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The history and the meaning of Einstein's Principle of Equivalence

Antonio Feoli*

Abstract

We review from a historical and a didactic point of view the Equivalence Principle, which was considered by Einstein as the corner stone of his new theory of Gravitation: the General Relativity. Before and after the enormous success of his theory, this principle was the subject of studies and discussions. Still today, after more than one century, the debate about its interpretation, application and generalization is very fertile. Einstein soon understood the revolutionary significance of his idea and defined it as "the happiest thought of my life".

Keywords: History of Physics, Didactics of physics, Theory of Gravitation.†

1 Introduction

General Relativity is the theory that explains the motion of the bodies in the gravitational field in a geometrical way as caused by the curvature of space-time. All the theoretical scheme is based on two fundamental postulates: the Equivalence Principle and the Principle of General Covariance. The first postulate is the argument of this review (that updates an invited lecture on this subject held at a meeting in 2005 [1]) because it represents one of the best ideas that human mind has ever produced, while the second principle is more mathematical but not less important, being used by Albert Einstein to write the field equations in the final form in November 1915 after ten years of hard work and failed attempts.

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*Department of Engineering, University of Sannio, Benevento, Italy; feoli@unisannio.it
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Since 1905, the annus mirabilis when Albert Einstein issued five fundamental papers for the History of Physics [2], he devoted all his efforts to try to make Newton's Gravitational Force consistent with his own Special Relativity [3]: in particular, he wanted to generalize Galileo's Principle of Relativity (which was valid for observers in straight line motion at constant velocity), even to accelerated observers. Galileo described his principle through the famous "mental experiment" of the ship‡. He suggested that we should perform a lot of experiments in a cabin below decks on a large ship. If we repeat the same experiment when the ship is standing still and when the ship is moving with constant velocity, we will always obtain the same result. From Galileo's reasoning we can conclude that *"there is no way to distinguish between what is experimented by an observer in the still ship and what is found by an observer in the ship in straight line motion at constant velocity".*

Einstein, in a lecture held in Kyoto [5], remembered when in 1907 [6] he had had the great idea to generalize this principle: "I was sitting in a chair in the Patent Office at Bern when all of a sudden a thought occurred to me:<<If a person falls freely, he will not feel his own weight>>" that is a freely falling observer feels no gravitational field. In this paper, we will try first to understand the meaning of this observation using the Galileo's method of the mental experiments, second to deduce the surprising consequences of this result on the role of space and time, on the vision of Gravitational theory and on the evolution of the entire Universe; finally, we will briefly discuss the limits and the violations of the equivalence principle.

2 The spaceship and the elevator

In order to explain his idea, Einstein resorted to a mental experiment which was very similar to Galileo's one. We can describe it in a simple way, using two observers (we will call them James and Tony). We will place one (James) in a lift still on the Earth and the other (Tony) in a spaceship far from stars and planets, so that it cannot undergo any gravitational influence. As in Galileo's ship example,

‡The Principle of Relativity in the words of Galileo [4]:"Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though doubtless when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still".

the two observers are not allowed to look out; moreover, it is supposed that the vacuum has been created both in the lift and in the spaceship.

James makes some experiments and realises that inside the lift the objects fall towards the floor with an acceleration $g=9.8m/s^2$. On the other side, Tony, in the spaceship far from any sort of matter and hence not subject to gravity, bounces inside the cabin together with all the objects which he has brought with himself and which are hanging in space.

In these conditions, is it possible to simulate gravity inside the spaceship, so that Tony can think he is on the Earth? Yes, it is. If we light the rockets of the spaceship with an acceleration $g=9.8\ m/s^2$, Tony will keep his feet anchored to the floor and will see the objects falling down just with the same acceleration as if he were on the Earth, even if, actually, they are not the objects that are falling on the floor, but it is the floor that is moving towards the objects. Whatever experiment Tony makes inside his accelerated spaceship, he will obtain the same results observed by James in the lift. So Einstein has reached his aim: he has found a principle which is similar to Galileo's one and is valid for accelerated observers.

The Equivalence Principle states that "*there is no way to distinguish between the effects observed in a constant gravitational field (lift) and the ones observed in a reference frame moving with uniform acceleration (spaceship)".*

In a similar way, we can simulate the absence of gravity, as Einstein had guessed in Bern Patent Office: we have only to cut the lift cables and to let it go into free fall, so that the situation becomes equivalent to the spaceship travelling with the engine switched off. Of course this is only locally true, i.e. if we limit ourselves to a small area of space where the acceleration of gravity (which depends on the distance from the centre of the Earth) is really constant and where we can neglect the tidal effects which exist in a real gravitational field but do not exist in a field simulated by an accelerated frame.

3 Galileo's experiment of the fall of heavy bodies

The validity of the equivalence principle is based on the experiment of Galileo who, after dropping different objects from the Tower of Pisa, discovered that, if the air friction can be neglected, the bodies reach the floor simultaneously. In the First Dialogue of his famous opera of 1638 [7] Galileo asserts that "...if the resistance of the medium was abolished, all the bodies of different materials would go down with the same speed". It means that all massive objects in free fall undergo the same acceleration in a gravitational field, regardless of their mass and composition. This experiment, which brought with it the equivalence between inertial mass and gravitational mass, was not given the due importance. Einstein noticed this "neglected clue […] shunned by everyone for three hundred years" [8] and made it the basis of his General Relativity. Let us try to understand why it is so important. We can imagine for a moment that Galileo was wrong, and gravity does not attract all the objects democratically with the same acceleration. Then the two observers, James and Tony, would not see the same results and the equivalence principle would be wrong. Actually, James in the lift would drop a hammer and a feather and would observe that the hammer is the first to reach the floor. On the contrary, in the spaceship they are not the objects which fall down, but it is the floor which moves towards the objects, so Tony cannot see the hammer falling before the feather, since the floor goes towards all the objects in the same way. To conclude, there is equivalence between James's results and Tony's ones only if Galileo was not wrong, otherwise the experiment of the fall of heavy bodies will give different results for the two observers. For the same reason, the equivalence principle would not work if we were making our experiments with an electric field rather than with gravity. Let us imagine to repeat Galileo's experiment with a charged hammer and feather that are placed in a constant electric field *E*. The force acting on the bodies is:

$$\vec{F} = q\vec{E} = m\vec{a} \Rightarrow \vec{a} = \frac{q}{m}\vec{E}$$

The term *q/m* will depend on the falling object; every object will have its own charge and its own mass and will be attracted towards the floor with an acceleration different from the others, so the hammer and the feather will not reach the floor simultaneously because the charge to mass ratios of the hammer and the feather are different:

$$\frac{q_h}{m_h} \neq \frac{q_f}{m_f}.$$

There is an interesting and noteworthy case where the Equivalence Principle is valid also for electromagnetism, because locally all the bodies have the same charge to mass ratio. This really happens inside the atom. If we consider only Coulomb's attractive Force exerted by the nucleus on the electrons, these latter have all the same charge and the same mass.

On the contrary, the Equivalence Principle can always be applied with the gravitational field: for example, near the surface of our planet, the acceleration

$$g = \frac{GM_T}{R_T^2} = 9{,}8 \text{ m/s}^2$$

does not depend on the masses of the falling objects, but on the mass and the radius of the Earth which attracts them. If all the bodies undergoing the influence of gravity fall from the same height, they will reach the floor of the lift in the same moment, as well as the spaceship floor, going upward, reaches all the bodies which are at the same height in the spaceship at the same moment. Now Einstein could start formulating his theory based on the Equivalence Principle, if he had not another objection to overcome.

4 The experiment with a beam of photons

If we think it over, there is actually a case where the two observers could obtain different results and Einstein's principle would not be valid. Let us resort to a mental experiment again. We will take a small laser pointer and fix it in the lift at 20 cm from the floor. We will switch it on and the photons will be projected on the opposite wall of the lift, where we will see a red point. The photons will be propagated in straight line as they have a zero mass and the gravity force does not influence them. We will take a ruler and, measuring the distance from the point to the floor, we would expect to find the value of 20 cm. However, let us see what happens in the spaceship when we repeat the same experiment. We will switch on the laser pointer and, as the rockets are working, in the time taken by the photons to reach the opposite wall, the floor will have moved a little upward. As a consequence, we will not expect the value of 20 cm, but we will obtain a slightly lower value (19,99…cm). Actually, we might not be able to measure the difference even by a precision instrument (it is of the order of 10^{-15}cm), but without any doubt the value is lower than 20 cm. So, Einstein's equivalence principle is violated, since we have found at least one experiment where the result obtained by Tony is different from the one obtained by James. Now we have a way to distinguish if we are in a real gravitational field or in a system with a constant acceleration.

On the ground of this argument, Einstein should renounce his principle, but he thinks that the principle is right, while the previous results are wrong. In particular, Einstein maintains that if the measure on the spaceship is 19,99….cm, then also the measure in the lift must necessarily be 19,99…cm. The course of the photons must slightly bend downward, i.e. also the photons must be influenced by the Gravitational Force.

By 1704 Isaac Newton had already wondered about the "weight" of light: " Do not bodies act upon light at a distance, and by their action bend its rays; and is not this action the strongest at the least distance?"

It is possible to answer the first question of Newton's "Optics", imagining that the photons themselves have a gravitational mass E/c^2 (hence they undergo the Newtonian acceleration of gravity) and calculating that, on its way towards the earth, a beam of light coming from a far away star, is deflected 0,87 arc seconds when it is skimming the Sun.

A similar approach was used by Einstein [9] in 1911 (he found 0.83 arcsec), but straightforward he realized that there may be an alternative explanation without any need to invoke a gravitational force.

From Fermat principle a beam of light always follows the path of minimal time to go from a starting point A to a final point B. The consequence is that the light beam travels along a straight line that is the shortest path between the points A and B. But in the example of the lift, the path of the light beam is curved. So "if a light beam can follow a curved path, then this curved path must be the shortest distance between two points – which suggests that space itself is curved [10]"

According to this revolutionary interpretation of gravity, the bodies in the lift fall downward and the photons curve, not because there is a force attracting them, but because they live in a "curved space-time". On the basis of this theory, the Sun curves the space around itself with its mass and the planets turn round it because they cannot do otherwise as they have to move in a curved space. The consequence of the curvature is that the distance between two points in space-time is not ruled any more by Pythagoras theorem in four dimensions

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

but it must be generalized finding some suitable functions of coordinates, for example f(x), g(x) etc in the formula:

$$ds^2 = f(x,y,z,t)dt^2 - g(x,y,z,t)dx^2 - h(x,y,z,t)dy^2 - n(x,y,z,t)dz^2$$

solving the new field equations.

Einstein took four years before writing, in November 1915 [11], the new equations which describe the gravitational field no longer as a force but as a curvature of space-time. The completely new feature of the theory of General Relativity is well summed up by Hawking's words: "Einstein made the revolutionary suggestion that gravity is not a force like other forces but is a consequence of the fact that space-time is not flat, as it has been previously assumed but it is curved or "warped" by the distribution of mass and energy in it. Bodies like the Earth are not made to move on curved orbits by a force called gravity; instead, they follow the nearest thing to a straight path in a curved space, which is called a geodesic [12]"

The physicists have to make an extraordinary conceptual jump, as Silvio Bergia comments on [13]: "It consists in supposing simultaneously that gravitation is not a force, that the motions it determines are free, in a way to be explored; and that a free motion must not necessarily be straight and uniform".

And now two simple observations. Even if Einstein's new idea seems to come from a consequence of Fermat principle, the curious thing is that the final result that he reaches corresponds exactly to the opposite of that principle. The curve of the shortest distance in space (the geodesic) corresponds in space-time to the one of the longest time.

The geodesic motion in a curved space-time is the analogue of the motion at uniform velocity along a straight-line in flat space and both of them occur in the absence of external forces. Even if Galileo certainly did not imagine a curved space-time, he was right in considering the motion of planets as a natural "free" motion (his idea of a "circular inertia").

In November 1915 Einstein [14] repeated the calculation of deflection of light beams using his new theory and obtained the double of the value found in 1911 by Newton's gravitation. So, the theory of gravitation reaches a crossroad with three possible ways out:

1) The photons have no rest mass, the gravitational force does not influence them and therefore they are not deflected when they go by the sun. The Equivalence Principle is not valid.

2) The photon's energy can be considered as a measure of its mass and massive particles travelling at almost the speed of light near the Sun will form a deflected beam, according to Newton's theory of about 0,87 arc seconds.

3) The gravitational force does not exist, but the photons, following the shortest way in the space-time curved by the mass of the Sun, form a deflected beam of 1.74 arc seconds, according with what had been predicted by Einstein's General Relativity based on the Equivalence Principle.

To settle the question it was necessary to wait until 1919, when there was the first solar eclipse, during which Eddington was the first to measure an apparent shifting of the position of the stars near the Sun finding a value of the deflection of the light beams close to 1,74 arc seconds. All the following experiments will always confirm Einstein's General Relativity, allowing also to explain the perihelion precession of Mercury. It will predict the existence of gravitational waves [15], permit the birth of Scientific Cosmology [16] and to devise a series of completely new phenomena among which the gravitational lenses and the black holes are the most extraordinary ones.

5 Discussion

After more than a century the Equivalence principle is still a subject of debate and a source of open questions. The historical evolution of the principle has also been studied in a more technical way [17] with respect to the didactic approach followed in this paper, together with the discussion about the critical points. Just to give some examples, we want to list the problems faced by our research group.

- Is the Einstein principle always and everywhere valid?

Of course, as each principle in physics, it has limits of applicability both in classical domain and in quantum regime. As we have underlined at the end of section 2, the principle works if we limit ourselves to a small area of space where the acceleration of gravity (which depends on the distance from the centre of the Earth) is really constant and where we can neglect the tidal effects which exist in a real gravitational field but do not exist in a field simulated by an accelerated frame. In the case of a real elevator, we can take for example two small balls (test particles) and put them on the top of the elevator at the maximal distance from each other and leave them fall in the Earth gravitational field until they reach the floor of the elevator. We have explicitly studied [17] this case considering a small elevator with a square floor surface whose side is 80 cm and height h = 220cm. We have calculated that a measure on the floor of the contraction of the distance between the two balls due to the tidal forces of the Earth field is about 2.76 X 10^{-5}cm hence very small.

- Instead of placing the elevator in a central field, is it possible to create a real uniform gravitational field?

The idea is the following: a massive body and the floor of the enclosure become closer and closer either because the body falls towards the floor attracted by an external mass (in the elevator on Earth), or because the floor moves towards the body accelerated by an external force (in the rocket in the space far from any massive object). In principle, a third way can exist: the space between the floor and the body can disappear if it undergoes a suitable contraction, that is the contrary of the cosmological expansion. So it is worthy to analyze this theoretical possibility suggesting a geometry that can simulate the effects of a real uniform gravitational field [18].

- Finally, is the equivalence principle valid in quantum mechanics?

The answer is that in Quantum Mechanics inertial and gravitational masses are not equivalent and this is confirmed by the experiment. For example, we have showed [19] the deep conflict between quantum theory and gravity analyzing a quantum bouncing ball system. In particular, we have emphasized graphically that the behavior of quantum particles in gravitational fields is mass dependent. Hence the Principle of Equivalence is not valid in the quantum regime and the problem of unification of Quantum Mechanics and General Relativity remains open as the one of the radiation emitted by a charged body in the Einstein’s lift [20].

These are only some of the possible questions. So many years of studies and experiments have not exhausted all the beauty, the astonishing consequences and the mysteries of a principle and a theory born on a day in 1907 from the idea of a man "sitting in a chair in the Patent Office at Bern".

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