary universe which is not void.

2.4 Theorem 3:

The void complementary is not void:

2.4.1 Proof:

It follows from the definition of complementary and in particular void complementary (everything that is not void). Therefore, the void complementary is not void.

Conclusion: The **void complementary** would reflect total information about any **potential** scenario, i.e. story. For example, total information about potential universes and their living creatures. Yet, these are only **potential** scenarios.

3. The Universe and its Complementary

For a potential scenario, i.e. a story, to be realized, a universe should exist in order to actual execute the story. So how universes exist?

3.1 The existence of Universes

As discussed in the previous section, the void complementary is a virtual space of total information about any potential scenario, yet it does not contain substance. In that sense, it is not a universe. There is zero matter in the void complementary. By saying: "there is zero matter", we agree that there is no matter, meaning null mater. Zero is one of the potential scenarios existing in the void complementary.

3.1.1 Why using zero and not other symbol, such as one, to symbolize null matter?

In terms of mathematics for the purpose of representing null matter e.g. zero matter we may as well use the number one. Zero is the idle element under the additive group (represented by the additive operation). In other words, any real number added to zero remains unchanged and any real number added to its inverse (i.e. negative) shall result in zero.

One is the idle number under the multiplication group (represented by the multiplication operation). In other words, any real positive number greater than zero which is multiplied by one, remains unchanged and any real positive number (A), greater than zero which is multiplied by its inverse number (i.e. by 1/A), results in one. Since there is a transformation from the additive group to the multiplication group (e.g. e^x) and vice versa from the multiplication group to the adaptive group (e.g. log(x)), it does not really matter if we shall use zero or one to represent our perception of null matter. Yet as said, zero is more intuitive and well established in most languages. For example, by saying: "this guy has zero money", it is clear that the guy has no money, in that senses he has null money. However, it is not intuitive and understandable to say: "this guy has one money" under the multiplication operation to indicate the same thing. So, as it seems, zero, the idle number under summation operation, is more institutive and understandable than using one as the idle number under the multiplication operation for our perception.

Humanity uses the idiom zero since ancient times. For example, the Egyptians have used the symbol “nfr (pronounced as nefer), to appreciate beauty, in accounting as zero balance and in construction as level zero.

Yet, when using the symbol zero, its meaning should be carefully understood. For example, when saying zero universes it should be understood there are no universes. By saying that there is zero life, it should be understood that there is no life. However, by saying the temperature today is zero, we understand that there is a temperature and its quantity is zero. This is not the same as saying zero temperature. In such case, it should be understood that the idiom temperature is null. The same applies to the saying zero matter, which means that there is no matter and clearly not that there is matter with mass zero (e.g. a matter that converted into light).

3.1.2 **Universes out of null matter**

Going back to our assumption that void complementary contains zero matter. This is the key point for understanding the paradox of matter out of nothing and what the outside of our universe is.

Zero Matter - **what does this mean?** If we ask someone: “Can you imagine zero? Can you tell us what do you see inside the zero?”. Actually, this question was asked and the answers were: "I see nothing", or "I see dark", or "I see white", or "I see circle". Yet, no one said, I see **(+2) + (-2)** hooked together indefinitely or in fact timelessly and always maintain zero. Recall, saying zero matter means no matter. This means that within zero there is no sensation of any physical force representing matter, like the force of gravity. So, if the right side of the equation (+2) + (-2) = 0, means no matter, no sensation of matter, then (+2) of the left side should represent something, some force, some sort of gravity in an amount equivalent to value 2. Generally, the following equation must satisfy that the total matter is zero:

A. $\{+U\} + \{-U\} = 0$

The right side means that there is zero matter, i.e. null matter. The left side is composed of the sum of two distinct parts. {+U} represents the inner universe. The inner universe is composed of positive particles (Definition 1 above) having positive mass including particles with mass zero (light). {-U} represents the complementary universe, i.e. the negative universe (Definition 2 above) which includes all negative particles having mass less than zero (not including zero).

3.2 **Definition3:** Inner universe:

The inner universe is the positive universe composed by positive matter as defined in definition 1.

3.3 **Definition 4: The Complementary Universe:**

The complementary universe refers to the negative universe composed by negative matter as defined in definition 2.

The complementary universe is everything which is not the inner universe such as the outside universe. We call negative particles lack of matter symbolized by negative mass. As noted, the left side of equation (A) is composed of two distinct parts indicating that the total matter of the inner universe and its complementary universe collapse into zero matter timelessly (i.e. null matter) representing the right side of the equation.

Once universe exists it should have complementary, it could be that the complementary universe is void, in that case, the universe should be complete, which is not, as discussed in theorem 2 and in the following theorem.

3.3.1 **Theorem 4:**

A universe complementary is not void (see also theorem 2)

3.3.1.1 Proof:

If the universe complementary is void, it means that the universe is complete. In that sense, the universe should reflect all possible matter, energy, velocity, volume, etc. Physics teach us that velocity is bounded, currently the boundary is defined to be speed of light. Thus, physics is not aware of particles traveling at speed higher than speed of light reaching speed of infinity. Furthermore, inner matter is composed by positive particles, there are no negative particles having negative mass. Therefore, just from this perspective the universe is not complete. Furthermore, the universe is evolving i.e. expanding and/or contracting, therefore, the universe is not complete and the ***universe complementary is not void***.

3.3.2 **Conclusion:**

The inner universe is expanding on the account of its complementary universe and vice versa. The complementary universe is everything that is not the inner universe and that includes the outside of the inner universe.

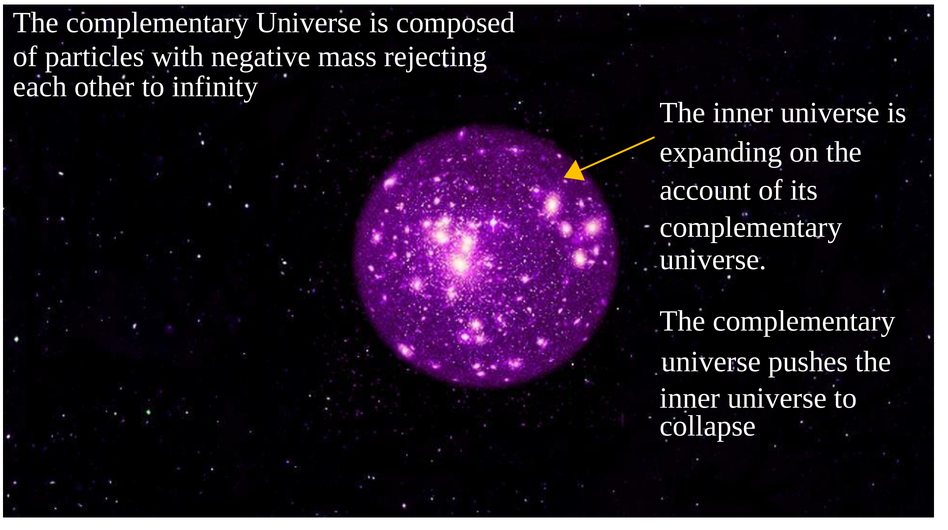

Figure 1: The inner universe is composed by positive matter while its complementary is composed by negative matter. The complementary is everything which is not the inner universe and that includes the outside of the inner universe.

Returning to the void complementary, Eq. (A) is one potential scenario describing universes out of zero matter i.e. null matter. In any situation both universes must obey the virtual metaphysical equation that total matter of the inner and its complementary universe (outer) must be zero resulting in zero matter. Yet, within the Universe {+U} matter exists in different scale, shape, density, temperature, pressure, etc., including matter with mass zero representing light. Life, as we are aware of, would be inside the inner universe {+U}.

3.3.3 **Anecdote: we are living within the equilibrium equation.**

As it may sound difficult to understand, we are living within one part of the equilibrium equation (A), in fact, within {+U}. Our body will be torn apart in {-U} due to its force of rejection (i.e. unattraction) causing particles to reject i.e. to push each other in a speed higher than speed of light, up to infinity, as explained in table 1 bellow.

The existence of matter is not magical, it is a logical virtual potential state of the void complementary (i.e. the metaphysical space). The matter cannot exist without having complementary universe representing anti or opposite characteristic such as mass, in the current perception that is negative mass.

The equilibrium equation (A) presents logically how matter exists out of zero matter. It is important to emphasize that it is not limited just to numbers. Other symbols or mathematical operation may be used as long as the totality of both universes, the inner universe and its complementary universe collapse into null matter i.e. zero matter. Nevertheless, as noted, zero is the most intuitive symbol to be used for this purpose.

4. Realization of stories within universe

4.1 **If a scenario exists, then what stops its realization?**

In fact, nothing. **Every story is being realized in a suitable universe, just because of its potential existence in the void complementary**. We, at the current situation, realize our own story. We do not know the end of the story but we are walking in the path of the story. While the story exists timelessly it is us determining and experiencing the story right now. Within the story we sense linear time. Yet from the perspective of the void complementary, before birth and after death represent lack of time.

Conclusion: Every story is being realized in a suitable universe, just because of its potential existence in the void complementary and every universe must obey the equilibrium equation (A) maintaining total matter zero. **We are executing our timeless story within the inner universe {+U}.**

4.2 **Definition 7: Illogical story or illogical scenario**

Illogical scenario means there is no equation of the form (A) that can realize the scenario.

Example: if the total matter resulting from eq. A is positive i.e. the sum results in a positive matter then such scenario is illogical scenario since the right side of eq. A is zero and the left size is greater than zero. The creation of matter must obey the metaphysical conservation law.

5. The metaphysical space

5.1 **Definition 5: Metaphysical virtual space:**

The Void and its complementary is considered to be the ***Metaphysical virtual space****.*

The Metaphysical space incorporates all information about any potential scenario including the null scenario or the null story.

5.2 **Theorem5:**

The metaphysical space is one complete space.

5.2.1 **Proof:**

a) If the metaphysical space is void then due to its voidness it is complete.
b) If the metaphysical space is “void complementary” then the complementary of the “void complementary“ is void. Thus, the “void complementary” is complete. Therefore, the metaphysical virtual space is complete (not evolving).
c) Uniqueness follows due to its completeness. If there are two metaphysical spaces, it means that the complementary of the “void complementary” is not void, which contradicts the completeness of the “void complementary” and if there are two voids it contradicts the voidness of the void. Therefore, the metaphysical space is unique.

5.2.2 **Conclusion:** The metaphysical space is unique complete virtual space. It does not evolve and it has no reason or desire to evolve. Yet, within our own story we evolve and our evolution reflects intention.
The paradox of nothing (void) and everything (void complementary) escort any one of us during our entire life, we feel there is something and yet we cannot touch it.

5.3 **Intention:**

Although there is no metaphysical reason for anything, from our perception it is suggested to provide reason to our existence. A personal reason such as:, I want to be educated, I want to be successful, I want to be musician, writer, inventor, I want to experience, I want to have family, children, I want to love and be loved, I want to empower my intention etc. Making beautiful timeless story is a wonderful reason.

5.4 **GOD**

One of the questions that is extensively being asked is: "what is GOD?" [9,10,11,16] and "does GOD intend?" In fact, the definition of GOD in the old testimony is a mathematical definition or at least logical definition. For example, GOD is one, complete, timeless or has front and rear. Such description suggests that GOD front is the void and the rear is the void complementary. Thus, the metaphysical virtual space perfectly matches the description of GOD as defined in the old testimony.

Does GOD intend? If considering GOD to be the Metaphysical space, then obviously GOD is not intending due to its completeness. Although, as said GOD is the totality, the other question is: "Is GOD aware?". Is our higher consciousness aware without physical body? This issue will be dealt in a different paper, yet, without any further explanation and to our best opinion the answer to both questions is yes and yet depends on each person's state of mind.

5.5 **Definition 6: The metaphysical conservation law:**

The metaphysical conservation law preserves null matter. That is to say, any form of eq. (A) must collapse into null matter. Existence of universes is a logical state that must obey the metaphysical conversation law.

5.6 **Anecdote:**

From the *metaphysical perception* the conservation law of null matter does not depend on coordinate system, gravitation, spin, electromagnetic, time and other physical perception. It depends only on the total matter of the inner universe and its complementary collapsing into null matter timelessly. All stories have been realized timelessly and therefore there is no meaning to time. For example, birth and death of a person, plant, sun, galaxy or of the entire universe are all timeless events. In order to understand the meaning timeless, consider the current potential brain memory, all memories up to the current time are potentially there while remembering does not depend on the sequence of the historical events. We can trigger the brain to remember events at age 10, then 5 then 18 and so on. Memories are timeless, although the story may associate to historical time, but the retrieving is independent on time. The metaphysical space incorporates all information about any potential event, we just trigger the brain to pull down timeless information.

6. The summation of total positive and negative matter and the outside of the inner universe

Electron, proton, neutron, atoms, etc., are all positive particles that we have defined to have positive mass in rest, and in the particular case of light, zero mass in rest. Positive particles are characterized by positive real number called mass, defined by the set:

6.1 $R_+ = \{ m \geq 0, m \in R\}$.

The question: "what is it outside the universe?" is considered logically under the assumption that the natural state is timeless void. It is important to note that voidness means indescribable void, as opposite to the quantum void [12,13,14,15,16,17] which is describable. In that sense, as explained in section 2 above, the universe complementary represents negative matter (Definition 2) so that the total matter of the inner universe and its complementary collapses timelessly into zero matter i.e. null matter based on the metaphysical conservation law.

Negative particles are characterized by negative real number representing negative mass as defined by the set

6.2 $R_- = \{ m < 0, m \in R\}$.

R_- is a reflection of R_+ centralized over zero mass which represents light. Reflection means both universes has total uncountable infinite real numbers representing positive and negative matter except of zero which is associated to the positive universe. Each universe has its own separate physics.

The virtual merger of all positive and negative matter i.e. the total sum of R_- + R_+ collapses into zero mass. But since all positive and negative matter vanished, that particular zero represents zero matter. One may say, the light is turning off, so zero mass becomes zero matter timelessly.

The equilibrium equation (A) in the form of R_- + R_+ is a virtual logical scenario i.e. state that enable the existence of universes. There is no description on how the summation is done, only that the outcome must be zero.

The virtual logical equilibrium equation (A) is not limited to summation operation i.e. “+”. Yet, as noted above, zero in most, if not all languages, is used to present nothing. This allow us to overcome language barriers. Other forms of equilibrium perception can be formed to replace equation (A) as long as metaphysical conservation law of null matter is preserved.

Cases where eq. (A) does not collapse into zero may indicate a magical creation wherein the right side of eq. (A) represents null matter and the left side of eq. (A) results in either positive or negative total substance. This cannot happen just because it is not logical. That is to say, only cases where eq.(A) collapse into zero reflect substance e.g. universes.

The complementary universe is everything which is not the inner universe and that include the outside of the inner universe. The inner universe is expanding into the complementary universe and if the inner universe is shrinking it is shrinking from its complementary universe or doing both locally (in some local area) or globally (over the entire inner universe).

6.3 Attraction vs. Rejection and their consequences

Particles of {+U} declare their existence by attracting other particles. We refer this attraction to the gravitation force; we may call it the force of LOVE. After all, our body is composed by particles as well. Attraction generate planets, suns and galaxies resulting in closed structure, while unattraction result in open structure with particles pushing each other to infinity in velocity much greater than speed of light up to infinity.

Attraction, i.e. gravitation causes particles to unify into solid structures while the initial explosion causes the universe to expand. Without expansion, the universe would not cool down after the big bang. Cooling down causes particles to unify into solid structures. As long as the universe is expanding it still in a process of cooling down. Once the universe will stop expanding, the complementary universe will start to push the inner universe to collapse into itself, meaning the inner universe will start to shrink.

The balance between the forces of the inner universe and its complementary universe determines the evolution of the universe to expand, shrink, develop deformed structures and so on. When the inner universe is expanding the complementary universe is kind of shrinking, but it never really shrinks since its boundary is open to infinity. Why? As mentioned, negative particles push each other to infinity, creating an open structure up to infinity. Positive particles within the inner universe attract each other creating closed structure. Unequalled local pressures from outside towards inside and from inside towards outside may result in local deformations as presented in figure 2 below.

In case the outer pressure overcome the overall expanding pressure, the inner universe may start to collapse. In such case, the universe may collapse into singularity which may result in new universal initialization i.e. like the big bang explosion with new initialization conditions [19,20,21,22,23]. These new conditions determine the way the universe will evolve, i.e. matter may diffused in different ways than the ones we are aware of today. Yet, starting the universe with a huge explosion would not be the only logical possibility, other theories may apply as well [1,2,3,4,5,12,13,14,16,17,23,24,25,26, 27].

In case the inner pressure overcome the outer pressure, the inner universe may continue to expand. Yet, the outside local pressures imposed onto the inner universe may not equality diffuse resulting in deformed shape (i.e. asymmetrical shape).

6.4 Dynamics: If all real positive numbers (mass ≥ 0) represent matter in the inner universe, what cause to a dynamic?

Dynamic is generated due to potential and potential results from the difference existing between various numbers representing different scale of matter (big versus small), different temperature (cold versus hot) or different pressure (high versus low). These differences are the natural state causing dynamic. **Furthermore, potential differences within the inner universe may result in disappearance and reappearance of particles or energy into the complementary universe and vice versa**. For example, two particles carrying positive mass under certain force, heat and pressure may result in negative mass. The outcome of this mass exchange is disappearance of a particle into the complementary universe. **Yet, since total matter in each universe is infinite and uncountable there will never be too short or too much matter.**

As discussed above, any potential scenario represents state of matter being realized in a specific universe. For example, universe that collapses into quantum scenario that cause the universe to restart itself in a big bang or other logical way or the universe may collapse into universal white dwarf (universal white dwarf means the entire universe become white dwarf) or into universal black hole (the entire universe become a blackhole), as long as these scenarios are logical there are suitable universes realizing these scenarios.

Currently, at this moment, we are experiencing a scenario that our universe is still expanding while cooling down after the big bang or another logical scenario. Whatever state the universe is, life can be realized only when the universe or part of it is at certain order enabling life. **No matter at which state this or any other universe is, the equilibrium equation (A), that total matter of the inner universe and its complementary universe collapse into zero, is fulfilled.**

6.5 Incomplete Physics:

From the equilibrium equation (A), the left side is composed by two distinct parts, each one has its own physics, would such physics reflect complete physics? This can definitely be concluded since the left side of the equilibrium eq. (A) is composed by two distinct parts, each part completes the other part, therefore, each part consists of incomplete matter, i.e. positive matter versus negative matter resulting in incomplete physics. This issue was also discussed in the previous theorems (2,3,4).

The collapse of the total inner universe and its complementary universe into total zero matter is not depending on a coordinate system, frequency, pressure, temperature, time, etc. It is a timeless state that depends only on the sum of the total matter of the inner universe and its complementary. Nonetheless, within the inner universe as well as its complementary universe (the outer) the sensation would be real physics associated with a perception of various spatial-temporal-frequency coordinate systems.

The characteristics of the inner universe {+U} and its complementary universe {-U} are summarized in the following table. Each physical characteristic of the inner universe has a complementary characteristic in the complementary universe and vice versa, except of the idle zero number that is associated to light within the inner universe and has no complementary characteristic in the complementary universe.

The merger of both physics (the inner and its complementary) collapses into the zero mass. As noted above the collapse of the total matter is into light, but since matter of the inner universe and its complementary vanish then that particular zero mass is in fact zero matter representing the null physics.

Inner Universe {+U}	**Remark**	**Complementary Universe {-U}**	**Remarks**
Positive matter i.e. mass $m \geq 0$	Uncountable endless particles of positive matter in various size, density or heat	Negative matter i.e. mass $m < 0$	Uncountable endless particles of lack of matter
Attraction (Gravity, Love)	Generating galaxies, suns and planets	Rejection	Pushing each other to infinity
Closed structure	Impossible to leave the universe.	Open Structure	No geometrical structure.
Bounded Velocity	According to Einstein Theory velocity is bounded by speed of light.	Unbounded Velocity	velocity is above speed of light and may approach infinity.
Light	Within the inner universe.	No light	Anticipating that all matter is spread up to infinity.
Other forces like spin, electromagnetic, nuclear, etc	Within inner universe	Not yet have been considered	Could be a distance collision that

			may result in various forces.
Pressure towards outside.	The general forces resulting from the initial explosion produces forces towards outside. When the universe is expanding the inner forces overcome the outer collapsing pressure. The universe may expand, shrink or collapse into singularity. This state of dynamic is depending on the balance of overall forces induces by inner universe and its complementary.	Pressure in all directions.	Complementary universe pushes the inner universe to collapse into itself.
Symmetrical vs. asymmetrical evolution.	The universe may evolve local structural deformation resulting from unbalanced local pressure induced by the inner universe and its complementary. Symmetrical structure would be when all forces are at equilibrium state.	No geometrical structure.	Due to open structure up to infinity where particles refuse to unify, there is no geometrical structure.
Substance may disappear and reappear. Negative matter may appear as long as the metaphysical conservation law is preserved	A collision of positive particles under certain forces, gravitation, heat, pressure, spin, electrical field, etc., may produce negative particle having negative mass. Such particle	Substance may! disappear and reappear.	It could be in a distance collision of negative particles which may result in positive particles (mass ≥ 0) which will vanish from the

	will vanish from the universe into its complementary universe.		complementary universe into the inner universe.
Never too short in substance.	Due to the fact that matter is represented by all real numbers greater or equal to zero.	Never too short in substance.	Due to the fact that lack of matter is represented by all real negative numbers.
Universal boundary.	The boundary is composed by zero matter, representing the void complementary. Considering the sum of $R_+ + R_-$ where $R_+ = \{m \geq 0, m \in R\ (Real\ numbers)\}$ and $R_- = \{m < 0, m \in R\ (Real\ numbers)\}$. Only sum resulting in zero are considered logically wherein, the collapsing into zero represent zero mass which is light, however, since all sort of matter (positive and negative) vanished, i.e. it is like turning off the light, this zero represents zero matter, i.e. null matter as well as null physics.		
The virtual metaphysical law of the preservation of null matter.	The virtual equilibrium equation {+U} + {-U} is timeless zero. It does not depend on any sort of coordinate system, gravitation, time, frequency, etc. It relays on the sum of all negative and positive matter resulting in null matter. Cases where the sum is not equal to zero are illogical and would not realize.		

Table 1: The characteristic of the inner universe vs. its complementary universe.

7. Universal Coordinate system and Time

A universe can be presented by various spatial-temporal-frequency coordinate systems. From the metaphysical perception there is no substance, therefore, it does not matter if we present particles as strings, wave, mass, spin, electromagnetic, etc., as it is related to our perception of the universe.

Currently there are two theorems describing the universe, general relativity and quantum mechanics. New theories, of supper strings, are emerging during recent decade, in an attempt to combine both theories into a single theory that will apply for large scale bodies as well as for tiny elements. The difference between the two currently existing theories results from the mathematical tools used for each. General relativity is supported by differential geometry, which relies on the basic assumption that all structures, i.e. manifolds, are smooth. These manifolds determine the local geometry resulting from the local gravitational forces of the local large-scale bodies (galaxies and suns). Differential geometry is a mathematical tool suitable to be used for macro physics but not for micro physics, as the movement of small particles, such as electrons, is so fast and so fluctuated that there is certain uncertainty to locate adequately a particle location its movement (momentum). We may find uncertainty in many aspects of predication. Uncertainty is not limited just to particle location vs. its momentum. In fact, uncertainty is in every aspect of life as well as of science. For example, in signal processing, sampling a signal over time more and more precisely requires more and more frequencies. To predict the future of a share price, to predict the next day and so on. Uncertainty, means incomplete. One may reduce the surprise of the next day by living highly predicted life, i.e. an average life, no surprises. On the other side making and unpredicted life so fluctuated shall result in complete unstable life. This issue is to be discussed in the following sections.

General relativity represents an average or smooth behaviour of large scale bodies resulting in small surprises and highly predicted location and momentum. On the other side, very tiny particles are high fluctuated represented by quantum mechanics theory, which may tell us the probability of a particle location versus its momentum. Both theories complete each other, as one is dealing with smooth behaviour and the other with fractional behaviour. A theory that unified macro as well as micro behaviour would provide a new perception of gravitational matter in every scale.

String theory is a one dimensional objects called strings. The idea is to describe how strings propagate through space and interact with each other. In scale larger than the string scale, a string looks just like an ordinary particle, with its mass, charge, and other properties determined by the vibrational state of the string. In string theory, one of the many vibrational states of the string corresponds to the graviton, a quantum mechanical particle that carries gravitational force. A string theory is a theory for quantum gravity.

String theory has been applied to a variety of problems in black hole physics, early universe cosmology, nuclear physics, and condensed matter physics, and it has stimulated a number of major developments in pure mathematics. Since string theory potentially provides a unified description of gravity and particle physics, it is naturally to consider it as the theory of everything. Nonetheless, it is not known to what extent string theory reflects the real world.

Superstring is supersymmetric string theory. It is the version of string theory that accounts for both fermions and bosons and incorporates supersymmetry to the model of gravity. (of note, fermion is a particle that has an odd half-integer spin (like 1/2, 3/2, etc., such as quarks, leptons and certain composite particles, such as protons and neutrons. Bosons are those particles which have an integer spin (0, 1, 2...)).

Development of a steady state quantum field theory resulted in infinite possibilities. Physicists developed a renormalization technique to overcome these infinities. Yet, this approach works for electromagnetic, strong nuclear and weak nuclear forces, but not for gravity. Therefore, unified theory for quantum gravity requires additional consideration. The work on string theory for all scale particles is not yet completed.

This paper is not aiming to discuss how to present matter in various gravitational resolution, rather, to provide a new perception on how matter exists out of the void, what is outside the universe and to argue that Einstein General Relativity theory may not be a complete.

Therefore, we simplified four-dimensional functional coordinate system, which describes universal global location and a certain global time reference. The components coordinate system X = (x,y,z) are functions of the local gravity, local inner pressure and local complementary pressure in the area surrounding the coordinate (x,y,z) neglecting other potential variables such a spin

A simple example is of a punched basketball from the outside (Picture 2.A) representing the complementary pressure which overcome the internal pressure towards the ball envelop. The coordinate (x,y,z) is no longer in its original location, it was moved to a new location as a result of the local physical forces imposed from the basketball complementary (e.g. the universe complementary) as well as of the basketball internal air pressure (e.g. initial big bang explosion) and of the local basketball structure such as the basketball material density nearby the local coordinate (x,y,z), e.g. this may be considered as the local gravitational forces of the surrounding area near the coordinate (x,y,z) .

Figure 2.A presents local deformation nearby the coordinate (x,y,z) where the deformation resulted from local unbalance outer (complementary) and inner pressures of the basketball. Picture 2.B shows that the entire basketball was deformed by unbalanced pressures induced from the outer and inner pressures of the basketball.

The example shown in Figure 2 emphasizes that coordinate system may be affected by various physical variables. Some of those variable forces were not considered in Einstein General Relativity theorem, particular the pressure induces from the complementary universe towards the inner universe trying to squeeze the inner universe to collapse into itself. Additional potential external variables related to spatial frequencies as well as phase (Eq. B, C bellow) have not been considered in Einstein General Relative theory due to the assumption that all forces are coming from within the inner universe itself without considering external forces except of the universal gravitational constant G, as shall be considered in the following section.

A B C

Figure 2:

a) A basketball was deformed locally as a result of the complementary pressure (outside the ball) resulting in a change of the coordinate location from a ball sphere into locally deformed ball sphere. The expansion of the inner universe initiated from potentially the big bang explosion is simulated by the inner air pressure inside the ball. In both cases the outcome results in expansion. The ball local surface density may relate to the local density of matter resulting in local gravitational forces or lack of gravitational forces in the surrounding area nearby the coordinate (x,y,z). The rest of the ball was not affected.

b) The entire basketball was deformed resulting from unbalanced external and internal pressures resulting from differences between the inner and the outer pressures. In a similar perspective, the inner universe can entirely be deformed due to the external forces versus the internal forces.

c) The inner positive universe after deformation versus the initial spherical shape. The spherical coordinate (x,y,z) is no longer located on the sphere, it has moved into different location. The deformation is a result of unbalanced external (complementary) and internal pressures.

The functional spatial coordinate system X – (x,y,z) may incorporate the following variables: local gravitation forces (Einstein Relative Theory), inner pressure induced from the universe origin at the current time near by the coordinate (x,y,z), complementary forces induced onto the inner universe, external spatial frequency and phase stamping the inner universe matter.

Any coordinate system may evolve (change) in time and therefore influence the spatial location. For example, a coordinate (x,y,z) at time t_k may disappear at time t_p as shown in figure 2C.

The time "t" is considered to be universal time reference initiated in the moment of a major event such as universal singularity resulting in a big bang or other event that may restart the universe or may provide time reference. Within the inner universe, each galaxy may have its own time reference relative to the day of its local creation.

7.1 **Does Einstein General Relativity Theory Complete?**

From our perspective the computation of the gravitational manifold based on Einstein General relativity does not take under consideration external opposite gravitational forces imposed onto the inner universe from its complementary. Such forces may push the universe to collapse into itself or may result in local structural deformation as shown in figure 2 above. Therefore, to our best opinion the General Relative Theory is not complete.

8. Where the universes are located?

Based on the above and considering the perception of the equilibrium equation (A), without derogating of any other potential perception of particles of matter, i.e. as a strings, etc., spatial-temporal-frequency and phase location may be added to provide perception of multi universes (inner and outer) by utilizing linear Schrodinger representation in the following way [6]:

B. $\{+U_ʒ(X, t, \Omega_i, \omega_i, a_i)\} = \exp(-iʒ(\Omega_i X + \omega_i t + a_i))\{+U(X,t)\}$

C. $\{-U_ʒ(P, k, \Omega o, \omega o, a_o)\} = \exp(-iʒ(\Omega_o P + \omega_o t + a_o))\{-U(Y,k)\}$

Wherein (B) and (C) must satisfy the following equilibrium state:

D. $\sum[\{+U_ʒ \text{ (Positive matter)}\} + \{-U_ʒ \text{ (Negative matter)}\}] = 0$

The pair (X,t) = (x,y,z,t) indicates functional coordinate system that depends on various variables such as the complementary forces which are imposed on the inner universe, inner gravitational pressure and topology

(P,k) = (Px,Py,Pz,k), is a global complementary functional coordinate system incorporating k time reference. However, it may be possible to use the same coordinate system (X,t) to describe the complementary coordinate system once identifying that the both universes share same origin. For coordinate X within {+U} the outer coordinate P results in null and for P within {-U} the inner coordinate X results in null. In addition, both universes can share same time reference. It just a perception how to look over things, nothing more.

Ω_i, Ω_o represent frequency vectors associated with the inner and outer global coordinate system X and P respectively. "i" stands for the inner universe and "o" stands form the complementary (outer) universe. The dot product $\Omega_i X$ refer to $(\Omega_x X + \Omega_y Y + \Omega_z Z)$. ωi, ωo represent frequency factors associated with the inner and outer global time reference t and k respectively. a_i , a_o refer to bias in the inner and outer universes respectively.

It is important to note that eq. (D) means that the total matter of both universe collapse into zero matter independently of any of spatial-temporal frequencies coordinate system incorporated within the inner universe coordinate system i.e. (X,t) or within the void complementary (outside the inner universe) i.e. exp(-iʒ(Ω_iX + ω_i t + a_i)). Furthermore, eq. (B) and (C) are potential logical i.e. feasible information of the metaphysical virtual space (i.e. void complementary) then such structures are realized.

Equation (B) indicates that it is possible to vibrate or influence matter, energy and time from the metaphysical space. General saying, it is possible to influence any universe. For example, exp(-iʒ(Ω_iX + ω_i t + a_i)) may vibrate the coordinate system "X" and time "t" or to multiply the universe by certain constant. Einstein relative theory have considered a universal constant G, such constant can be achieved by multiplying the universe with G = cos(a_i).

From the metaphysical virtual space, as consciousness intention, it is possible to influence matter within the universe.

Figure 3 below presents inner universes {+Uʒ} located on a two-dimensional distinct virtual surface hologram (for simplicity) with each disk located on virtual phase axis wherein the inner universes satisfy Eq. (D).

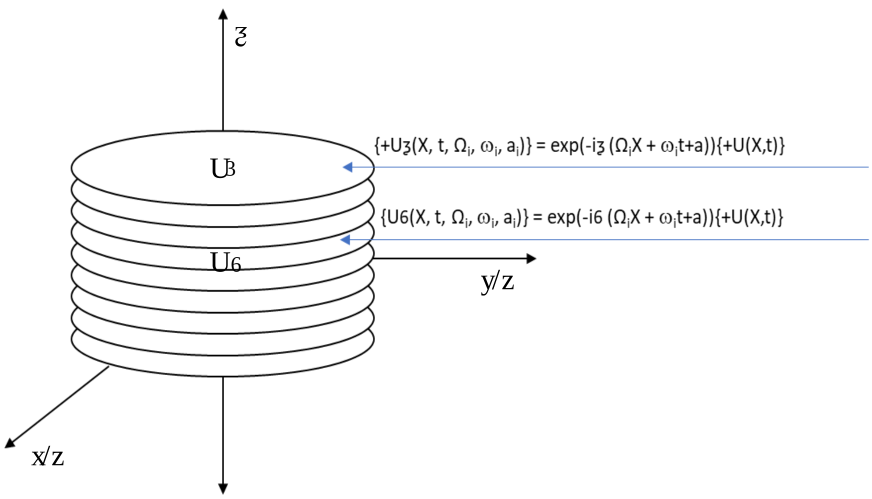

Figure 3: Each inner universe {+U(X,t)} is located over distinct virtual phase hologram surface located at ʒ. Due to drawing limitation of multi axis the Global coordinate system (x,y,z) is projected onto 2D coordinate system (X/Z, Y/Z)

As can be seen, traveling between distinct positive inner universes requires to locate the right phase and the time of the desired inner universe. However, since these parameters are controlled virtually from the metaphysical space, it is suggested that one must be pure consciousness with powerful intention to locate the appropriate universe and its desired phase and time. Yet, pure consciousness is not part of the current paper, it will be discussed in another paper.

Moving from one desired phase to another desired phase is possible by using the following equations ([6]):

E. $\exp(-i\eta(\Omega_i X+\omega t+a))\{+U_{ʒ}(X,t,\Omega_i, \omega_i,a_i)\} = \{+U_{\Theta}(X,t,\Omega_i, \omega_i,a_i)\}$ where $\Theta = \eta+ʒ$.

While we are located within the inner universe and within a physical body, in order to travel between positive inner universes located on two distinct phases (figure 3), it would require to generate wormhole connecting the two universes as presented in Figure 4 below. Matter within universe A may be associated with frequencies Ω_{iA} and ω_A and matter within universe B by the frequencies Ω_{iB} and ω_B . Therefore, it may require the body to adjust itself when moving between universes.

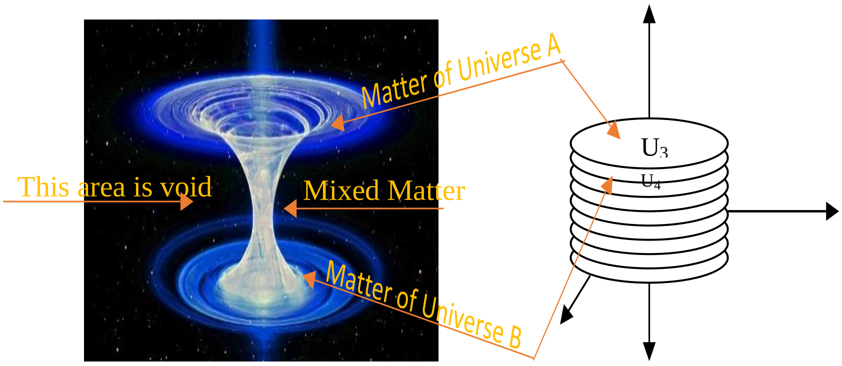

Figure 4: A wormhole connecting two distinct positive inner universes each one is located on distinct phase hologram surface. Matter of the inner universe A is protracted into inner Universe B and matter from inner universe B is protracted into inner universe A. Outside the wormhole there is void since matter exists only within universes, wherein each universe A and B obeys the virtual equilibrium equations (A) and (D). As noted, the conservation metaphysical law does not depend on the coordinate system nor on the added frequencies or time.

Based on eq. (A, B, C, D) above, we summarise that universes do not have physical location; each inner universe is located virtually on a distinct hologram surface. Only inside one of the parts of the equilibrium equation, for example within {+U(X,t)} there is a physical sensation of matter, location and time.

9. An anecdote about brain neural network

Basic neural operation would be one of the following states: idle, receiving charge, transmitting (firing) charge.

Computer memory is composed by zeros and ones coded in bits and bytes. What information these zeros and ones telling us without executing a dedicated software to synchronize these zeros and ones into harmonic information.

What does a neuron tell in idle, receiving or firing state? As of the case of the software computer synchronizing the zeros and ones into information, the brain synchronizes neuronal activity into information. What it means synchronize neural activity?

Figure 5A demonstrates matches thrown randomly on a table. Order is achieved by setting groups of matches according to their common angle.

Matches at identical angel form a synchronized group with a synchronization angle equal to Δ. The group intensity is considered the total matches within each group.

Figure 5**:** A: Disorder Matches B: Each group is represented by the angel and the number of matches.

Considering single match as a neuron, the group angel may represent the synchronization element and the number of neurons within the group or the total potential charge fired by the group as the group intensity.

The brain neural synchronization may be presented in a similar way to universes virtual location by the linear Schrödinger representation as follows ([6], [7], [8])

F. $$\exp(-i\eta(\lambda X + \mu t + a))N_{\Delta}(X,t)$$

wherein $N_{\Delta}(X,t)$ presents a neuron associated with a synchronized group of neurons having synchronization factor Δ, virtually stored over a hologram surface $\eta = \Delta$. The coordinate $X = (x,y,z)$ represents the neuron spatial location. λ represents spatial frequency vector associated with the neuron at location X while the λX refer to vector dot product. μ is a frequency associated with time and “a” shift phase constant.

For example, all neurons $N1.34(X,t)$ having synchronized factor $\Delta = \eta = 1.34$ form a synchronized group of neurons. That synchronized group of neurons form a neural pattern related to certain event.

Additional variables may include in the mathematical consideration, such as the group intensity, i.e. number of neurons participating in the group, total temporal charge released by the group (i.e. temporal charge means charge per unite time) and certain constrains such as the group topology, for example, group of neurons having identical η and are within $L = \{ (X,Y): |X-Y| < d, d>0 \}$. Other topologies may consider such as association rather than distance. It is important to note that single neurons may synchronize with various groups of neurons.

Information is retrieved by linear combination of groups of synchronized neurons virtually stored over various distinct hologram surfaces. The way information is stored is that each group of synchronized neurons has its own neural connections reflecting certain distinct phase hologram surface Δ. The synchronized Δ group is virtually stored over the hologram surface marked by Δ. A virtual linear combination over various hologram surfaces retrieve the information stored. This means, to remember is to recreate the neural patterns associated with the historical events.
Pictures 5 presents the general idea of synchronize temporal groups of neurons.

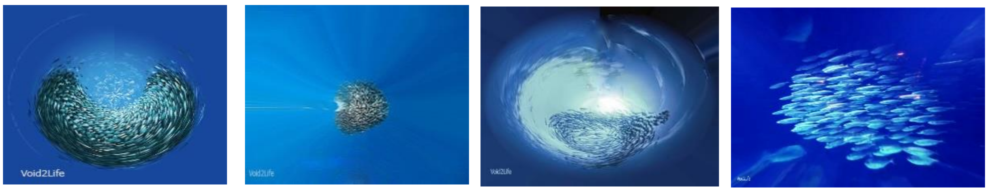

Figure 6: Each picture represents different synchronized school of fish. Considering each fish as a single neuron associated to its current synchronized group. Each synchronized group has its own pattern.

As noted above, single neuron can participate in various temporal groups depending on the stimulus. This characteristic enables large variation of synchronized groups of neurons.

9.1 How do we "download" information?

The retrieving process is achieved by recreating or re-stimulating existing groups of neurons in a certain linear combination. We may consider the retrieving process as linear combination of synchronized neurons stored virtually over different hologram surfaces.

Although memory depends on existence of synchronized groups of neurons, unexperienced sensation such as images, voices, smell, taste and feeling can be created by synchronizing new groups of neurons without experiencing real sensation. This may occur in dreams, whether as random or intended dreams, wherein groups of neurons are being synchronized without being experienced in reality.

Whenever existing groups of synchronized neurons are recreated, it is treated as memory, otherwise it is considered as new learning, imagination, ideas, dreams, illusion and so on. Groups of synchronized neurons create a neural pattern and these pattern associate with certain sensation including internal control of the human body.

Intention is the key for downloading metaphysical information. Intention may trigger the neurons to synchronize in such a way that it provides the desired sensation. It is beyond the scope of this paper addressing issues relating to the pure consciousness coupling with the brain neural network.

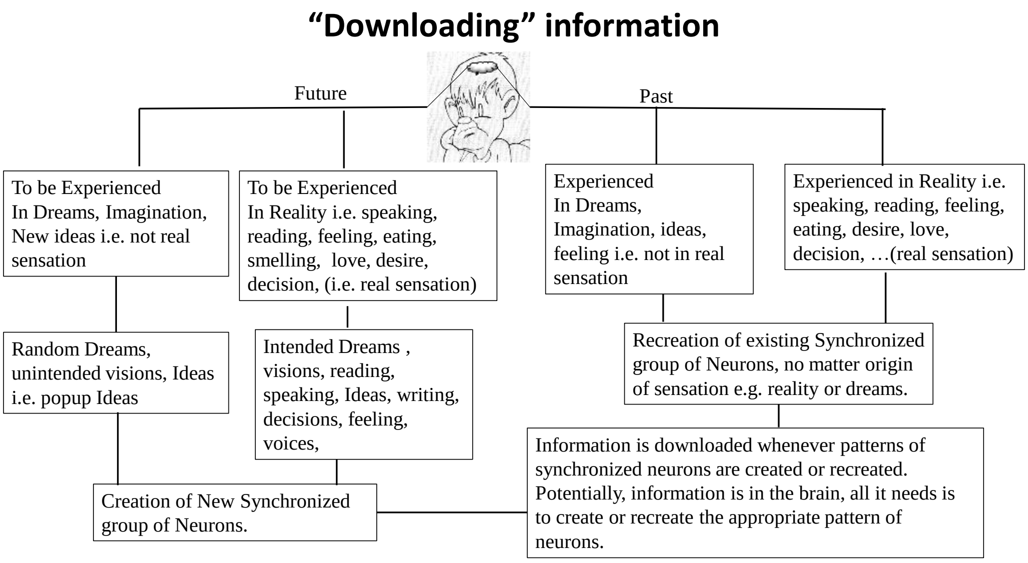

Table 2: Past depends on recreating neural patterns acquired during life. These patterns may have been acquired by real experience or by other sensations like dreams, imagination. Future is the potential to create new patterns of groups of synchronized

neurons either by real sensation or virtual sensations such as in dreams or imagination whether randomly or intended. Whenever neural pattern is created sensation is established no matter the source (real life experience, popup idea, dreams, imagination, etc.). Memory can fade if not being used or if there is certain biological hardware problem like blood supply.

9.2 **Why we do not remember all our experiences all the time?**

Although all memories are potentially existing within the brain neural network up to the current moment, we do not experience the entire memory every moment, unless the brain is triggered to remember i.e. to recreate or re-stimulate the neural patterns associated with that event.

Why the computer does not play its entire memory all the time, unless there is a request. The simplest answer would be minimal energy spending. There is no need to overload the system, e.g. the system may crash and/or there is no request. We remember when we need to remember, when the brain is trigger to retrieved information. We can remember in disordered manner, age 20 then age 5 then age 10 and so on. Once stories are stored as brain neural patterns, time is irrelevant.

Considering the metaphysical virtual space, total potential information is there, once the brain creates synchronize neural patterns information is retrieved or downloaded from the metaphysical space. Retrieving order is irrelevant just as retrieving order of brain memories is irrelevant. Potentially everything is there and we just need to trigger the brain to pull down information.

Based on the above, it seems brain neural synchronization and universes virtual location may have similar hologram surface presentation. When saying everything is in the mind, it is pretty good suggestion. Potentially there is no limitation to generate groups of synchronize neurons reflecting information beyond reality.

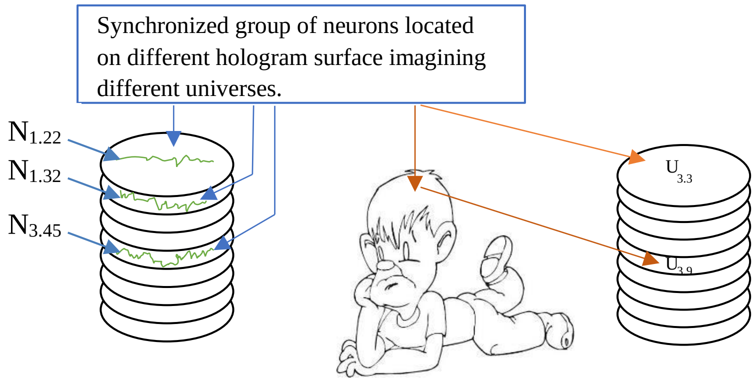

Figure 7:

Synchronized groups of neurons virtually stored on hologram phase surfaces in a similar way the universes are virtually located on hologram phase surfaces.

10. Conclusions

Void is the natural state and due to the existence of the void, a void complementary exists. The void complementary does not incorporate substance. The void complementary represents the total information about any potential scenario, i.e. any potential story. A potential story can be realized only within real universe.

The existence of universes (substance) is a logical state of the void complementary in which the existence of matter must satisfy the metaphysical conservation law of null matter. The metaphysical conservation law was presented by an equilibrium equation of two sort of matter, the positive matter having mass greater or equal to zero and the negative matter having mass less than zero wherein the total matter i.e. the positive and the negative collapses into zero matter timelessly.

The equilibrium equation exists as potential information in the void complementary and it is realized since there is nothing stopping its realization. As noted, the void complementary incorporates all possibilities, for each possibility there is a universe realizing it, otherwise, the possibility would be illogical and therefore would not realize.

The inner universe is expanding into its complementary universe, wherein, the complementary universe carries matter with opposite characteristic in order to satisfies the metaphysical conservation law of null matter.

We anticipate that negative particles reject (push) each other to infinity, while positive particles attract each other representing the force of gravity or force of love.

Although, the equilibrium equation (A) is based on the additive group due to the perception that the total matter (i.e. sum) of the inner universe and its complementary universe is timeless zero. It is possible to symbolize the equilibrium eq. (A) by other group structure or other symbols as long as the perception is logic in the sense that the outcome shows null matter.

With regard the complementary universe, it is most likely impossible to make life in the form we are aware of, due to the dispatching structure, while within the inner universe particles of matter attract each other. As it seems we are living within the inner universe of the equilibrium equation (A).

The metaphysical virtual space is defined to be the void and the void complementary representing timeless, complete and unique virtual space. At current moment, we are determining and experiencing our metaphysical timeless story within the inner universe. Life may exist within each part of the equilibrium equation. Yet, most likely life as we aware of, cannot exist in the complementary universe, the body will be torn apart.

We have discussed the idea that there are infinite uncountable universes, each located on a virtual hologram surface. From the metaphysical point of view a way to travel between inner positive universes requires consciousness intention. However, traveling between positive universes from within requires to generate sort of wormhole that connect between the two distinct positive universes.

We approached the idea how information is created and retrieved by the brain neural mechanism, wherein, synchronized groups of brain neurons have virtual storage structure as distinct hologram surfaces in a similar way to virtual storage (location) of universes over distinct holograms surfaces. Whenever brain neural patterns are created or recreated i.e. as groups of synchronized neurons, metaphysical information is popping up.

Following this paper, we plan to expose the next paper discussing the question, what is pure consciousness? Does pure consciousness can be aware without physical body. Furthermore, experiencing the metaphysical space out of the body, is it a real sensation, dream or a wishful thinking or maybe only those who have experienced pure consciousness metaphysical tour can tell us.

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Some critical remarks on the epistemology of functional magnetic resonance imaging (fMRI)

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Abstract
The article examines epistemological and ontological underpinnings of reasearch performed by means especially of functional magnetic resonance imaging (fMRI). It takes as its guiding line the thesis, set forth by Rom Harré, that instruments such as barometers or thermometers do not cause the states they measure into existence, whereas apparatuses cause the material states into existence which are subsequently processed (treated, measured, etc.) according to suitable methods (e.g. algorithms). Accordingly, when the objects of examination (brains, e.g.) are subjected to 2 or more Tesla in fMRI (a strength of magnetic field never occuring in earthly nature), the technical means literally create the states to be examined.
Close examination of the functioning of fMRI indicates that brain states, e.g., are not simply read as degrees of temperature or measured on some scale. Thus, mental functions as fMRI outputs remain invisible, for the outputs have been semantically processed on the basis of quantum mechanical events according to translation procedures built into the fMRI device.
Keywords: apparatus; instrument; measurement; imaging; experiment; fMRI.[3]

1. Introduction

The *decade of the brain* would have been inconceivable without some groundbreaking technical means such as functional magnetic resonance imaging (fMRI). This technique yields images of human and non-human brains with colorful blobs that are supposed to (strictly speaking)

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differentially *signal* areas of cerebral activity. Though differing in outlook and size, all MRI machines are doing the same job when performing imaging tasks especially for cognitive neuroscientists. These machines are conventionally described and propagated as devices designed for *capturing* areas of brain activity.[4] 'To capture' is merely a different word for 'to measure.' However, the process of measuring brain activity by way of fMRI (if we stick to the conventional terminology) differs greatly from measuring temperature by means of quicksilver thermometers or from measuring atmospheric pressure by means of barographs. The difference turns out to be not of linguistic, but rather of *ontological* as well as of *epistemic* nature. In what follows, we reconstruct the said difference and thereby focus on the distinction, suggested by Rom Harré (2003), between instruments and apparatuses.

Why deal with fMRI techniques against the background of the instrument vs. apparatus distinction? The outputs of fMRI, the brain images with colorful blobs, have induced some neuroscientists to believe that scanners permit them to "watch the mind at work", as Hobson & Leonard (2001, 14) put it. This and similar claims indisputably enhance fascination by, and admiration for, fMRI based neuroscience in the general public. However, they also have been contradicted by other neuroscientists who concede that "fMRI is not and never will be a mind reader" (Logothetis, 2008, 869). This contradiction cannot be solved by better techniques, better equipment, better computer programs, or better experimental designs. The purpose of our contribution is to argue for the need to avoid the contradiction at stake from the outset by reflecting on what fMRI data are.

More often than not, neuroscientists interpret fMRI outputs in a way that what they *see* does in fact also occur in nature. Whenever a fMRI scanner is operating, the device is said to provide a "window into the brain" (Parry & Matthews 2002, 50). To put it differently, a fMRI scanner is perceived and used as if if its purpose were to lift some kind of curtain in order to disclose the brain *as it really is*. Following a proposition of Harré (1998, 353-354), we believe that many neuroscientists uncritically adopt the classical account of scientific experimentation according to which experiments reveal some aspect of the world *as it is* (here: the brain and/or brain function *as is*). However, the underlying assumption then entails that the experimental setup can be *eliminated from the interpretation* for good, in the very same sense that thermometers are eliminated from the act of collecting data by reading temperatures.: When the thermometer within a glass of water displays 25° C, one interprets this as the device showing the real state of something real in the real world (i.e. the water in the glass) and not as the state of the device itself. In other words, we would say that "the temperature of the water has 25° C"

[4]See Siemens's homepage: https://www.healthcare.siemens.de/magnetic-resonance-imaging as well as Phillips's homepage: https://www.usa.philips.com/healthcare/solutions/magnetic-resonance.

and not "the subjects of the thermometer is 25° C" because the top of its quicksilver column has come to rest at the 25° C mark. According to this underlying assumption of *transparency*,[5] the experimental procedure can be *epistemologically discarded*. Applying the principle of transparency to any fMRI scanner generated data output, one expects that the device reveals some state of affair on something in the natural world *as it is*, in the very same way as the thermometer gets epistemologically discarded in the above example of taking the temperature of the water. The interpretation of fMRI data along these lines as revealing *brain activity as it is* is likely to have contributed to fMRI's growing popularity and its suggestive power in the larger public. In the remainder of this contribution, we argue that such a transparency favoring interpretation of fMRI is both mistaken and misleading. Hence, if follows from this thesis that the underlying epistemology of fMRI − an epistemology not yet adequately described − must be different. Following Harré's (2003) conception of scientific experimentation, we will endeavor to show that fMRI provides models of brain physiology by means of a sophisticated technique of data production, whose relation to the natural phenomena (the phenomena of brain physiology) is opaque and indissolubly melted with the machinery's workings in their entirety.

2. (Functional) Magnetic resonance imaging

Our main argument rests on the basic difference between instruments (such as thermometers, seismometers, barographs, myographs, and similar devices), on the one hand, and apparatuses (such as MRI-machines or particle accelerators and similar devices), on the other hand (according to Harré 2003).

The key issue is as follows: in what sense does an apparatus such as a MRI device truly differ from an instrument such as a thermometer? There is no simple, straightforward answer to this question. Technical information is indeed not only useful, but truly necessary in order to understand one characteristic feature of MRI machines, viz. that these machines first *cause material states* into existence which are subsequently processed physically and digitally. In contradistinction, thermometers *do not cause* material states that they are expected to measure. To put it differently: understanding the ontological and epistemic specificity of a device such as a MRI machine, one needs to know first what it does, technically speaking, in order to grasp and further process analytically what it has previously done.

[5] This is both Harré's (1998, 355) terminology and the argument that rests on the concept of transparency.

So let us look for a while into the standard MRI device in action in a neuropsychological lab or in a neurological/neurosurgical ward. The object of investigation is living matter (the brain of a human or non-human animal)[6] subjected to a magnetic field of one or more tesla (T).[7] The intensity of magnetic fields within the range of 1,5 T to 4 T or more produced by the magnetic coil of the scanner is *absolutely uncommon* in nature (of the planet we inhabit), for one T corresponds to the intensity of earth magnetism multiplied by 20,000. Which is to say that a human being undergoing MRI (and/or fMRI) examination is exposed to 30’000 or 40’000 or 60’000, and sometimes even stronger natural magnetic fields that cause innumerable single modifications in the living matter of the head.[8] *But note*: the brain (or, for that matter, the hip, the gut, etc.) is *not* (i.e. without intermediate procedures) made the key target object of MRI machines in action; rather, any magnetic field of e.g. 2, 2.5, 3, or 4 T aims at changing the behavior of protons. Thus, the immediate target objects are the protons and their changing behavior in atoms in molecules within bunches of molecules within tissues within organs such as brains within organisms such as humans.

Indeed, it is a fact (well confirmed by modern physics, but here grossly simplified in its rendering), that the subatomic particles called protons may also be defined as displaying the properties of magnets. An external magnetic field thus causes their orientation to change accordingly. Moreover, protons possess a spin, which adapts to the presence of a magnetic force. These two properties are at the base of well working MRI machines. The protons inside the living organism, excited by the oscillating magnetic field, emit, in reaction to the ‘artificial’ magnetic field, radio frequency signals that are registered by the receiving coil of the scanner. The signals thus received are processed in such a way as to encode position information obtained by means of gradient coils, the function of which consists in varying the magnetic field.

Though quite incomplete, the basic information on MRI given here is sufficient to purport our thesis that scanners are not instruments that permit one to proceed to any ‘brain reading,’ and even less so to any ‘mind reading.’ Scanners *cause* phenomena into existence that are destined to be processed so that information extracted from the processing be then conveyed to observers. What observers ‘read,’ if they read anything in the strict sense of the term, on screens or photograph-like outputs, are visualizations of proton configurations that are brought to behave according to what we might call a ‘physico-technico-digital experimental protocol.’ Thus, in MRI, *nature as it is* and

[6] However, MRI is also good for the study or the diagnosis of other parts of a living body, such as the belly of a dog, the broken knee of a hippopotamus, or the painful bladder of a patient in some hospital.
[7] Experimental MRI machines of the most recent generation work with up to 8 T; we discard them here as well as so called ‘open’ MRI machines which run on low magnetic fields.
[8] In comparison: any human being, and thus any human brain, is exposed e.g. in California, USA, to a magnetic field of 35 microtesla.

nature as read by observers are, strictly speaking, *ontologically incompatible* with one another. They are nevertheless *epistemologically related to one another* by means of highly complex ensembles of physical equipment, signal creating and signal receiving devices, signal processing, statistical algorithms, and visualization programs implemented into the scanner and its assistant devices such as desktops to be looked at, but designed so as to turn the final output into nicely readable images.

The presence of protons in organic molecules determines the amount of signals received by the receiving coil. This is to say that tissues, differing from one another by their chemical composition, and thus by the amount of protons ready to specifically react to the magnetic field produced by the machine, emit signals of different radio frequency depending on their chemical properties. In the case of fMRI, the key neurological theorem underlying the imaging procedures tells us that active brain regions demand higher affluence of blood than less active regions. Increments of blood in circumscribed brain regions cause an increase of signals emitted by the protons in the molecules of both the living matter in action in that cerebral area and the blood whose quantity has increased due to the cerebral activity in the respective region.

To sum up: In fMRI studies, the data that *in the end yield* brain images with colored blobs signalling cerebral activity, are neither neurocognitive nor neuroontological, but, as one could say, artificially brought about by a device that triggers signals according to theorems of quantum mechanics. It follows from these considerations that fMRI 'embodies' materially very complex properties that hinder one to treat scanners as members of the species of instruments according to Harré's (2003) definition explained above. In addition, the setup of scanners requires from experimental subjects to adopt a very unnatural behavior.[9]

3. The concept of experimentation

It is a remarkable fact that philosophers of science would focus upon the relationship between theory and experiment in order to understand whether, and how, theories can be tested. Whereas the members of the Vienna Circle, e.g., aimed at showing that theories are amenable to verification by observations, Popper's critical rationalism held that

[9] Note also that the subjects in fMRI studies are compelled to rest motionless in a narrow scanner bore while signaling the solution of tasks by minimally moving nothing but some fingers. Such restrained behavior is highly 'unnatural,' but essential for the successful performance of neuropsychological experiments under fMRI condition. But this 'unnatural' pattern of behavior is fundamentally different from the 'unnatural' action of a magnetic field of one or more T upon protons in the brain of experimental subjects, although both − the pattern of behavior and the magnetic field-brain complex within the scanner bore − are paradigmatic for the laboratory setting as such.

theories need to be formulated in such a way that they lend themselves to falsification. The controversy was based primarily on predicate logic, for the key question was whether, and how, general scientific propositions ("all x are F") could be deduced from single experimental observations ("some x are F"). Controversies of this kind have long prevented philosophers of science from paying due attention to other aspects of scientific research, especially to various work processes in laboratories, field research, and other places of data triggering and collecting, as well as to the role(s) of modeling in cases where direct observation is strictly impossible.

Positivists such as the members of the Vienna Circle were skeptical regarding propositions reaching beyond the limits of our perception. They claimed it would be inaccurate to assert that the temperature outside is 20º C, but accurate to assert that the thermometer displays 20º C on its visible scale. In contradistinction, contemporary realism, which is the dominant view among most scientists, claims that we can *know*, on the basis of observation(s), that things such as temperature, gravity, electrons, natural selection or neural networks *actually exist*. Contemporary realism also claims that we can corroborate *theories* beyond observation.

In spite of these and similar claims, many realists pay a minimal amount of attention to *how* experimental data relate to the world researchers are supposed to elucidate. Experimental procedures risk thus to be taken to be indispensable in principle. Hence, they are mostly considered to be *epistemologically* irrelevant, for they are said to *reveal by themselves some aspect of the natural world as it really is* without adding anything to, or modifying somehow, the aspects of the world they are revealing.

The case of fMRI is likely to show that this way of considering experimental procedures is short-sighted. Since fMRI procedures create the phenomena to be studied, one ought to dismiss the idea that such phenomena are simply there to be collected. To put some words from Ian Hacking's remarkably apt argument: "To experiment is to create, produce, refine and stabilize phenomena. If phenomena were plentiful in nature, summer blackberries there just for the picking, it would be remarkable if experiments didn't work. But phenomena are hard to produce in any stable way." (Hacking 1983, 230) And since experimental results depend on the phenomena at hand, results are also not just there to be picked like summer blackberries. Or with reference to Harré (2000, 274), let us say that scientific results cannot just be "read off the world."[10]

[10] Many phenomena studied in scientific research, rather, are products emerging from of complex processes by which (a part of) nature is made readable by human beings within their conceptual, linguistic, and technical/instrumental frames.Without discussing Harré's approach in detail, it follows from the main line of his arguments that experimental results are to be conceived as *interpretations* and that no interpretation is ultimately exclusive.

Harré also argues that there exists a (direct, simple) causal relation between aspects of the world and experimental apparatuses such as MRI machines (this also holds, of course, for fMRI studies, which rest on the use of MRI machines, as shown above). Hence, phenomena observed, registered, and measured in experiments may *essentially depend upon* apparatuses, whenever such apparatuses are used. This is to recapitulate that such phenomena are *actively construed.* Apparatuses are nonetheless part and parcel of the material world: they only exist in material form. Though *causality* holds *within* experiments, the relationship between experimental apparatus and world is and remains in part a *semantic* one. Apparatuses are ultimately, and in general, designed to help creating *models*. The latter are neither *true* nor *false* − they are more or less *adequate* or *representative* for the aspects of the world under examination. Such kinds of models (contrary to architectural or anatomical models) are successfully designed if and only if they permit one to control, and actively manipulate, these aspects much more significantly than one would be able to do *in nature*. Thus, phenomena yielded in experiments by means of apparatuses are intrinsically bound to the latter by which they are produced. A straightforward *causal relation to* the world *beyond the apparatuses* is no longer warranted. In contradistinction, what apparatuses *do* reveal is a single disposition or a set of such dispositions, i.e. of possibilities of nature. In other words, experimental phenomena reveal "what Nature is capable of *in conjunction with apparatus*" (Harré 1998, 369, our emphasis). Thus, transparency may be achieved rather between an experimental apparatus and nature's disposition(s) captured by that apparatus, than between the experimental apparatus and nature's actual states.

Harré's approach also offers an explanation for the relative stability of experimental results. Indeed, nature's laws causally impinge upon apparatuses themselves as part of nature, in spite of the fact that these laws cannot be determined by way of experimentation based on apparatuses.

An apparatus models nature more or less *representatively*, not directly, i.e. purely *causally*. Thus, experimenting turns out to be a way of systematically analyzing similarities and differences between a model and the aspect of the world the model is said to represent. However, apparatuses differ in the ways in which they embody analogies to the aspect(s) of nature, which they are expected to represent. In fact, the underlying relationship between apparatus and nature is intricate, for there exist apparatuses that lack even the slightest analogue in nature. The Wilson cloud chamber as a particle detector is thus a device that, as such, has no equivalent in 'raw' nature, although it helps to materially model some physical processes which take place in 'raw' nature. Likewise, submicrometre cylindrical cavities in metallic films for experimentally exploring properties of light are a device that, due to its properties, is radically non-existent in 'raw nature,' although it permits to study, and thus to model, the behavior of light in nature (see Ebbesen et al. 1998).

Hence, any apparatus fulfilling the criterion set by Harré's definition is to be seen as a material model of some structure or process occurring in nature. It "reproduces an *instance* of a natural *regularity* that exists in the real world" (Harré 2010, p. 35). Above all, an apparatus is not just taken, or derived immediately, from nature, since it is something which is designed, has been constructed from scratch, and therefore has to have additional properties. It not only has to be economical, but also purified and standardized in order to allow reliable replications. This presupposes the availability of technical means industrially supplied[11].

To sum up, two important points obtain from these considerations:

(a) Since the apparatus partakes of the material world and is causally subjected to manipulations, it should also be considered as something that affects the material world. Thus, the constructive character of apparatus-bound experiments yields experimental results that are inextricably bound to the technical arrangement at stake. This holds for fMRI as well, as this technique can be interpreted as an apparatus (not an instrument) in Harré's sense.

(b) The question as to how far we may back-infer from apparatuses to nature itself remains dependent upon a subtle balance between abstraction and verisimilitude. And this entails that the 'art of the experimenter' turns out to be a decisive factor in scientific research making more or less extensive use of apparatuses (Harré 2010, 36). To put it differently, "modeling is a scientific technique that requires a good deal of intuition and insight to be really effective" (Harré ibid, 36). It is never an affair that rests on some easy matter of *facts*.

Furthermore, Harré distinguishes two types of apparatuses:

First, working models of natural processes: There exist working models of natural processes *within* the material systems for which they count as models. Example: model organisms deliberately designed by genetic means such as a variety of rats exhibiting specific phenotypical traits, e.g. outstanding memory abilities or the propensity for fear. Although bred in a laboratory, such organisms could in principle have developed in nature without human intervention. A model organism *qua* apparatus of this sort is the material model of a piece of nature in 'domesticated' form. It is simpler, so to speak purer, more regular and thus more easily manipulable. In one word, it's easier to experiment with. Although ontologically different from that part of nature under scrutiny, but similar to all other types of apparatus, model organisms

[11] This is a serious point as it presupposes a certain technological development of a society in order to provide technical devices such as MRT scanners.

rest on the simplified version of a set of phenomenona. In this case, experimenters do not aim at explaining the effects produced by the environment on the model organism *qua* apparatus, since they operate by transferring a piece of nature into the lab. This allows one to draw sometimes strong back-inferences from the experimental results to nature outside the laboratory (although back-inferences depend on how relations of similarity and difference are weighted by research aims).

Apparatus as apparatus-world-complex: An apparatus of this type *creates* phenomenona that *would not, and cannot, occur in nature*, *it literally produces artefacts*. As it is not transcendent to the world, but has to be made epistemically operant in order to solve e.g. some strong theoretical paradox (such as the particle-wave-dualism in quantum mechanics). A lot of experimentally induced phenomena result from such apparatus-world-complexes (AWC). They turn out to be hybrids of human design and nature. This was, paradigmatically, the case in Humphry Davy's isolation of sodium. As is well known, sodium does not exist in isolated form in nature. When dealing with "some chemical agencies of electricity", he discovered that applying electricity to various chemical compounds, such as sulphate of sodium and phosphate of sodium (among several other similar compounds) (Davy 1807, 18), pure sodium obtained by decomposition *within the borders* of the apparatus, i.e. in the AWC (Davy ibid., 12). To describe Davy's experiments in other terms, the "powerful electrical machine" designed by a certain Mr Nairne (Davy ibid., 31) was set up so that it constituted a model of a fragment of nature where, due to manipulations, decomposition of chemical compounds containing sodium atoms occurred. However, without this specific AWC, without this piece of experimentally 'domesticated' nature, where laws of nature did neither fail nor even slightly change, the "powerful machine" that had created a micro-universe where dispositions of natural matter could manifest themselves previously unheard of and unobserved, pure sodium would not have been isolated. The decomposition of sodium compounds was artificial, i.e. experimentally and artfully induced in, through, by, and thanks to Davy's AWC.

Back-inferences to nature are often difficult to draw in AWCs, for their ontological status as compared to the ontology of nature 'pure and simple' is unclear. Indeed, the true contribution of an apparatus to the production of phenomenona is not transparent from the outset (cf. again Davy's extensive reports on his experiments on the chemical agencies of electricity). Therefore, AWCs cannot be conceived of as instances that unmediatedly actualize properties of nature, that is, the principle of actualism has to be dropped. Instead, we can only speak of potentialities of nature that are made available by AWCs, or of affordances, to refer to

Harré's terms.[12] Indeed, each affordance is relative to what humans intend to do with it − it "would not have existed without human action to bring it into being" (Harré 2010, 37).

4. Functional magnetic resonance (fMRI) scanners as AWCs[13]

Let us go back to the issue of fMRI. It follows from the critical reconstruction of Harré's approach that he conceives of experiments as instances that necessarily remain related to nature (rather than being just merely symbolized within the scientific discourse − according to some postmodernist approaches), while rejecting at the same time the traditional claim of transparency. To put it in simpler words: experimental apparatuses do not reveal states in the world *as these states are in and by themselves*. As indicated above, Harré argues for the non-eliminable, constructive character of experimental apparatuses; at the same time, however, he emphasizes that there is a world 'out there' to which apparatuses necessarily relate. Neither the world itself nor apparatuses by themselves reveal the phenomena considered to be relevant for the elaboration of scientific results, since both indissolubly melt into some AWC.

Concerning fMRI scanners, we claim that the most popular and common experimental devices of cognitive neuroscience are members of the AWC-type. The AWCs are intelligently designed and carefully manufactured machines that create, as one could say, pieces of 'domesticated' tissues ready to be used as material models of real, organic, living tissue to which they refer. Nowhere in nature do we encounter differences in blood magnetization due to a magnetic field whose force is thousands of times stronger than magnetic fields to be observed on earth's surface. As AWCs, fMRI scanners (in conjunction with MRI scanners) model the anatomy of the brain as well as

[12] Harré refers to James Gibson's idea of affordance.

[13] The italian physiologist Angelo Mosso (1846-1910) is given credit for having devised the first instruments to non-invasively measure the redistribution of cerebral blood in response to cognitive tasks. Therefore, he is often seen as a father of functional magnetic resonance imaging (Sandone et al., 2014). Originally, Mosso had devised a plethysmograph to register alternations in cerebral blood flow from the dura mater in patients with skull defects. He related these alternations to different states of vigilance in the subjects or to cognitive tasks such as listening to their names or to a striking clock (Zago et al., 2009). In order to do similar studies in healthy subjects, Mosso devised a "human circulation balance." Subjects lay on a wooden table that was balanced on a pivotal point. In a sophisticated experimental procedure, Mosso aimed at demonstrating that cognitive and affective tasks were leading to increased blood flow to the brain, thereby changing the equilibrum in which the lying subject had been set towards head side. Interestingly, Mosso's device would be different from fMRI as it would have to count as an *instrument sensu* Harré rather than an *apparatus* (such as fMRI) (for the instrument vs. apparatus distinction, see above). In both Mosso's plethysmograph and his circulation balance there is a direct causal connection between states in the world (changes in cerebral blood flow) and changes in the instrument (amplitude or tilt of balance, respectively).

cerebral functions by means of highly sophisticated techniques of data production, where the relationship to natural phenomenona (the physiology in nature's brains) is opaque and by material necessity linked to the structure of the machinery. Thus, the assertion according to which neuroscientific data are "extracted from the physical world by technical measuring devices such as [...] functional MRI scanners" (Metzinger, 2004, 591) is totally misconceived if based on a realist interpretation of fMRI data, to say the least, and at worst simply wrong.

Due to their design, MRI scanners make good candidates for Bohrian apparatus-world-complexes. As we have argued above, back inferences from AWCs to the natural world beyond human intervention are inherently problematic, as their status with respect to nature remains opaque. Of course, brains in laboratories causally affect fMRI data. These data, however, do not allow us to directly conclude what brains in nature actually do, as the technique of MRI scanners produces phenomena not to be found in nature. Thus, the relationship between fMRI data and natural brains is not a causal one. It has to be judged on the basis of *adequacy*, i. e. of whether the lab situation of a magnetic field of e.g. 3 T provides an appropriate model for brains active outside the scanner in a natural magnetic field of 35 microtesla.

The best way to make sense of the epistemological status of Bohrian apparatuses (such as MRI and fMRI scanners) is to assume that they actualize a *potentiality* of the world, not an *actuality*. Thus, the fMRI scanner may help to systematically manipulate parts of the world (i.e. brains) for good or for bad, but they seem hardly apt to straightforwardly reveal by themselves, if considered to be self-controlled agencies, anything about the *essence* of the mind (i.e. about one part of the world as it *is*).

5. Conclusion

When apparatuses (in Harré's sense) such as MRI, fMRI, and similarly conceived equipments are at stake, their functioning and their output(s) entail some far-reaching consequence, as the present analytic reconstruction is meant to show. The relation holding between the observing, measuring, experimenting subjects, on the one hand, and the target objects, on the other hand, reveals itself to be epistemically significantly different from the relation holding between the said subjects and their instruments vis-à-vis the target objects given (so to speak) without some intervention caused by the means of observation and measurement. The distance between two points in space is not altered when being measured by a folding rule; human heartbeat is in itself not altered by the stethoscope. However, neither colliding subatomic particles in helium chambers nor activated brain states are (literally) seen when examined on displays, films, and other means. Which is to say that the verbs to see, to observe, to measure are semantically and epistemically flexible and need to be specified according to the context of research procedures and equipments.

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Linking Ontology, Epistemology and Research Methodology

Mukhles M. Al-Ababneh*

Abstract

The purpose of this paper is to offer insights that can help researchers to link ontology, epistemology and research methodology. This paper outlines the links among ontology, epistemology and research methodology by exploring ontological, epistemological and methodological perspectives in the research. It discusses how ontological and epistemological issues influence research methodology by providing a clear understanding of different research methodologies based on ontology and epistemology. Furthermore, attention is given to research aspects such as the elements of the research process, research philosophy, research approach, research strategy, the choice of method, and research design.

Keywords: ontology; epistemology; research methodology.§

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1 Introduction

This paper discusses the research process from ontological, epistemological and methodological perspectives to explore and understand the social phenomena. The goal of this review is to provide a clear understanding of different research methodologies based on ontology and epistemology. The research methodologies can generally be subdivided into quantitative and qualitative research methodology. Quantitative methodology generates knowledge by investigating things which we could measure in some way. Qualitative methodology increases understanding of why things are and the way they are in the social world, and why people act the ways they do.

The differences amongst research methodologies are based on a philosophical and theoretical view of research that guides researchers in their social science research. Furthermore, researchers can select the research methodology in social science and that depends on philosophical issues, which are related to ontology (the nature of reality) and epistemology (the nature of knowledge). This paper is an attempt to make the reader aware of the research methodologies and their basis of difference from ontological, epistemological and methodological perspectives.

2 Research Process

The research process consists of four elements, namely epistemology, theoretical perspective, methodology, and methods that inform one another, as shown in Figure 1. In other words, epistemology informs the theoretical perspectives, these perspectives determine research methodology, and then methodology governs and chooses the methods of research (Crotty, 1998).

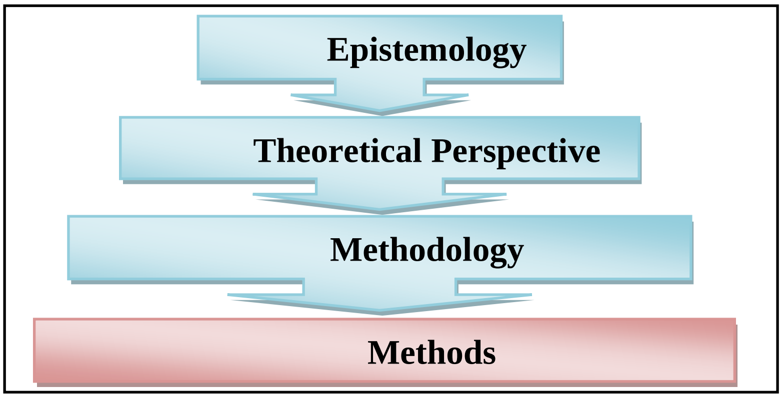

Figure 1: The Basic Elements of Research Process (Crotty, 1998, p.4).

Crotty (1998: p.3) defined the basic elements of research process as the following:

- Epistemology: "The theory of knowledge embedded in the theoretical perspective and thereby in the methodology".
- Theoretical Perspective: "The philosophical stance informing the methodology and thus providing a context for the process and grounding its logic and criteria".
- Methodology: "The strategy, plan of action, process or design lying behind the choice and use of particular methods and linking the choice and use of methods to the desired outcomes".
- Methods: "The techniques or procedures used to gather and analyze data related to some research question or hypothesis".

The first element, epistemology is described as inherent in the theoretical perspective as "a way of looking at the world and making sense of it". Major types of epistemology are: objectivism, constructionism, and subjectivism. The second element, a theoretical perspective describes the philosophical stance that lies behind the chosen methodology. It grounds the main assumptions of choosing methodology. Main types of theoretical perspective are: positivism (and post-positivism), interpretivism, critical inquiry, feminism, postmodernism, etc. The third element, research methodology represents the strategy and plan of action, which refers to the research design that shapes chosen research methods. Methodology provides a rationale for the choice of methods and the particular forms in which the methods are employed. Major types of methodology are: experimental research, survey research, ethnography, phenomenological research, grounded theory, heuristic inquiry, action research, discourse analysis, feminist standpoint research, etc. The last element, research methods refer to the techniques or procedures that including certain activities to collect and analyse data based on research question or hypotheses. Some of major research methods such as: sampling measurement and scaling, questionnaire, observation, interview , focus group, case study, life history, narrative, visual ethnographic methods, statistical analysis, data reduction, theme identification, comparative analysis, cognitive mapping, interpretive methods, document analysis, content analysis, conversation analysis, etc. (Crotty, 1998).

On the other hand, Saunders, Lewis, & Thornhill (2009) explained the research process as an onion including several important layers; each layer leads to another till the centre of research onion including the choice of data collection techniques and data analysis procedures as shown in Figure 2. The first layer represents the research philosophy which relates to the nature and development of knowledge. Thus, the researchers can adopt different research philosophies as positivism, interpretivism, realism, and pragmatism. After selecting research philosophy, the researchers have to select research approach in the second layer either deductive or inductive. In the third layer, different research strategies such as experiment, survey, case study, action research, grounded theory, ethnography, and archival research, can be applied to answer the research questions. Research method in the fourth layer can be different types to collect data, namely mono method, mixed methods, and multi-method. Time is important aspect in the research in the fifth layer, and therefore the research can collect data just once over a short period of time 'Cross-sectional' or collect data several times over a period of time 'longitudinal'. Finally, the centre of onion is the core of research that including data collection techniques and procedures of data analysis.

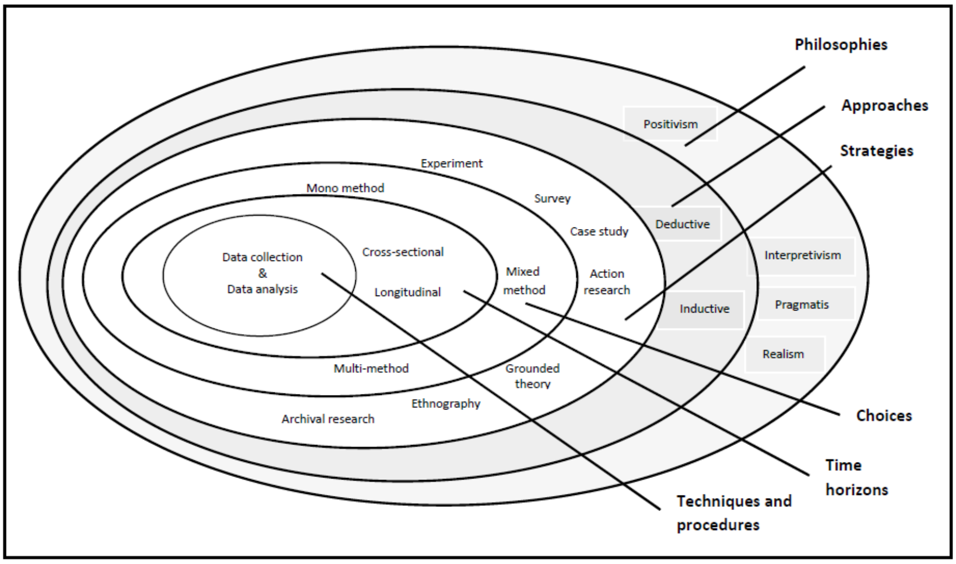

Figure 2: The Research Onion (Saunders et al., 2009, p.108)

2.1 Epistemology

Epistemology involves knowledge and embodies a certain understanding of what is entailed in knowing, that represents how we know what we know (Crotty, 1998). Epistemology deals with 'the nature of knowledge, its possibility, scope and general basis' (Hamlyn, 1995: p. 242) as cited by Crotty (1998: p.8). Furthermore, Maynard (1994) as cited by Crotty (1998: p.8) explains that 'epistemology is concerned with providing a philosophical grounding for what kinds of knowledge are possible and how we can ensure that they are both adequate and legitimate'. Epistemology is concerned with the acceptable of knowledge in the study field (Saunders et al., 2009).

Major types of epistemology are objectivism, constructionism, and subjectivism (Crotty, 1998). Objectivism means that meaning and meaningful reality exists as such apart from the operation of any consciousness (Crotty, 1998), and it represents "the position that social entities exist in reality external to social actors" (Saunders et al., 2009: p.110). Constructionism refers to the meaning that comes into existence in and out of human engagement with the realities in the world due to there is no truth waiting to discover it as well no meaning without a mind. This view supports that subject and object emerge as partners in the generation of meaning. Finally, subjectivism refers to the meaning comes from anything but the object to which it is ascribed, that means the object itself makes no contribution to the meaning that is imposed on the object by the subject (Crotty, 1998). Saunders et al. (2009: p.111) considered subjectivist view as "social phenomena are created from the perceptions and consequent actions of social actors". On the basis that the phenomena to be investigated exist independent of consciousness and individuals are in direct contact with reality through sensory perception, and therefore this research leans more towards an objectivist epistemology.

2.2 Research Philosophy (Theoretical Perspective)

A theoretical perspective describes the philosophical stance of informing and determining the research methodology (Crotty, 1998). Research philosophy is an important element as it is useful for determining which research design is going to apply and why (Easterby-Smith, Thorpe, & Lowe, 1999), while Saunders et al. (2009) considered research philosophy as a researcher thinking about the development of knowledge. However, there are four types of the research philosophy based on researchers' views about the research process: positivism, interpretivism, realism, pragmatism (Saunders et al., 2009). Whereas, Collis and Hussey (2003) classified the research paradigms into two types: the positivistic paradigm and the phenomenological (or interpretivist) paradigm. More specifically, paradigm refers to "the progress of scientific practice based on people's philosophies and assumptions about the world and the nature of knowledge" (p.46). In other words, people's beliefs about the world will impact research design and the procedures of research (Collis and Hussey, 2003).

2.2.1 Positivism

Positivism provides assurance of unambiguous and accurate knowledge of the world. It refers to something that is posited (i.e. something that is given). Positivism is interested in the development of a comprehensive social that apply the scientific method to the study of society and human beings for their benefit. Positive science based on direct experience, not speculation. Knowledge in this science is grounded firmly and exclusively in something that is posited, and it does not arrived at speculatively. Therefore, positive science (or positivists) is defined as what is posited or given in direct experience is what is observed in scientific methods. Contemporary positivism is linked to empirical science as closely as ever (Crotty, 1998). The Positivist philosophy adopts the philosophical stance of the natural scientist, and the results of this research philosophy can be law-like generalisations, similar to the results obtained by physical and natural scientists (Saunders et al., 2009). Positivism presents scientific discovery and technology the driving force for progress. Scientific knowledge is both accurate and certain, which represents the confidence in science. Positivism is objectivist completely. Objects in the world from the positivist viewpoint, have meaning prior to, and independently of, any consciousness of them. As well, it is necessary to maintain the distinction between objective, empirically verifiable knowledge and subjective, unverifiable knowledge (Crotty, 1998). In the positivism philosophy, researchers deal with issues objectively without impacting the real problem being studied. Thus, positivism philosophy needs very well structured methodology, quantifiable observations and statistical analysis (Remenyi, Williams, Money, & Swartz, 2005). Consequently, positivism supposes that researchers make an objective analysis and interpretation for collected data (Saunders et al., 2009).

2.2.2 Interpretivism

Phenomenology (or interpretivism) is another theoretical perspective that emerged in contradistinction to positivism to understand and explain human and social reality. The interpretive approach looks for culturally derived and historically situated interpretations of the social life world. Whereas, the positivist approach follows the methods of natural sciences by way of purportedly value-free, detached observation, identifying universal characteristics of human-hood, society and history that provides explanation and consequently predictability and control (Crotty, 1998). This philosophy looks to the social world of business and management science as too complex to be treated as a physical science due to complex management studies in the social world will be lost if its complexity is reduced to law-like generalisations. Interpretivism philosophy considers the situation in each business is unique, and differs from other situations. This methodology is not an appropriate method for generalisation due to the changing state of business organisation and various interpretations by people as well as the complexity and uniqueness of the world (Saunders et al., 2009). This interpretivist philosophy develops knowledge in a different way by focusing on a subjective and descriptive method to deal with complicated situations rather than an objective and statistical method (Remenyi et al., 2005). Social research is complex and does not lend itself to theorising by definite laws such as other research in science. Social research requires investigating behind law-like generalisations due to the complexity of the social sciences, and the philosophy of this research is interpretivist (Saunders et al., 2009).

2.2.3 Realism

Realism is another research philosophy relates to scientific enquiry. The Realist philosophy is based on the belief that reality exists in the world, and this reality is independent of human thoughts and beliefs. Realism is opposed to idealism due to the existence of reality is independent. Realism is one type of epistemology, and therefore it is similar to positivism that assumes a scientific approach to develop knowledge. However, there are two types of realism, namely *direct realism* 'what we see is what we get', and the researchers see the real world accurately; and *critical realism* 'what we see is not what we got' researchers see the world as sensations, not the real things directly, that requires more criticism in the reality (Saunders et al., 2009: pp.114-115).

2.2.4 Pragmatism

Pragmatism declares that the reality exists in the world, and it supports the objective nature of science. As well this philosophy assumes that individuality may impact how people perceive the world, and therefore research is subjective. The view of this philosophy brings multiple explanations and interpretations for science. This philosophy uses both objective and subjective criteria (Saunders, Lewis, & Thornhill, 2003). Hence, the pragmatist philosophy is between positivist and interpretivist research philosophy, it refers to there is no one appropriate philosophy and therefore researchers can adopt more than one research philosophy. Pragmatism argues that is possible to work with variations in epistemology (Saunders et al., 2009).

According to Creswell's (1994) classification, he refers that positivistic paradigm as quantitative paradigm, and phenomenological paradigm as qualitative paradigm (Collis and Hussey, 2003). Whereas, Collis and Hussey (2003) summarised the main features of positivistic paradigm and phenomenological paradigm related to research methodology and method. Table 1 shows the features of main paradigms.

Concept	**Positivistic Paradigm**	**Phenomenological Paradigm**
Data collection technique	Quantitative	Qualitative
Epistemology	Objectivist	Subjectivist
Practice	Scientific	Humanistic
Research philosophy	Experimentalist, Traditionalist	Interpretivist

Sample size	Large samples	Small samples
Research approach	Hypothesis testing	Generating theories
Collected data	Highly specific and precise	Rich and subjective
Location	Artificial	Natural
Reliability	high	low
Validity	low	high
Generalizability	Generalises from sample to population	Generalises from one setting to another

Table 1: The Features of the Main Research Paradigms (Collis and Hussey, 2003, p.47, 55).

On the other hand, Saunders et al. (2009) compared between the four research philosophies in terms of Ontology 'the nature of reality', Epistemology 'the acceptable knowledge', Axiology 'the role of values in research', and data collection techniques, as shown in Table 2.

Concept	**Positivism**	**Interpretivism**	**Realism**	**Pragmatism**
Ontology	External, objective and independent of social actors.	Socially constructed, subjective, may change, multiple.	Is objective. Exists independently of human thoughts and beliefs or knowledge of their existence (realist), but is interpreted through social conditioning (critical relist)	External, multiple, view chosen to best enable answering of research question.
Epistemology	Only observable phenomena can provide credible data, facts. Focus on causality and law like generalisati	Subjective meanings and social phenomena. Focus upon the details of situation, a reality details, subjective meanings	Observable phenomena provide credible data, facts. Insufficient data means inaccuracies in sensations (direct	Either or both observable phenomena and subjective meanings can provide acceptable knowledge

	ons, reducing phenomena to simplest elements.	motivating actions.	realism). Alternatively, phenomena create sensations which are open to misinterpretation (critical realism). Focus on explaining within a context or contexts.	dependent upon the research question. Focus on practical applied research, integrating different perspectives to help interpret the data.
Axiology	Research is undertaken in a value-free way, the researcher is independent of the data and maintains an objective stance.	Research is value bound, the researcher is part of what is being researched, cannot be separated and so will be subjective.	Research is value laden; the researcher is biased by world views, cultural experiences and upbringing. These will impact on the research.	Values play a large role in interpreting results, the researcher adopting both objective and subjective points of view.
Data Collection Techniques	Highly structured, large samples, measurement, quantitative, but can use qualitative.	Small samples, in-depth investigations, qualitative.	Methods chosen must fit the subject matter, quantitative or qualitative.	Mixed or multiple method designs, quantitative and qualitative.

Table 2: Comparison of the Research Philosophies (Saunders et al., 2009, p.119).

Based on the features of the main research paradigms as discussed by Collis and Hussey (2003) as shown in Table 1, and further comparison of the research philosophies as explained by Saunders et al. (2009) is presented in

Table 2. The researcher found that the criteria of a positivist philosophy are suitable to the research objectives such as: independence of researcher, exploration of the relationships and causality between variables, objective criteria, deductive approach, quantitative and qualitative measurements, and generalisation. Consequently, this research selected positivism philosophy to understand the causation among variables, and to explain antecedents related to those variables causally (Crotty, 1998).

3 Research Design

Research design is a crucial part of any research as it is concerned with turning research questions into projects. Research design is important in deciding the research processes and elements such as research methods, research strategy, and sampling (Robson, 2002). Figure 3 displays the components of research design.

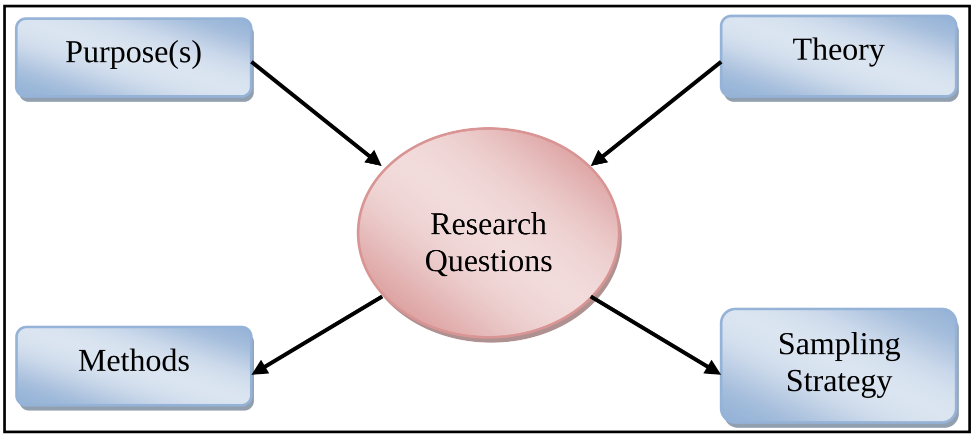

Figure 3: A Framework for Research Design (Robson, 2002, p.82).

The choice of research design depends on the purpose(s) of research, and hence there are four types of research design which are: exploratory study, descriptive study, explanatory study (Saunders et al., 2009), and emancipatory study (Robson, 2002).

1. Exploratory Study: This type of studies focuses on investigating what is happening, asking questions, seeking new insights, assessing phenomena in a new light, as well generating ideas and hypothesis for future researches. An exploratory study is characterised as a flexible design (Robson, 2002). The exploratory study is conducted when there is no information is available or little information is known about how similar research has been conducted in the past. Therefore, exploratory study provides a better understanding of the nature of the problem

being researched since very few studies have been conducted in the same area (Sekaran, 2003). This study is useful for clarifying and understanding of an imprecise problem, it can be conducted based on three main ways are: a search of the literature, interviewing experts, and conducting focus group interviews (Saunders et al., 2009).

2. Descriptive Study: This study displays an accurate profile of persons, situations or event. This type requires collecting a lot of information about the situation that will be studied. The descriptive study may be flexible and /or fixed design (Robson, 2002). Descriptive study is conducted in order to determine and describe the characteristics of the variables in the situation. Therefore, the descriptive study aims to provide researcher a profile or describe aspects of the phenomena being researched at different levels such as individual, organisational, industry-oriented, and other perspective (Sekaran, 2003). This study is considered as a piece of, or a forerunner to exploratory research, and therefore it is necessary to have a clear picture of phenomena before conducting exploratory study (Saunders et al., 2009).
3. Explanatory Study: This study seeks an explanation of a situation or problem being studied and not necessary to be in causal relationship, and explain of patterns relating to studied phenomenon. This study may be flexible and /or fixed design (Robson, 2002). Explanatory study investigates the relationship between variables of phenomenon in order to establish a causal relationships between variables (Robson, 2002; Saunders et al., 2009). Other authors called this type of study as hypotheses testing. Hypotheses testing are usually conducted to explain the nature of the specific relationships, or indicate the difference among groups of independent variables, as well as explain the variance in the dependent variables or to predict outcomes (Sekaran, 2003).
4. Emancipatory Study: This study is not common but it creates opportunities and the will to engage in social action. This study is a flexible design (Robson, 2002).

4 Research Approach

The research approach can be classified into two approaches: deductive approach, and inductive approach. The deductive approach should be used when research focuses on developing theory and hypotheses, and designs a research strategy to test hypotheses. The inductive approach should be used when collecting data and developing a theory as a finding of the data analysis (Saunders et al., 2003). It is necessary to match research philosophies and research approaches; the deductive approach relates more to the positivist philosophy and the inductive approach to the interpretivist philosophy

(Saunders et al., 2009). A deductive approach was chosen in this study by using theoretical arguments based on existing phenomena and testing hypotheses (Blaikie, 2000). This approach is used to describe the causal relationship between variables, testing hypotheses, and generalising the regularities in human social behaviour (Saunders et al., 2009).

On the other hand, Creswell (1994) identified that there are two types of research paradigms based on the assumptions of the paradigms. Firstly, the quantitative paradigm is termed traditional, positivist, experimental, or empiricist paradigm. Secondly, the qualitative paradigm is termed, constructivist approach or naturalistic, interpretative approach, or post-positivist or postmodern perspective. Table 3 explains the main paradigm assumptions for quantitative and qualitative showing.

Assumption	Question	Quantitative	Qualitative
Ontological Assumption	What is the nature of reality?	Reality is objective and singular, apart from the researcher.	Reality is subjective and multiple, as seen by participants in a study.
Epistemological Assumption	What is the relationship of the researcher to that researched?	Researcher is independent from that being researched.	Researcher interacts with that being researched.
Axiological Assumption	What is the role of values?	Value-free and unbiased.	Value–laden and biased.
Rhetorical Assumption	What is the language of research?	Formal. Based on set definitions. Impersonal voice. Use of accepted quantitative words.	Informal. Evolving decisions. Personal voice. Accepted qualitative words.
Methodological Assumption	What is the process of research?	Deductive process. Cause and effect. Static design-categories isolated before study. Context-free. Generalisation leading to	Inductive process. Mutual simultaneous shaping of factors. Emerging design–categories identified during research process. Context-bound. Patterns, theories developed for

		prediction, explanation, and understanding.	understanding.
		Accurate and reliable through validity and reliability.	Accurate and reliable through verification.

Table 3: The Assumptions of Quantitative and Qualitative Paradigm (Creswell,1994, p.5)

In order to explain the relationship between the study's variables, therefore, a deductive approach can be used to describe causal relationships between variables, and measure facts of variables quantitatively (Saunders et al., 2003). To analyse that association a quantitative methodology can be used to test hypotheses in a cause-and-effect relationship by using a deductive approach (Creswell, 1994). Indeed, qualitative paradigm can be used as a part of quantitative paradigm.

5 Research Strategy

The research strategy is a general plan of how to answer the research question(s) that includes clarify research objectives, specify the sources of data collection, and consider research constraints. There are various research strategies are: experiment, survey, case study, grounded theory, ethnography, action research (Saunders et al., 2003), and archival research (Saunders et al., 2009). However, Saunders et al. (2009) reported that no specific research strategy is better than any other strategies, and therefore selecting research strategy is based on research question(s) and objective(s), research philosophy, and the extent of existing knowledge.

Survey strategy is a common strategy in social research due to it authoritative by people, and this strategy is related to deductive approach (Saunders et al., 2009). Survey strategy is considered a positivistic methodology. There are two types of survey: a descriptive survey is concerned with identifying the frequencies among participations related to specific issues for one time or several times for comparison. The other type is an analytical survey is concerned with investigating the relationship between various variables (Collis and Hussey, 2003). In addition, survey strategy tends to be used in the exploratory study and descriptive study, and therefore this strategy allows researcher to collect quantitative data and analyse these data quantitatively through descriptive and inferential statistics (Saunders et al., 2009). The questionnaire is not the only data collection technique that belongs to survey strategy, and other data collection techniques can be used as structured observation, and structured interviews (Saunders et al., 2009). A survey strategy can collect primary data more efficiently, accurately, inexpensively, and quickly (Zikmund, 2003).

6 Research Method

The research methods can have three techniques for collecting data are mono method which refers to use a single data collection technique (quantitative or qualitative); multi-method refers to those combinations between more than one data collection technique (Tashakkori and Teddlie, 2003); and mixed-method which refers to use both quantitative and qualitative data collection techniques (Saunders et al., 2009). Multi-method research can be either quantitative 'multi-method quantitative study' or qualitative 'multi-method qualitative study', while mixed-method can be classified into two types are 'mixed method research' uses quantitative and qualitative data collection techniques with relative analysis technique for each data, 'mixed model research' combines quantitative and qualitative data collection techniques and analysis procedures (Saunders et al., 2009). Multiple methods are considered very useful for research due to they provide better opportunities to answer research questions, and better interpreting for research findings (Tashakkori and Teddlie, 2003).

One of the main reasons for using a mixed-method design is triangulation (Saunders et al., 2009). Furthermore, triangulation methodology is a multi-method research using more than one approach for collecting data in order to enhance confidence in the results. Triangulation refers to the attempt to obtain the right data by combining different ways of looking at it (method triangulation) or different findings (data triangulation) (Silverman, 2010). Similarly, Saunders et al. (2009) viewed triangulation as the use of different techniques for data collection within one study to ensure that the data are telling the researcher what he/she think they are telling he/she. For instance, the collected qualitative data through semi-structured interviews may be a valuable way of triangulating collected quantitative data through a questionnaire. Thus, the various data collection techniques (data triangulation) can obtain the right data, and more specifically.

Regarding time horizons, there are two approaches for collecting data are cross-sectional studies, and longitudinal studies. Cross-sectional approach is a positivistic methodology designed to obtain data from different contexts at the same time. In this approach, data is collected just once over a short period of time, it takes a snapshot of an on-going situation. Therefore, cross-sectional approach conducts to investigate the existence of correlations among variables in large samples, and it is the most common approach in the literature. On the other hand, longitudinal approach aims to investigate the dynamics of problem continuously for several times. This approach allows researchers to investigate the changes related to problem being researched over the time. Longitudinal approach is often related to positivist methodology, it based mainly on a qualitative approach (Collis and Hussey, 2003).

On the basis that the phenomena (i.e. service quality and customer satisfaction) to be investigated as a research. The research leans more towards an objectivist epistemology. Whereas, the theoretical perspective of this research tends to be a positivist philosophy, and a positivist philosophy is suitable to the research objectives to understand the causation among variables, and to explain antecedents related to those variables causally. For example, one study may use a deductive approach to describe causal relationships between variables, and measure facts of variables quantitatively to explain the relationship between those variables. Both quantitative and qualitative methodologies can be used to test hypotheses in a cause-and-effect relationship by using a deductive approach. The research can apply an objectivist view as epistemology, positivist philosophy as theoretical perspective, survey strategy as research methodology, and quantitative method of statistical analysis and qualitative method of content analysis as research methods as shown in Figure 4.

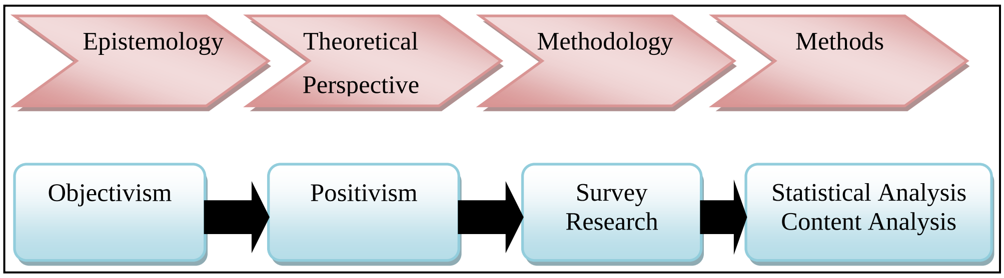

Figure 4: The Elements of Research

As shown in Figure 4, survey strategy is considered a positivistic methodology and it related to deductive approach. Thus, survey strategy allows the researcher to collect quantitative and qualitative data and analyse quantitative data quantitatively through descriptive and inferential statistics through descriptive survey (it concerned with identifying the frequencies among participations related to specific issues), and analytical survey (it concerned with investigating the relationship between various variables), while qualitative data will be analysed qualitatively through content analysis. On the other hand, a holistic view of the research design, more specifically, the research can use positivism as research philosophy, deductive as research approach, survey as research strategy, mixed methods as research method, cross-sectional approach to collect data once, and finally, structured interviews (qualitative data) collaborate with questionnaire (quantitative data) to achieve triangulation, and therefore data can be analysed by using both statistical analysis and content analysis.

7 Conclusions

The two major forms of research methodology are quantitative methodology, which is grounded on positivist paradigm and qualitative methodology, which is grounded on interpretivist paradigm and quantitative methodology, which is grounded on positivist paradigm. Qualitative methodology is concerned with understanding the meaning of social phenomena. On the other hand, quantitative methodology is concerned with attempts to measure social phenomena by collecting and analyzing data. A clear understanding of the philosophical orientation can guide researchers to conduct their research. Hence, linking research and philosophical traditions helps researchers to clarify their research's theoretical frameworks. The research framework includes the beliefs about the nature of reality (ontology), the theory of knowledge that informs the research (epistemology), and how that knowledge may be gained (methodology) that present different research methodologies used in research.

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Mathematics and the Liberal Arts

Anthony Shannon[1]

Abstract

The Liberal Arts deal with the human being as a whole and hence with what lies at the essence of being human. As a result, the Liberal Arts have a far greater capacity to do good than other fields of study, for their foundation in philosophy enables them to bring students into contact with the ultimate questions which they are free to accept (or reject). Even if these questions have little or no 'market value', it should be obvious that the way they are taught and learned is going to have a powerful impact upon the future of the students and society. It is suggested here that mathematics has an integral role in the study of the liberal arts in a first degree at a university where the 'meal ticket' is subsequently studied in the graduate or professional school.

Keywords: Metaphysics, liberal arts, trivium quadrivium, humanism.[2]

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1 Introduction

The English musical composer, Emily Howard had a piece, "Calculus of the Nervous System", performed at the 2012 BBC Proms, inspired by Ada Lovelace, Lord Byron's daughter, who tried to develop a mathematical model for how the brain gives rise to thought and nerves to feelings. Emily has combined cello-playing with her academic love of mathematics. She is a graduate of the University of Oxford (Lincoln, 1997). She says that "many of my compositional techniques are derived from mathematics or science, and these ideas give structure to my work. 'Calculus of the Nervous System' works as a sequence of memories emerging. Like thoughts there is no continuous narrative; memories occur and recur in different ways. Questions about memory processes play an increasingly important role in my work" [1]. Mathematics and the perception of beauty is just one part of the jig-saw which liberal arts form in education.

If the question had been "can mathematics have a place in the liberal arts?", then the recent paper [2] should have answered the question for you. "Does Mathematics have a Place in the Liberal Arts" on the other hand can only be answered after we have considered the nature and scope of the liberal arts. As an academic with interests in number theory and logic [3], I am in favour of both mathematics and the liberal arts, for their own value as well as for their applications and general utility and inherent beauty.

What this paper attempts is to unravel the role of the liberal arts and to illustrate their value in education *through* liberal arts as well as through training *in* mathematics. *Why* this is attempted is because of a danger of university research league tables encouraging the development of highly trained technocrats at the expense of deeply educated professionals. *How* this is done in this paper will be quantitatively more about liberal education, but, in a sense, qualitatively it will be on mathematics spurred on by the challenge of 'to what mathematics should the educated professional of the twenty-first century be sensitized?"

2 Liberal Arts

The Liberal Arts deal with the human being as a whole and hence with what lies at the essence of being human. It is a common mistake to blame the current materialism and moral decline of the Western world on its extraordinary technological achievements, as if a scientific-technological outlook on life were incompatible with the supremacy of spiritual values. The decline has occurred partly because of the gradual displacement and internal disorientation of properly conceived liberal arts programs which should occupy the foundation of secondary and tertiary education.

I can recall a philosophy teacher of mine who considered both *liber* (free) and *libra* (balance) in the etymology of “deliberate” in lectures on free-will. There is a sense too in which these words can be applied to Liberal Education which should free the mind to be open to a balanced view of the things that matter in life so that people can make decisions freely with information [4]. Nussbaum puts it well when she says that “cultivated capacities for critical thinking and reflection are crucial in keeping democracies alive and wide awake” [5].

According to Professor David J. Walsh, one often finds in American higher education no clear idea of the end result to be aimed at. In most universities there is "an assemblage of incoherent, fragmentary disciplines and sub-disciplines... without any clearer guidance than some vague commitment to methodological requirements within the separate fields" [6]. In short, there is no "unifying sense of direction” [7]. But this phenomenon is not only a crisis of educators; it is a "crisis of knowledge." Contemporary education is frequently confused because, despite often vast knowledge in particular fields, many scholars lack knowledge of what matters most of all, namely the purpose of human existence, a question from which too many wish to flee.

Again according to Walsh, "the clearest evocation of paradigmatic excellence" has traditionally been found within the cluster of disciplines called the Liberal Arts. Today, however, "education in the Liberal Arts has sadly very little to do with the formation of existential purpose; ... it has generally devolved into an increasingly irrelevant discussion of 'ideas,' 'theories,' methods and techniques." In other words, teachers of the Liberal Arts have lost their sense of vocation. Too often, so-called liberal studies can merely be a smorgasbord of subjects from which students can choose and which they tick off as they meet degree requirements. There is little sense of their inter-relations and intimate connections which a genuinely integrated program can provide.

3 Liberal Education

Great literature speaks both to the heart and to the mind, as do all the arts when true to their proper nature. Great literature conveys a vision of truth and beauty and moral excellence capable of raising the spirit of the reader to unsuspected heights, even in the most unpromising circumstances [8]. This has been demonstrated time and time again. One recalls the dramatic effect the reading of Cicero's (now lost) work, *Hortensius*, had on the young Augustine, kindling in him a passion for wisdom that was to inspire his whole life [9]. For Cicero the liberal arts formed the basis of one's "*humanitas*."

In ancient Greek and Roman cultures the liberal arts referred to intellectual arts as distinct from the mechanical arts, the arts of the hand. It may be that the Greeks, in particular, exaggerated the distinction between the two kinds of art, and that the ancients in general demeaned manual work as servile. This appeared to be a mistake to many who came later, but they should not forget that the Greek conviction that reality is intelligible made possible the modern scientific revolution of which we are the heirs [10]. In ancient times the seven liberal arts were the trivium and the quadrivium united by metaphysics and theology, with Truth, Beauty and Goodness as the supporting tripod.

- The *trivium* consisted of
  - Grammar → basic systematic knowledge, the 'what'
  - Rhetoric → the 'how'
  - Logic → the 'why'

- The *quadrivium* consisted of
  - Arithmetic → number in itself
  - Geometry → number in space
  - Music → number in time
  - Astronomy → number in space and time

- Illuminated by
  - Metaphysics → ultimate reality
  - Theology → ultimate end

- Permeated by
  - Truth
  - Beauty
  - Goodness.

We see how the quadrivium was fundamentally the mathematical foundation for the trivium. What should a modern quadrivium consist of for the educated professional of the twenty first century? Perhaps, by analogy with the ancient quadrivium:

- Discrete mathematics
- Projective geometry
- Astronomy
- Statistics?

This can, and should be, a source of debate and argument!

The modern trivium can take care of itself in a sense, especially if we include fuzzy logic! Educated people like to think that they are literate, but do not seem to mind boasting at dinner parties that they are not really numerate. Nevertheless, contrary to the popular impression that the arts are a "soft option," true liberal education is actually very demanding. For this reason, few people have attained so noble and so realistic an understanding of human affairs as for instance, Thucydides and Aristotle. Educational reformers would do well to consider including *The Peloponnesian War, The Nicomachean Ethics,* and *The Politics* in the reading program of students who are bent upon careers in public (or civil) service which, fundamentally, is a noble vocation [11].

In the final analysis, Liberal Arts education is moral education - moral "in the most fundamental sense of forming the core of the personality, that underlying sense of direction and purpose from which the entire life's pattern of thoughts, decisions and actions arise" (Walsh, 1985). The intense moral vision of a single great artist has far more potential for changing hearts and minds than the pietistic clichés of a thousand preachers. For the artist speaks with the authority conveyed by suffering in the cause of truth, beauty and goodness and the realisation of one's personal mediocrity and even nothingness, a recognition that inspires struggle to rise above all limitations.

But perhaps teachers of the Liberal Arts have not met so well their task of putting students in direct contact with the minds of such great men and women. Have they, perhaps, interposed themselves, their lecture notes, their textbooks, between their students and true genius? Of course, if there is a fault to be found it is not theirs alone. Sometimes, too, required assignments are so heavy that students have little time to "waste" with books that require meditation more than memorization.

An even greater obstacle is the general lack of interest in today's world in genuine reading. No one doubts the capacity of most students to "read" - even far into the night – in order to pass an impending examination or to submit an urgent assignment. But librarians can attest that they rarely encounter university students engaged in serious leisure reading. Whether electronic or paper, newspapers, paperback romances or thrillers, and spiritualistic morale-boosters probably constitute the main course, and even the only course, of any extracurricular reading students do. Walking city streets is now hazardous because so many are being entertained by their "smart" phones! [*cf.* 12].

What *is* to be deplored, however, is the narrowly pragmatic attitude which reduces all of education to the "training of manpower" in the service of the national economy. For that attitude loses sight of the priceless value of every unique individual; no person can live "by bread alone." Degrees are often seen only as meal tickets for earning a living, not also as means for living.

But can "liberal arts" and "liberal education" be defended in a time of serious post-Covid-19 economic recession? Is it, perhaps, at best a luxury that society cannot afford at the present time? Yet Professor Christopher Wolfe of Marquette University defines liberal education as "an introduction to the ordered pursuit of the truth about reality". It "seeks knowledge of the inter-related and integrated whole, as against the study of isolated bits of information about the whole." Its aim is "knowledge for its own sake, a good in itself (like health), something intrinsically desirable" [13].

Thus, peculiar to the liberal arts is their cathartic function: they help people to look at reality – and especially at themselves - in the face. This not only dispels illusions but also does the constructive service of inspiring magnanimity and many other virtues which are indispensable in one way or another to development to development as a person, and therefore as a citizen.

The liberal arts also make an effective and unique contribution to human communication. We do not need to be reminded how many official pronouncements and statements written by public functionaries are notorious for their ambiguity, vagueness, and banality. By contrast, we also know how one well-conceived cartoon can get more across than dozens of speeches. It is precisely because good art is such an effective communicator that there are circumstances when it is the best way to engage in social criticism and the best defence against tyranny. As Aleksandr Solzhenitsyn has said, "Wherever else it fails, Art always has won its fight against lies, and it will always win... The lie can withstand a great deal in this world but in cannot withstand Art" [14].

4 Metaphysics

Nevertheless, if the arts undoubtedly express values and make them shine, they cannot create or justify those values. The great artist is simply the one who can see more deeply into reality than the rest of us, who can catch some hidden truth, and successfully translate it into understandable language. The arts are not an end in themselves; they naturally point us toward a realm above them, to philosophy and its pursuit of the ultimate causes of all things. It is the "first philosophy" (metaphysics) which leads us to the foundation of those values which the liberal arts endeavour to explain and illustrate.

Metaphysics is at the heart of that liberal education which prepares the student for living. "Metaphysics is not just one among many subjects in philosophy. Unlike other fields, such as epistemology and aesthetics, metaphysics takes priority. Furthermore, metaphysics is a unique field of knowledge for culture and civilization. Arguably, the very identity and well-being of a civilization depends on whether it accepts metaphysics as a fundamental way of knowing… metaphysics, in its effort to grasp reality, is ultimately responsible for *explaining* how reason knows what is real” [15].

Metaphysics provides the rational basis and the ordering scheme by which all of the arts and sciences can be understood in their rightful internal autonomy. It does this by clarifying the first principles which each discipline takes for granted as it pursues its proper objects [16]. Here they have the reason why Aristotle reserved for metaphysics the title of Wisdom. This was later expanded by the medieval thinkers into the maxim, "*non est consenescendum in artibus*" (one does not grow old in the arts) [17]. If, as they thought, the beginning of wisdom is to be found in the liberal arts of the *trivium* and the *quadrivium,* Wisdom itself was only to be reached (on the level of human knowledge) in metaphysics.

The discipline of philosophy enjoys so little esteem and is understood so poorly that most people will laugh at the assertion that wisdom is only to be found within it. It cannot be surprising that philosophy invites little respect when the Liberal Arts in general are not regarded as very serious subjects. But if this matter is so important, so fraught with consequences, we must try to pursue the reasons for such superficial views. Why have education specialists and educational reformers failed to appreciate the supremely formational role of a liberal education capped by philosophy? Why do so many supposedly well-educated people resort instead to "spiritualistic" sects based principally on sentiment?

History can help us here by contrasting the esteem in which the liberal arts were held in remote times and places. Raymond Klibansky is referring in the following passage from the early medieval School of Chartres: “The seven liberal arts together give man both knowledge of the divine and the power... to express it. But in doing so, they fulfil at the same time another purpose. They serve *ad cultum humanitatis*, that is, they promote the specifically human values, revealing to man his place in the universe and teaching him to appreciate the beauty of the created world” [18].

And what was the “humanism” which the medievalist Richard William Southern considered not only typical of the period from 1100 to 1320 but even as the catalyst of what later came to be known as the Renaissance? That humanism connoted for Southern an emphasis on human dignity and reason, a recognition of the order and intelligibility of nature [19].

Sometimes historians are accused of advocating a return to the past. That, of course, is neither possible nor desirable [20]. But one can advocate the recovery of an attitude which is open to past achievements and in particular to the understanding of wisdom as the most perfect, the most noble, the most useful, and the most joyful of all human pursuits.

In a pragmatic and relativistic society such as ours, the effort to restore metaphysics and a realist philosophy in general might seem doomed from the start. Nevertheless, one who is disposed to make the attempt can surely gain inspiration from the reflection that reformers always have to swim against the tide of contemporary fashion and prejudice. A remarkable example of upstream swimming is provided by Robert Maynard Hutchins, President of the University of Chicago between 1929 and 1951, and an advocate with Mortimer Jerome Adler of “The Great Books of Western Civilization” [21]. A few words on Hutchins can serve to emphasise these points in the context of this paper [22].

Soon after becoming President of the University of Chicago, Hutchins took the highly unorthodox and unusual decision “to leave undone a vast number of things that university presidents can do” in order to dedicate himself to self-education and research. He had become convinced that despite being a graduate of Yale University and Yale Law School he was still “uneducated,” and that “as a man to whom had been confided one of the major educational posts in the country, he should begin by trying to get an education.” He thus set about educating himself “with the help of some great books that have endured, and with twenty to thirty undergraduates as fellow students, selected from each incoming class for their promise.” This was the beginning of a four-year course in the Great Books.

This self-imposed course enabled him to bring about within the University of Chicago an intellectual and educational revolution that would make that university one of the best, not only in the United States but in the world. (Gress has a slightly different view on the Great Books approach to Western Civilisation which is well worth noting but it would be too much of a digression to pursue here [23].) Hutchins stressed that the great works of the distant past provide a scale of values against which to judge the activities and educational offerings of his own age, and that these older values have an intellectual and scientific solidity which the contemporary pragmatist and relativist mentality can hardly imagine. He believed that metaphysics, as conceived by Aristotle, can help people have the foundations for the acquisition of wisdom in ways that applied studies, which are concerned with the particular, cannot.”

5 Conclusion

Henri Bergson once said, “The body, now larger, calls for a bigger soul.” This remark seems very relevant to education in the western world today. The material structure of education is now much larger and more elaborate than it was when I was an undergraduate. But our material growth has not been matched by a corresponding spiritual growth. I assert that one of the reasons for this is the neglect of the liberal arts and failure to delve deeply into the purpose of education which has characterized educational planning in recent years. The ultimate purpose should be the starting point of such planning!

But to put this situation right, it will not be enough merely to give more prominence to arts subjects (it is not the quantity that matters); it will be necessary to see the liberal arts – and the sciences, as well – as parts of an integrated larger whole, capped by philosophy ("first philosophy"). Philosophy and mathematics can be natural companions in undergraduate education, and I was fortunate enough to have experienced this at the University of Sydney in the 1950s.

In this way it is possible to achieve what otherwise might seem impossible, namely "not growing old in the arts," but remaining true to the dreams of their youth [24], being truly young at heart. In the last analysis, immersion in the liberal arts in a connected way is a strong foundation for any profession and is at the heart of the humanities, a means of living a full human life.

All of this begs the question about who can teach in an integrated liberal arts program? Based on the foregoing such teachers should not only be scholars in one of the strands of the Liberal Arts but also be able to demonstrate an appreciation of the integration of the strands appropriate to a Liberal Arts program. The whole program has to be greater than the sum of the parts, in much the way that strands of string can produce a strong rope. It is not a matter of ticking the boxes as subjects are passed. This happens in a la carte liberal studies courses in some universities.

In answer to the question which was posed at the beginning of this paper it should be clear by now that the answer is "yes", but it depends on how it is taught, who teaches it, and how it is integrated with the rest of the curriculum. An example of this would be someone like the great British mathematician, G.H. Hardy, a number theorist who tried to put rigour into analysis, who was a pure mathematician whose guiding light was beauty and elegance, with strong links to history and logic, necessary ingredients for passion and inspiration in a teacher [26]. Ironically, much of the work of this very pure, pure mathematician was superseded by the Bourbaki movement, and he is most likely to go down in history for an application [27]!

(The sentiments and structure of the reflections in this conclusion are based on John Joiner White, a founder of Strathmore University in Kenya, that country's first multiracial educational foundation.)

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Low Solar Activity, Winter Flu Conditions, Pandemics and Sex Wars: A Holistic View of Human Evolution

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Abstract

The current spread of coronavirus has caught our modern world by surprise, which leads to widespread panic, fear and confusion. However, if we view the unfolding of these events from a scientific historical perspective of past human evolution, we may discover the reoccurring patterns of the environmental conditions that give rise to such epidemics. Hence, we can figure out better methods to prepare and react to the infectious agents that spread diseases that have shaped the course of human history before. Here, I propose a holistic view of human evolution, with an interdisciplinary approach that studies how cyclic variation in solar UV energy affects the evolution of viruses and shapes the symbiotic dynamics of human life on earth.
Keywords: Epidemics; cosmo-climatology; evolutionary biology.†

1. Introduction

In the *Nature* article, "How Pandemics Shape Social Evolution," Laura Spinney (2019) reviews Frank Snowden's book, *Epidemics and Society: From the Black Death to the Present* (2019), that looks at how infectious diseases have affected human culture. Spinney writes on the cyclic versus linear view of human historical progress in respect to dealing with epidemics:

> *When will we learn never to declare the end of anything? Only 50 years ago, two prominent US universities closed their infectious-disease departments, sure that the problem they studied had been*

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> *solved. Now, cases of measles and mumps are on the rise again in Europe and the United States, new infectious diseases are emerging at an unprecedented rate, and the threat of the next pandemic keeps philanthropist Bill Gates awake at night.*
>
> *So it's a shame that to make this point,* Epidemics and Society, *Frank Snowden's wide-ranging study on this rolling human reality, repeats the urban myth that in 1969, US surgeon-general William Stewart said, "It is time to close the book on infectious diseases, and declare the war against pestilence won." Even though Stewart never said this, it's clear that there was a pervasive, dangerously complacent attitude in the late 1960s. International public-health authorities were predicting that pathogenic organisms, including the parasite that causes malaria, would be eliminated by the end of the twentieth century. Snowden's broader thesis is that infectious diseases have shaped social evolution no less powerfully than have wars, revolutions and economic crises.*

2. Solar cycles and pandemics

My recently published article, "Solar Cycles, Light, Sex Hormones and the Life Cycles of Civilization: Toward Integrated Chronobiology" (Barzilai, 2019), asserts a causal relation between solar activity cycles and the human sexual dynamics through the epochs of history. Here, I would like to further expand on this theory to propose the causal relation of viral pandemics and plagues and the central role they play in shaping human evolution through the epochs of history.

This article on astrobiology, "Sunspot Cycle Minima and Pandemics: A Case for Vigilance at the Present Time" (Wickramasinghe et al., 2017) suggests that past recurring cases of pandemics are caused by period of low solar activity:

> *Direct records of sunspots and the solar cycle have been maintained in astronomical observatories for about 1610AD, while indirect records derived from 14 C analysis of ice cores go back to about 900AD. Minima in the sunspot cycle present conditions conducive to the entry of viruses and bacteria to the Earth and also for mutations of already circulating pathogens. Three grand minima of solar activity on record – the Sporer minimum (1450-1550AD), Maunder minimum (1650-1700AD) and the Dalton minimum (1800-1830) have all been marked by a preponderance of pandemics – Small Pox, English Sweats, Plague and Cholera.*

> *The sunspot numbers recorded for the present period 2002-2017 include the deepest sunspot minimum (Cycle 23-24) since records began, and a trend to declining numbers throughout the cycle. The same period has seen the resurgence of several pandemics – SARS, MERS, Zika, Ebola, Influenza A. We consider it prudent to take note of these facts whilst planning future strategies for pandemic surveillance and control*
>
> *Lists of conspicuous pandemic and epidemic events during the period 2002-2015:*
>
> *2010-2016 Scarlet fever (S. pyogenes)*
> *2015 Zika*
> *2014 Ebola*
> *2013 Influenza A H7N9*
> *2012 MERS*
> *2009 Influenza H1N1(China, India, Sri Lanka)*
> *2002 SARS*
> *The Influenza subtype H1N1, which was involved in both the 1918/1919 pandemic and the epidemics in 1976/1977, reappeared in 2009 in India, China and elsewhere. In 2017 it was raging across Sri Lanka and neighboring countries. In 2013 the Influenza subtype H7N9 appeared first in birds and later spread globally.*

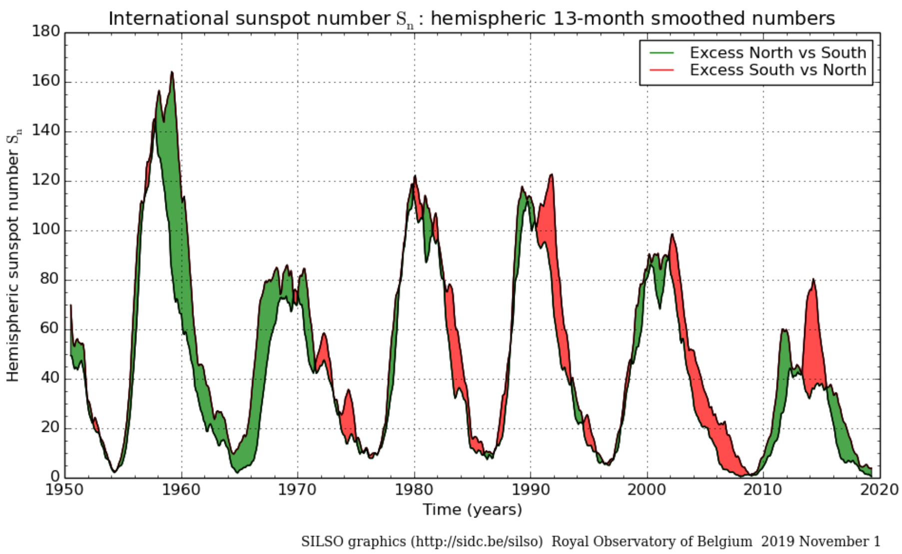
According to this analysis of cycles of solar trends, we are now at a very high risk of a new pandemic, similar to the Spanish flu pandemics of 1918, which occurred in similar solar minima cyclic conditions to current events. The article, "Origin and Virulence of the 1918 'Spanish' Influenza Virus" (Taubenberger, 2006) states:

> *The "Spanish" influenza pandemic of 1918–19 caused acute illness in 25–30 percent of the world's population and resulted in the death of up to an estimated 40 million people. Using fixed and frozen lung tissue of 1918 influenza victims, the complete genomic sequence of the 1918 influenza virus has been deduced. Sequence and phylogenetic analysis of the completed 1918 influenza virus genes shows them to be the most avian-like among the mammalian-adapted viruses. This finding supports the hypotheses that (1) the pandemic virus contains genes derived from avian-like influenza virus strains and that (2) the 1918 virus is the common ancestor of human and classical swine H1N1 influenza viruses.*

In 2098 NASA published this report on a substantial drop in solar activity (Philips, 2009), which led to the 2009 Influenza H1N1 outbreak. NASA asserted that the solar conditions mirrored those leading to the environment of 1918 great pandemic:

> *2008 was a bear. There were no sunspots observed on 266 of the year's 366 days (73 percent). To find a year with more blank suns, you have to go all the way back to 1913, which had 311 spotless days. Prompted by these numbers, some observers suggested that the solar cycle had hit bottom in 2008.*
> *Careful measurements by several NASA spacecraft have also shown that the sun's brightness has dimmed by 0.02 percent at visible wavelengths and a whopping 6 percent at extreme UV wavelengths since the solar minimum of 1996. These changes are not enough to reverse global warming, but there are some other, noticeable side-effects.*

As we see in the following chart, solar activity has been in a declining trend since its peak the late 1950s. In the late 1970s this trend caused the HIV global pandemic, the SARS in 2002 and H1N1 outbreak in 2009, occurring with a global recession, and the 2019 Coronavirus epidemic, both near historically significant solar minimums. Actually, the optimistic view described by Spinney (2019) of humanity's victory over epidemics in the 1950s and into the 1960s, was the result of high solar activity leading to decline in the prevalence of epidemics.



SILSO graphics (http://sidc.be/silso) Royal Observatory of Belgium 2019 November 1

In addition, major historical pandemics and plagues, such as the Justinian plague of 536 CE during the cold period called the Dark Ages, and the Black Death of 1350s, are related to low levels of solar activity:

> *Winter was coming. In AD 536, the first of three massive volcanic eruptions ushered in a mini ice age. It coincided with an epidemic of the plague, the decline of the eastern Roman Empire, and sweeping upheavals across Eurasia.*
> *Now we have the first evidence that the disruption to climate continued a lot longer than a decade, as was previously thought. The extended cold period lasted until around 660, affecting Europe and Central Asia, and perhaps the rest of the world too. (Sarchet, 2016)*

The Black Plague in the fourteenth century occurred during the Little Ice Age, a period of cooling that began after the rise of civilization from the Dark Ages into the Medieval Warm Period. Starting in the thirteenth century, global cooling began due to low solar activity over the next centuries (1200-1450). This also led to the invasion of Mongol hoards in a series of devastating wars of conquest, murder, and rape that destroyed much of Chinese and Islamic civilizations as well as Russia in Eastern Europe. This is described in this article, “Little Ice Age Wetting of Interior Asian Deserts and the Rise of the Mongol Empire” (Putnam, et al., 2016):

> *Wetter-than-present conditions characterized the core of the inner Asian desert belt during the Little Ice Age, the last major Northern Hemispheric cold spell of the Holocene. These wetter conditions accompanied northern mid-latitude cooling, glacier expansion, a strengthened/southward-shifted boreal jet, and weakened south Asian monsoons. We suggest that southward migration of grasslands in response to these wetter conditions aided the spread of Mongol Empire steppe pastoralists across Asian drylands.*

Now that the relation between low solar activity and pandemics has been demonstrated, we shall continue to explore the possible mechanism that causes these effects.

3. Cosmo climatology: effects of solar activity on our climate

Henrik Svensmark, director of Center for Sun-Climate Research in the Danish National Space Center, has discovered a way in which low solar activity affects our climate. This is caused by decline in solar shielding of cosmic radiation that reaches earth, leading to greater cloud cover, leading to more winter conditions that bring greater levels of precipitation, as described in the article “Cosmoclimatology: A New Theory Emerges (Svensmark, 2007):

> *The title reflected a topical puzzle, that of how to reconcile abundant indications of the Sun's influence on climate, with the small 0.1% variations in the solar irradiance over a solar cycle measured by satellites. Clouds exert (on average) a strong cooling effect, and cosmic-ray counts vary with the strength of the solar magnetic field, which repels much of the influx of relativistic particles from the galaxy. The connection offers a mechanism for solar-driven climate change much more powerful than changes in solar irradiance.*

4. The effect of UV solar radiation on regulating germs in the atmosphere

The reason that flu pandemics usually occur during winter is that solar radiation causes both greater humidity in the air, which serves to dampen particles floating in the air into the ground, and also strong UV radiation kills germs and viruses in the atmosphere, reducing the risk of infection. The article, “Inactivation of Influenza Virus by Solar Radiation” (Sagripanti & Lytle, 2007), proposes that high solar activity during the summer is responsible for low cases of influenza:

> *Influenza virus is readily transmitted by aerosols and its inactivation in the environment could play a role in limiting the spread of influenza epidemics. Ultraviolet radiation in sunlight is the primary virucidal agent in the environment but the time that influenza virus remains infectious outside its infected host remains to be established. In this study, we calculated the expected inactivation of influenza A virus by solar ultraviolet radiation in several cities of the world during different times of the year. The inactivation rates reported here indicate that influenza A virions should remain infectious after release from the host for several days during the winter “flu season” in many temperate-zone cities, with continued risk for reaerosolization and human infection. The correlation between low and high solar virucidal radiation and high and low disease prevalence, respectively, suggest that inactivation of viruses in the environment by solar UV radiation plays a role in the seasonal occurrence of influenza pandemics.*

The following quote from the blog post, "The Reason For The Season: Why Flu Strikes in Winter" (Foster, 2014), further elaborates on the scientific findings that "The Flu Likes Cold, Dry Weather":

> *For many years, it was impossible to test these hypotheses, since most lab animals do not catch the flu like humans do, and using humans as test subjects for this sort of thing is generally frowned upon. Around 2007, however, a researcher named Dr. Peter Palese found a peculiar comment in an old paper published after the 1918 flu pandemic: the author of the 1919 paper stated that upon the arrival of the flu virus to Camp Cody in New Mexico, the guinea pigs in the lab began to get sick and die. Palese tried infecting a few guinea pigs with influenza, and sure enough, the guinea pigs got sick. Importantly, not only did the guinea pigs exhibit flu symptoms when they were inoculated by Palese, but the virus was transmitted from one guinea pig to another.*
>
> *Now that Palese had a model organism, he was able to begin experiments to get to the bottom of the flu season. He decided to first test whether or not the flu is transmitted better in a cold, dry climate than a warm, humid one. To test this, Palese infected batches of guinea pigs and placed them in cages adjacent to uninfected guinea pigs to allow the virus to spread from one cage to the other. The pairs of guinea pig cages were kept at varying temperatures (41°F, 68°F, and 86°F) and humidity (20%-80%). Palese found that the virus was transmitted better at low temperatures and low humidity than at high temperatures and high humidity.*

Radiologists have also discovered that artificial far-UVC radiations can be usefully applied to eliminate viral and bacterial infectious agents from the air, causing minimal damage to the skin, inventing the groundbreaking medical device of *Far*-UVC light: A new tool to control the spread of airborne-mediated microbial diseases:

> *Airborne-mediated microbial diseases such as influenza and tuberculosis represent major public health challenges. A direct approach to prevent airborne transmission is inactivation of airborne pathogens, and the airborne antimicrobial potential of UVC ultraviolet light has long been established; however, its widespread use in public settings is limited because conventional UVC light sources are both carcinogenic and cataractogenic. By contrast, we have previously shown that far-UVC light (207–*

222 nm) efficiently inactivates bacteria without harm to exposed mammalian skin. This is because, due to its strong absorbance in biological materials, far-UVC light cannot penetrate even the outer (non living) layers of human skin or eye; however, because bacteria and viruses are of micrometer or smaller dimensions, far-UVC can penetrate and inactivate them. We show for the first time that far-UVC efficiently inactivates airborne aerosolized viruses, with a very low dose of 2 mJ/cm2 of 222-nm light inactivating >95% of aerosolized H1N1 influenza virus. Continuous very low dose-rate far-UVC light in indoor public locations is a promising, safe and inexpensive tool to reduce the spread of airborne-mediated microbial diseases. (Welch, et al., 2018)

5. Red queen hypothesis: sexual origins of social conflict

As the integrated chronobiology (Barzilai, 2019) article suggests, the low solar activity environment also causes social and political conflict driving the course of our sociocultural evolution. The 1918 Spanish flu pandemic coincided with WWI, the communist revolution in Russia in 1917, and the formation in the Nazi-Socialist party in Weimar Germany in 1919. We have been experiencing similar trends in recent years with the rise of far-right nationalist and far-left communist political movements throughout the world, leading to a conflict between inclusive globalism trends and xenophobic nationalist figures.

These political movements express opposing social attitudes, as communism seeks to unite all of the human race under one empire that dissolves national and ethnic boundaries, is ideologically conflicting with Nazi-Socialism that seeks for one race to establish dominance and even eradicate other ethnic groups. However, both ideologies seem to similarly arise in period of increasing social tensions that lead to conflict and phase transitions in human social organization. This in-group versus out-group dynamics may be a product of sexual drives as a means of social evolution in reaction to changing environmental conditions.

For example of the of war for empire as a means to achieve sexual recombination of genes across different population groups, it has been established through genetic science that great conquerors in history, such as the powerful Mongol leader Genghis Khan, have used their empires to father many children with numerous wives and concubines to spread their seed through their great territories. This has been described in the article on Genghis Khan’s genetic legacy’s competition (Callaway, 2015):

> *The Mongolian leader left a strong footprint in the Y chromosomes of modern descendants — but he was not the only one.*
>
> *Millions of men bear the genetic legacy of Genghis Khan,the famously fertile Mongolian ruler who died in 1227. Researchers have now recognized ten other men whose fecundity has left a lasting impression on present-day populations. The team's study points to sociopolitical factors that foster such lineages, but the identities of the men who left their genetic stamp remains unknown.*
>
> *The case for Genghis Khan's genetic legacy is strong, if circumstantial. A 2003 paper led by Chris Tyler-Smith, an evolutionary geneticist now at the Wellcome Trust Sanger Institute in Hinxton, UK, discovered that 8% of men in 16 populations spanning Asia (and 0.5% of men worldwide) shared nearly identical Y-chromosome sequences. The variation that did exist in their DNA suggested that the lineage began around 1,000 years ago in Mongolia.*

This suggests that social tensions between human groups that lead to geopolitical conflicts and even world wars, may arise by sexual dynamics of in-group versus out-group genetic populations. Hence, these sociocultural militarist trends may originate in the evolution of sexual reproductive strategies that evolved in reaction to viral epidemics in our biological history. Sex wars that rise in reaction to changing environmental conditions evolved as a biological adaptation to parasitic infections, may have originating in mutating viruses and bacteria attacking a specie population (during periods of low solar activity). The Red Queen hypothesis explains the origins of sex as a product of the need for sexual recombination of genes in a specie's populations in order to combat mutating infectious disease agents:

> *Most organisms reproduce through outcrossing, even though it comes with significant costs. The Red Queen hypothesis proposes that selection from coevolving pathogens facilitates the persistence of outcrossing in spite of these costs. We utilized experimental coevolution to test the Red Queen hypothesis, and found that coevolution with a bacterial pathogen (Serratia marcescens) resulted in significantly more outcrossing in mixed mating experimental populations of the nematode Caenorhabditis elegans. Furthermore, we found that coevolution with the pathogen rapidly drove obligately selfing populations to extinction, while*

> *outcrossing populations persisted through reciprocal coevolution. Thus, consistent with the Red Queen hypothesis, coevolving pathogens can select for biparental sex. (Morran et al., 2011; Brockhurst et al., 2014)*

The mutual role that sex wars and pandemics play through human history can be well illustrated during the dreadful period of Black Plague, in which germs were also used for biological warfare by the Mongols in the rage for global conquest to establish a huge empire:

> *On the basis of a 14th-century account by the Genoese Gabriele de' Mussi, the Black Death is widely believed to have reached Europe from the Crimea as the result of a biological warfare attack. This is not only of great historical interest but also relevant to current efforts to evaluate the threat of military or terrorist use of biological weapons. This theory is consistent with the technology of the times and with contemporary notions of disease causation; however, the entry of plague into Europe from the Crimea likely occurred independent of this event. (Wheelis, 2002)*

6. Human evolution, retroviruses and punctuated equilibrium

The classical Darwinian theory of evolution suggests the evolution is a gradual process of accumulation of mutations through natural selection and the competition for survival of the fittest individual organisms. However, this view was contested during the period of low solar activity during the 1970s from two rising fronts.

In 1972, paleontologists Niles Eldredge and Stephen Jay Gould published a landmark paper "Punctuated Equilibria: An Alternative to Phyletic Gradualism," developing their theory by studying fossil records. They revolutionized our conception of evolution by stating that it is composed of periods of prolonged stability, called *statis*, punctuated by sudden events of significant evolutionary change, in which species split into new distinct species, rather than by gradual transformation.

The second revolution was formalized by renowned evolutionary biologist, Lynn Margulis, also during this period, offering a new revolutionary theory of *symbiogenesis*, causing different individual organisms to unite through symbiotic or parasitic relationship into a single greater complex new organism. Her now accepted theory of endosymbiogenesis states that complex eukaryotic cells were formed by symbiosis of ancient prokaryotes that led to the mitochondria, functioning as the energy producing organelles in our animal cells.

This new paradigm of rapid evolutionary transformations can now be explained through the discovery of retroviruses that enter a cell, and have the ability transform their RNA into the DNA of that cell, through a reverse transcription process, hence changing the genome of that cell. The article, “Retroviruses Shows That Human-Specific Variety Developed When Humans, Chimps Diverged” states the theory that retroviruses are a meaningful component in the giant leaps required for human evolution:

> *The idea of a relatively sudden genetic change that alters evolution isn’t new. Scientists, such as the late Stephen Jay Gould, proposed a mechanism called “punctuated equilibrium” more than two decades ago. This idea, not yet completely accepted by scientists, proposes that evolution has depended more often on sudden and unexpected changes in genomes rather than a simple Darwinian paradigm of gradual evolutionary change due to extremely long-term natural selection. (University of Georgia, 2002; see also Gemmell et al, 2015)*

Another article, “Transposable Elements and an Epigenetic Basis for Punctuated Equilibria” (Zeh, et al., 2009) proposes that retroviruses and epigenetic switches play a vital role in the punctuated equilibrium theory of human evolution:

> *Evolution is frequently concentrated in bursts of rapid morphological change and speciation followed by long-term stasis. We propose that this pattern of punctuated equilibria results from an evolutionary tug-of-war between host genomes and transposable elements (TEs) mediated through the epigenome. According to this hypothesis, epigenetic regulatory mechanisms (RNA interference, DNA methylation and histone modifications) maintain stasis by suppressing TE mobilization. However, physiological stress, induced by climate change or invasion of new habitats, disrupts epigenetic regulation and unleashes TEs. With their capacity to drive non-adaptive host evolution, mobilized TEs can restructure the genome and displace populations from adaptive peaks, thus providing an escape from stasis and generating genetic innovations required for rapid diversification.*

7. Conclusion

A holistic view of human evolution as a product of sexual reproductive strategies evolved in reaction to changing environmental conditions and changing parasitic threats can form a new paradigm to explain and deal with current epidemics that threaten our modern civilization. This interdisciplinary perspective, integrating energy trends and their effects on biological processes, can provide the ultimate theoretical framework required to better treat these epidemics from a holistic viewpoint.

Medical authorities accustomed to reductionist Western science that is mostly focused on biological organic systems, have hence invested their resources of fighting contagious epidemics to developing vaccines when these dangerous arise. However, the development process of specific vaccinations and other treatments to cure these agents can demand very long periods, in which the epidemic can already cause great numbers of fatalities. Therefore, in addition to these reductionist methods, a holistic approach that seeks to understand the underlying evolutionary mechanism that leads to greater environmental risk of spreading pandemics can result in better results in dealing with these conditions.

As referred to before, new advancement in radiology can supply some required general solutions, as proposed in the article, "Germicidal Efficacy and Mammalian Skin Safety of 222-nm UV Light" (Buonannoa et al., 2017), suggesting we can utilize artificially produced light to counter the spread of viruses, when solar light is too low during cold and dry winter-like conditions*:*

> *We have previously shown that 207-nm ultraviolet (UV) light has similar antimicrobial properties as typical germicidal UV light (254 nm), but without inducing mammalian skin damage. The biophysical rationale is based on the limited penetration distance of 207-nm light in biological samples (e.g. stratum corneum) compared with that of 254-nm light.*

This opens exciting new frontiers in the mutual progress of philosophy, science and medicine to better the human condition, as the Enlightenment philosopher, Francis Bacon, the father of empirical science, who proposed, "an entirely new system based on empirical and inductive principles and the active development of new arts and inventions, a system whose ultimate goal would be the production of practical knowledge for 'the use and benefit of men' and the relief of the human condition" (Internet Encyclopedia of Philosophy, 2020).

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1918-2018: CANTOR and infinity in today's high school

Carlo Toffalori*

Abstract

In the first centenary of Cantor's death, we discuss how to introduce his life, his works and his theories about mathematical infinity to today's students.

Keywords: proper and improper infinite, cardinal number, countable set, continuum, continuum hypothesis.

Sunto

Nel primo centenario della scomparsa di Cantor, si discute come presentare la sua vita, le sue opere e le sue teorie sull'infinito agli studenti di oggi.

Parole chiave: infinito proprio e improprio, numero cardinale, numerabile, continuo, ipotesi del continuo.†

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Carlo Toffalori

1. L'infinito pensabile

"Dal Paradiso che Cantor ha creato per noi, nessuno deve poterci mai scacciare": l'aforisma di David Hilbert nel saggio *Sull'infinito* (Hilbert, 1978) è talmente conosciuto che citarlo ancora una volta può sembrare perfino scontato. Non vedo tuttavia modo migliore di celebrare la ricorrenza 1918-2018 del primo centenario della scomparsa di Georg Cantor e la sua teoria dei numeri transfiniti – *"il fiore più bello dello spirito matematico e in generale una delle più alte prestazioni dell'attività puramente intellettuale dell'uomo",* per affidarsi nuovamente alle parole di Hilbert. Non che la strada cantoriana verso l'infinito, cioè il duplice concetto di numero ordinale e cardinale, sia l'unica scientificamente possibile. Esistono autorevolissimi approcci alternativi quali

- i *numeri iperreali*, o anche solo *iperinteri*, dell'analisi non standard di Abraham Robinson (Robinson, 1996), (Keisler, 1982),
- i *numeri surreali* di Conway (Conway, 2001), che del resto accolgono tra di sé pure gli ordinali,
- *numerosità* (Benci et al., 2012) e *grossoni* (Sergeyev, 2015)

e via dicendo (Lolli, 2012), (Benci & Freguglia, 2017). A proposito: nel 2018 ricorre un altro importante centenario, quello della nascita di Abraham Robinson, poi prematuramente scomparso nel 1974. Prendiamo comunque atto che neppure in tema di infinito c'è in matematica una via regia, meno che mai univoca. Tuttavia la teoria di Cantor è la prima, rivoluzionaria, coraggiosa, memorabile trattazione dell'argomento, e ha influenzato profondamente la matematica del Novecento e dei nostri giorni.

Scrive Ernst Zermelo, nella sua prefazione all'edizione completa delle opere di Cantor da lui curata nel 1932 (Cantor, 2012): *"Nella storia delle scienze è un caso veramente raro che un'intera disciplina di importanza fondamentale sia dovuta all'opera creativa di una sola persona. Questo caso si è verificato con la teoria degli insiemi, create da Georg Cantor"*.

Lo stesso Cantor rivela piena coscienza, e la giusta fierezza, dell'originalità della via da lui intrapresa. Altri prima di lui avevano iniziato a saggiarla: Galileo, solo per fare un esempio, quando aveva osservato certi paradossi dell'infinito (Galilei, 2011), e tra questi che i numeri naturali n sono tanti quanti i loro quadrati n^2, stante la corrispondenza biunivoca $n \to n^2$ che associa gli uni agli altri, e che ciò nonostante li contengono come sottoinsieme proprio. Ma poi, di fronte a simili stranezze, lo scienziato aveva ritenuto *"inconveniente"* addentrarsi nell'indagine. Cantor invece perseguì tenacemente la sua strada, convinto che all'infinito potenziale di Aristotele e a quello inaccessibile di Dio un terzo si potesse affiancare, da lui definito *"proprio"*, *"pensabile"* dalla nostra mente e come tale obiettivo sensato di una investigazione scientifica; che numeri transfiniti si potessero concepire e studiare e, conseguentemente, conteggi infiniti si potessero eseguire.

Agli entusiasmi di Hilbert si contrapposero tuttavia i dubbi e le critiche di chi riteneva improponibile uno studio matematico dell'infinito, come quello stesso Kronecker che di Cantor studente universitario era stato professore: *"Dio creò i numeri interi, tutto il resto è opera dell'uomo"* è la celebre frase che gli viene attribuita, pronunciata probabilmente durante un convegno a Berlino nel 1886. Come pure si possono citare le riserve espresse da un collega famoso come Poincaré all'ambiguità e alla vaghezza del concetto di insieme, così come inizialmente elaborato da Cantor. Scriveva allora Poincaré nel 1908 (Poincaré, 1909): *"Per conto mio io penso, e non sono il solo, che l'importante è di non introdurre mai che delle entità che si possano definire completamente con un numero finito di parole"*.

2. Cantor oggi

Lasciamo pure da parte tutte queste polemiche, ormai largamente superate, e veniamo piuttosto al centenario del 2018. Domandiamoci in particolare quale attualità manifesti il messaggio di Cantor nella scuola di oggi, e quanto sia, non dico raccomandabile, ma almeno appropriato parlarne. Sembra infatti, a proposito degli insiemi, che ridurne l'insegnamento ai diagrammi di Eulero-Venn e ad esercizi su unioni, intersezioni e complementi sia di nessuna utilità per gli studenti e che d'altra parte la teoria cantoriana vera e propria sia troppo complicata per arrischiarsi a introdurla nella scuola superiore. Del resto i ragazzi di oggi sembrano prediligere a speculazioni troppo astratte l'efficienza dei computer e degli smartphone, e la loro capacità di immagazzinare ed elargire quantità formidabili di dati e informazioni. È difficile dar loro torto. Ammoniva l'Amleto di Shakespeare: *"Ci son più cose in cielo e in terra, Orazio, che non ne sogni la tua filosofia"*, e noi potremmo rivolgere idealmente la stessa osservazione a Cantor. Potremmo anzi aggiungere che ancor più cose esistono nel software di un computer.

O non è forse vero il contrario? Scriveva Cantor nelle *Grundlagen*[‡] che il nostro pensiero può eseguire conteggi determinati anche su insiemi infiniti, dove invece a un calcolatore non è dato arrivare. Del resto, quante operazioni elementari può compiere in un secondo il supercalcolatore più potente? Sembrerebbe 10^{17}, cioè un uno con 17 zeri dietro, stando almeno alle informazioni desumibili in rete. Quanti secondi poi sono passati dall'inizio del

[‡] Per gli articoli di Cantor citati nel testo ci affidiamo alla traduzione italiana di Gianni Rigamonti in (Cantor, 2012) o, nel caso dei *Beiträge*, a quella inglese in (Cantor, 1955). Rimandiamo invece a (Lolli, 2013) per maggiori dettagli sull'opera complessiva di Cantor e sui concetti matematici trattati in questa nota.

mondo a oggi? Secondo la teoria del Big Bang, non più di 10^{19}. Dunque un supercalcolatore che avesse lavorato senza sosta per tutto questo tempo non sarebbe arrivato a 10^{36} operazioni. Aggiungiamo che una stima delle particelle elementari dell'universo – le *"cose in cielo e in terra"* di cui parla Amleto – le fissa in al massimo 10^{82}. Ne consegue che se ognuna di esse ospitasse un calcolatore dell'ultimissima generazione, e se questi computer avessero preso a operare tutti assieme all'inizio del mondo e ancor oggi proseguissero il loro lavoro, le operazioni elementari che avrebbero eseguito non supererebbero le 10^{118}. Ma la mente umana, perfino in un ragazzo con una minima dimestichezza con le potenze, può concepire e calcolare pure 10^{120}, cioè cento volte tanto, e molto oltre. Anche i computer, in verità. Ma la mente può immaginare addirittura l'infinito. Hans Enzensberger, l'autore del *Mago dei numeri* (Enzensberger, 2005), scrive a questo proposito nell'altra sua opera *Gli elisir della scienza* (Enzensberger, 2004): *"Pare essere un'idea fissa della pedagogia che i bambini non siano capaci di pensiero astratto. Mentre si tratta di una convinzione infondata. E' semmai giusto il contrario. Il concetto dell'infinitamente grande e dell'infinitamente piccolo è, per esempio, immediatamente accessibile a livello intuitivo per qualunque scolaro di nove o dieci anni"*.

Forse non solo ai bambini, ma pure agli adolescenti sono concesse la medesima capacità di stupirsi e la voglia di approfondire. Esistono in effetti testimonianze di esperienze didattiche che introducono ai ragazzi delle superiori i grandi numeri – come le gocce d'acqua di un oceano o i granelli di sabbia di un deserto – e giungono loro tramite agli infiniti di Cantor. Lo stesso *Mago dei numeri* di Enzensberger, pur rivolgendosi a un ragazzo che all'inizio del libro dichiara di odiare *"qualsiasi cosa abbia a che fare con la matematica"*, tuttavia lo accompagna un poco alla volta non solo all'infinità dei numeri interi, ma anche agli arcani di questa infinità, mostrandogli come al suo interno i pari e i dispari, apparentemente la metà degli elementi possibili, siano in realtà tanti quanti la loro totalità – un caso analogo ai quadrati di Galileo. Di Cantor il *Mago dei numeri* riferisce anche, brevissimamente, il *pulviscolo*, cioè l'insieme più che numerabile di misura nulla che si ritaglia progressivamente dall'intervallo $[0,1]$ rimuovendo anzitutto la parte centrale $]1/3,\ 2/3[$, ripetendo poi l'operazione nelle ali rimanenti $[0,\ 1/3]$ e $[2/3,1]$ e via dicendo.

La strategia per accostare i ragazzi al mondo di Cantor è chiara e definita. Si comincia dall'evidente constatazione che gli usuali numeri naturali consentono di contare gli elementi di un insieme finito, ma non oltre. Si rileva poi, e si conferma facilmente con gli esempi, che due insiemi finiti condividono il numero degli elementi se e solo se si possono mettere in corrispondenza biunivoca. Si osserva infine come quest'ultima proprietà, cioè l'esistenza di una biiezione, si possa considerare, stabilire o smentire, anche per coppie di insiemi infiniti. E allora perché non introdurre i nuovi numeri – in questo caso i *cardinali* – come le classi della relazione di equivalenza tra insiemi che ne associa due esattamente quando si trovano in corrispondenza biunivoca? Troppo complicato per un ragazzo? Ma l'introduzione di numeri più familiari, quali i reali, i razionali, gli stessi interi, se svolta in dettaglio, con la cura dovuta, non è altrettanto astratta, se non di più?

Confrontare, dunque, invece di contare. Anzi confrontare per contare. Riprendendo – noi se non i nostri studenti – le parole famose di Cantor all'inizio dell'articolo *Contributo alla teoria delle molteplicità* del 1878: *"Mi sia concesso, se due insiemi M e N possono essere associati l'uno all'altro in modo univoco e completo, elemento per elemento (cosa che se è possibile in una maniera lo è sempre anche in molte altre), di dire d'ora in poi che tali insiemi hanno uguale potenza, o anche che sono equivalenti"*.

Cantor sottolinea opportunamente subito dopo che insiemi infiniti apparentemente distinti e lontani, addirittura l'uno sottoinsieme proprio dell'altro, possano condividere la stessa potenza – oggi si preferisce dire: la medesima cardinalità. Eventualità impensabile tra gli insiemi finiti, cui basta aggiungere o togliere un elemento per modificarne il numero. Eppure eventualità che ricorre largamente all'infinito, come mostrato dal *Mago dei numeri* e presagito da Galileo, al punto da avvalorare il sospetto che tutti gli insiemi infiniti siano, proprio in quanto tali, in corrispondenza biunivoca tra loro e quindi in definitiva esista un'unica cardinalità infinita. Le dovizie di esempi che intervengono a sostenerlo – di coppie di insiemi infiniti in biiezione tra loro – ne includono alcuni storicamente famosi e sorprendenti, come i due proposti nell'articolo cantoriano del 1878

- $\mathbb{N}$ e $\mathbb{N}^2$ (la così detta *biiezione di Cantor*),
- $\mathbb{R}$. e $\mathbb{R}^2$ (il risultato che provocò il commento meravigliato che Cantor espresse al suo interlocutore Dedekind in una lettera del 1877, *"je le vois, mais je ne le crois pas"*),

come pure altri talora più semplici,

- $\mathbb{N}$ e $\mathbb{N} \setminus \{0\}$ (l'*albergo di Hilbert*),
- $\mathbb{N}$ e $\mathbb{Z}$,
- $\mathbb{N}$ e $\mathbb{Q}$,
- la retta reale e un suo qualsiasi intervallo anche minuscolo,
- $\mathbb{R}^2$ e $\mathbb{C}$.

Tra l'altro, le corrispondenze biunivoche che attestano questi risultati vengono espressamente prodotte caso per caso, fornendo in particolare per quegli insiemi, come $\mathbb{N}^2$, $\mathbb{Z}$ e $\mathbb{Q}$, che esse collegano a $\mathbb{N}$ altrettanti algoritmi effettivi di numerazione, cioè di codifica mediante numeri naturali: circostanza che ha evidenti ricadute nella scienza dei moderni calcolatori, per i quali l'informazione è spesso tradotta in stringhe di cifre, dunque proprio in forma di numero. Molti degli esempi precedenti si possono allora proporre agli studenti con finalità anche pratiche, specie nei casi già segnalati di $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{N}^2$ e $\mathbb{Q}$.

La corrispondenza biunivoca tra retta e segmento aperto è poi un semplice esercizio di trigonometria, poiché si affida alle proprietà della funzione tangente, ristretta all'intervallo $]-\frac{\pi}{2},\frac{\pi}{2}[$, e della sua inversa, l'arcotangente. La biiezione tra $\mathbb{R}$ e $\mathbb{R}^2$ (o $\mathbb{C}$), ossia, per dirla in termini geometrici, tra retta e piano, o anche tra segmento e quadrato, è invece più delicata, ma non per questo meno capace di stupire – e non solo Cantor.

A proposito dei reali, non va poi dimenticato che proprio a Dedekind e a Cantor va attribuito principalmente il merito di averli introdotti rigorosamente a partire dai razionali nel 1872, il primo attraverso le *sezioni*, il secondo attraverso le *successioni di Cauchy*. I numeri reali, e specificamente il metodo di Dedekind, si insegnavano una volta nelle scuole secondarie. Chissà se lo si fa ancora. Confido di sì, perché gli irrazionali, e il teorema di Pitagora che per primo li insinua, sono una svolta della storia del pensiero classico, come giustamente sottolineato da Hardy in (Hardy, 2012). Non "razionali", cioè frazionari, meno che mai interi, tant'è che ciascuno di loro richiede una rappresentazione decimale – una sorta di carta di identità – *infinita* e imprevedibile. Numeri anch'essi nuovi, *"divino portento"* secondo Platone, opera sospetta dell'uomo stando invece a Kronecker. Tuttavia *ispirati dalla natura*, come la $\sqrt{2}$ del teorema di Pitagora, o il numero aureo, o il π che è il rapporto costante tra la misura di una circonferenza e quella del corrispondente diametro.

2. Diagonalizzando

Eppure già nel 1874, nell'articolo *Su una proprietà della classe di tutti i numeri reali algebrici*, Cantor aveva fornito (per dirla nei termini del lavoro successivo del 1878) la prova dell'esistenza di almeno due potenze, ovvero cardinalità, infinite distinte

- quella *numerabile* di $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, …
- la *potenza del continuo* di $\mathbb{R}$, $\mathbb{C}$, …

Non c'è infatti biiezione, e in verità neppure funzione suriettiva, possibile di $\mathbb{N}$ su $\mathbb{R}$. Per la cronaca, la dimostrazione più famosa di questo teorema, quella col metodo della *diagonalizzazione*, compare in un articolo del 1890-91, *Su una questione elementare della teoria delle molteplicità*, che prova addirittura l'esistenza di non solo due, ma di un'infinitudine di cardinalità infinite. L'argomento originario del 1874 è altrettanto semplice e ingegnoso, ma meno conosciuto, oscurato forse dal taglio stesso dell'articolo, che sin dal titolo riserva la sua enfasi a questioni di algebra e polinomi. Come che sia, è lecito sostenere che la teoria dei numeri cardinali infiniti prende il suo avvio nel 1874. I numeri transfiniti, non solo i cardinali ma anche gli ordinali, si svilupperanno e affineranno poi in lavori successive di Cantor, soprattutto nelle già citate *Grundlagen* del 1883 e nei *Beiträge* (i *Contributi alla fondazione della teoria dei numeri transfiniti*) del 1895-97. Teorie estremamente sottili e complicate, certo inadatte ai ragazzi, forse da vietare sotto i 18 anni… Eppure perfino a quelle età si potrebbero forse evocare

- $\aleph_0$, alef con zero, il primo cardinale infinito,
- oppure ω, omega, il primo ordinale infinito,

e con loro tutti *"i vasti numeri che un uomo immortale non raggiungerebbe nemmeno se consumasse la sua eternità contando"* (Borges, 1994, *La cifra*, *Nihon*). Allo stesso modo si potrebbero azzardare e discutere esempi suggestivi, osservare quindi

- che $\aleph_0 + 1 = \aleph_0$ (l'argomento dell'albergo di Hilbert), perché l'infinità di $\mathbb{N}$ resta inalterata aggiungendole un nuovo elemento,
- oppure che $1 + \omega = \omega \neq \omega + 1$, perché l'ordine abituale di $\mathbb{N}$ rimane lo stesso se gli si premette un minimo, ma cambia se gli si aggiunge un massimo.

Tra l'altro, sulla prima uguaglianza $\aleph_0 + 1 = \aleph_0$ si basano un tentativo di spiegazione matematica del paradosso di Zenone su Achille e la tartaruga e di conseguenza, nelle aule scolastiche, un possibile collegamento con la filosofia. La spiegazione è quella presentata da Bertrand Russell (Russell, 1970), non in verità la più convincente sull'argomento. Procede tuttavia così. Finché le tappe successive della rincorsa di Achille alla tartaruga restano un numero finito n, l'animale mantiene comunque il suo vantaggio, seppure ridotto. Infatti alla posizione n di Achille corrisponde la $n + 1$ della tartaruga, e $n \neq n + 1$. Ma arrivati al passo $\aleph_0$ si ha appunto $\aleph_0 + 1 = \aleph_0$ e Achille corona l'inseguimento.

Tornando ai reali e ai naturali e al teorema cantoriano del 1874, si potrebbe poi accennare il mistero dell'*ipotesi del continuo*. In effetti, una volta stabilito che le potenze di $\mathbb{N}$ e $\mathbb{R}$ sono distinte, è lecito, anzi naturale, domandarsi se esistono sottoinsiemi infiniti di $\mathbb{R}$ che hanno una cardinalità diversa dall'uno e dall'altro, né numerabile né continua, oppure se, viceversa, ogni sottoinsieme infinito di $\mathbb{R}$ si trova in corrispondenza biunivoca o con $\mathbb{N}$ o con $\mathbb{R}$. L'*ipotesi del continuo* corrisponde alla seconda di queste affermazioni.

Per anni Cantor tentò senza successo di dimostrarla. Nel 1900 Hilbert pose la questione al primo posto della sua famosa lista di 23 problemi e ancora nel 1925 provò ad abbozzare una possibile soluzione. Ma poi bisognò attendere il 1963 e Paul Cohen per avere una risposta, che poi non è così definitiva come taluni credono, e solleva anzi nuovi dubbi e nuovi interrogativi. Prova infatti che l'ipotesi del continuo non si può dirimere sulla base degli assiomi usuali della matematica, quelli proposti da Zermelo e Fraenkel all'inizio del Novecento. Dunque o si rinnovano questi assiomi, aumentandone la potenza, o ci si rassegna ad accogliere l'ipotesi stessa, o la sua negazione, al loro interno.

4. Matematica, follia e libertà

"[L]'essenza della matematica […] sta proprio nella sua libertà": la frase di Cantor, tratta dal capitolo 8 delle *Grundlagen*, è diventata un aforisma famoso. Esprime però della matematica un'immagine anticonvenzionale, lontana da quella rigida e coercitiva che in genere le si attribuisce. Ribadisce anzi Cantor: *"la matematica merita – e lo merita essa sola – il nome di libera, un attributo che, se stesse a me scegliere, io preferirei a quello ormai usuale di pura"*. Cantor va pure oltre, negando che la stessa libertà sussista per la matematica più legata al mondo, all'esperienza sensibile, alla natura: *"se la matematica ha il diritto di muoversi in piena libertà e senza alcun vincolo metafisico, non posso invece riconoscere lo stesso alla «matematica applicata»"* che è invece priva del *"soffio vivificante del libero pensiero matematico"*. Insomma: la mente, quando guidata dalla realtà, perde la sua autonomia. Ora, non c'è dubbio che, nei passi citati, Cantor elogi anche se stesso e la capacità del suo pensiero di immaginare numeri che nessuna natura suggerisce, di abbozzare e scolpire teorie solide e profonde laddove – all'infinito – nessuna percezione visiva e sensoriale può fornire sostegno. Ma, al di là di questo, o forse proprio per questo, sembra davvero il caso di discutere con gli studenti il tema della libertà della scienza, in particolare della matematica pura, intesa come libertà di pensiero.

Come anche è raccomandabile dibattere con i ragazzi, e semmai contraddire, uno stereotipo diffuso su ricerca e ricercatori. Sembra infatti che le teorie scientifiche, come proposte e talora imposte a scuola, siano altrettanti sistemi dogmatici, rifiniti e inappuntabili. Tanto vale per la matematica e in particolare per l'aritmetica transfinita e per l'insiemistica, nel modo in cui vari solidi manuali le presentano. Allo stesso modo, lo scienziato è spesso percepito come genio infallibile, dedito a confezionare teorie irreprensibili sin dal loro concepimento. Cliché sbagliatissimi, specie nel caso di Cantor. La sua teoria degli insiemi nasce tra speranze e delusioni, plausi e critiche, errori e rettifiche, attraverso idee e concetti che, prima solo abbozzati, acquistano progressivamente e faticosamente un'identità e una forma, e talora, come per l'ipotesi del continuo, restano sospesi per anni in attesa di risposta. Credo che sia giusto sottolineare con gli studenti questo travaglio della ricerca, che è poi uno dei fattori del suo fascino. Niente meglio della vita di Cantor può esemplificarlo, quando ci testimonia l'angoscia per la difficoltà dei problemi da risolvere, la rabbia per l'ostilità dei colleghi, la conseguente voglia di distrarsi inseguendo interessi extra-matematici, filosofici e paraletterari e, soprattutto, il progressivo affievolimento della stagione creativa, fino alle progressive crisi nervose – frutto forse dell'usura delle ricerche svolte, e delle incomprensioni sopra accennate, o di una qualche predisposizione naturale, o di quel legame sottile che sembra collegare talora il genio alla follia. La mente di Cantor progressivamente cedette negli ultimi anni dell'esistenza, che egli trascorse spesso ricoverato in cliniche. E in uno di questi ricoveri morì, appunto agli albori del 1918.

Credo che pure questa sia storia della scienza, ed è bene che i ragazzi la conoscano. Ma al di là del decadimento fisico e mentale degli ultimi anni, Cantor consegna a noi e loro un patrimonio scientifico emozionante. Viene quasi voglia di celebrarlo ancora, magari affidandosi nuovamente a Borges, quando in (Borges, 1997), *Storia dell'eternità*, *La dottrina dei cicli*, evoca appunto *"Cantor e la sua eroica teoria degli insiemi"*. La scuola non dovrebbe trascurare questa eredità. Tuttavia, perché questa esortazione non resti soltanto un libro dei sogni, dovrei indicare come e dove parlare agli studenti di questo retaggio. Osservo allora che colleghi ben più autorevoli di noi auspicano da tempo un rinnovamento dei programmi di matematica, che li aprano, non dico solo a Cantor, ma certo alla matematica dell'Ottocento e del Novecento. Nell'attesa che questo auspicio si realizzi, segnalo le occasioni – come i laboratori PLS, o il Liceo Matematico – che già ora consentono di introdurre in modo adeguato agli studenti il fascino di questi argomenti.

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