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From Continuous to Discrete via V&V Bar

Thomas Vougiouklis[1]

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Abstract

The 'continuous' and the 'discrete' in nature and in science live and fight forever. The questionnaires and the Lickert scales are indispensable and widely used tools in social sciences research. Vougiouklis & Vougiouklis bar is a new tool introduced as an alternative to Lickert scales. We believe that such an alternative might offer some solutions to problems that crop up during the fight between continuous and discrete. Nevertheless, the greatest contribution of the V&V bar is that it offers the researchers freedom in all stages of the research procedure using a questionnaire.
Keywords: continuous, discrete, questionnaires, Lickert scale, V&V bar

1 Continuous–Discrete

Mathematical models are widely used in almost every field of empirical research to reinforce the reliability of each individual research. This is because mathematicalisation of a problem could make its results recognizable and comparable with other results. In other words, representing an actual research object or a phenomenon with numbers and figures or graphs might be the

[1] Emeritus Professor of Mathematics, Democritus University of Thrace, Greece; tvougiou@eled.duth.gr.

simplest and the most recognizable way of reading the actual results.

The *discrete* in mathematics starts from *characteristic function* which in Set theory states that an element belongs or not to a set. It is the analogous in Logic Theory to true – false. Discrete objects can be easily achieved by computers.

The *continuous* appears in nature in all phenomena (Αριστοτέλους 2003) but, when necessary, we can easily transfer it into a discrete. The *numbers* and the *order* help in transforming the continuous into discrete and Geometry configurates the continuous. Similarly, analysis can do this by using the *limit*, as well. However, there is always a fight between the *point* and the *atom*, characteristic representatives of the continuous and discrete respectively. Therefore, sometimes we believe that atoms were invented in order to be transferred. Finally, it is very hard to approach the real numbers by rational numbers.

2 Hyperstructures

A relatively new branch of algebra is the theory of hyperstructures introduced by F. Marty in 1934. The hyperstructure theory, or multivalued theory or polysemy theory, is strongly related to Fuzzy Theory as well. Therefore, there are a lot of applications on other sciences including the social ones. For basic definitions and applications on the related theory one can refer to books as (Corsini 1993; Davvaz & Leoreanu 2007; Vougiouklis 1994) and related papers as (Chvalina, Hoskova 2007; Maturo, Sciarrra, Tofan 2008; Vougiouklis 1991; 2009; 2011). We focus on the large class of hyperstructures called H_v-structures introduced in 1990 by Vougiouklis, which satisfy the weak axioms where the non-empty intersection replaces the equality.

Basic definitions on the topic are the following:
In a set H equipped with a hyperoperation (abbreviation, hyperoperation = hope)

$$\cdot : H\times H \rightarrow P(H) - \{\emptyset\}.$$

we abbreviate by WASS the weak associativity:

$$(xy)z \cap x(yz) \neq \emptyset, \quad \forall x,y,z\in H$$

and by COW the weak commutativity:

$$xy \cap yx \neq \emptyset, \quad \forall x,y\in H.$$

The hyperstructure $(H,\cdot)$ is called H_v-semigroup if it is WASS, it is called H_v-group if it is reproductive H_v-semigroup, i.e., $xH = Hx = H$, $\forall x\in H$.
The hyperstructure $(R,+,\cdot)$ is called ***H_v-ring*** if both $(+)$ and $(\cdot)$ are WASS, the reproduction axiom is valid of $(+)$ and $(\cdot)$ is *weak distributive* with respect to $(+)$.

Motivations (Vougiouklis 1994):

1. The quotient of a group with respect to an invariant subgroup is a group.
2. The quotient of a group with respect to any subgroup is a hypergroup.
3. The quotient of a group with respect to any partition is an H_v-group.

The main tool to study hyperstructures are the fundamental relations β^*, γ^* and ε^*, which are defined, in H_v-groups, H_v-rings and H_v-vector spaces, respectively, as the smallest equivalences so that the quotient would be group, ring and vector space, respectively. A way to find the fundamental classes is given by theorems as the following (Vougiouklis 1994).

Theorem. Let $(H,\cdot)$ be an H_v-group and denote by $\boldsymbol{U}$ the set of all finite products of elements of H. We define the relation β in H by setting $x\beta y$ if and only if, $\{x,y\}\subset \boldsymbol{u}$ where $\boldsymbol{u}\in\boldsymbol{U}$. Then β^* is the transitive closure of β.

An element is called *single* if its fundamental class is singleton (Vougiouklis 1994).

Fundamental relations are used for general definitions. Thus, an H_v-ring $(R,+,\cdot)$ is called ***H_v-field*** if R/γ^* is a field. Then the H_v-vector space can be defined.

Let $(H,\cdot)$, $(H,*)$ be H_v-semigroups defined on the same set H. The hope $(\cdot)$ is called *smaller* than $(*)$, and $(*)$ *greater* than $(\cdot)$, if and only if, there exists an automorphism $f\in Aut(H,*)$ such that $x\cdot y\subset f(x*y)$, $\forall x,y\in H$. Then we say that $(H,*)$ contains $(H,\cdot)$. If $(H,\cdot)$ is a structure, then it is called *basic* structure and $(H,*)$ is called *H_b-structure*.

The Little Theorem. Greater hopes than the ones which are WASS or COW, are also WASS or COW, respectively.

A very large class of H_v-structures is defined for any given operation on a set together with any map on the set.

Definition. Let $(H,\cdot)$, be a groupoid (resp., hypergroupoid) and $f: H\to H$, be any map. We define a hope (∂) on H, called theta-hope or *∂-hope*, as follows

$$x\partial y= \{(x)\cdot y, x\cdot f(y)\},\ \forall x,y\in G \quad (\text{resp. } x\partial y= (f(x)\cdot y)\cup(x\cdot f(y)),\ \forall x,y\in G)$$

If $(\cdot)$ is *COW*, then ∂ is *COW*. If $(\cdot)$ is associative, then ∂ is WASS.

A very interesting and 'strange' very large class of H_v-structures, introduced by Vougiouklis in 1988, is the following:

Definition. An H_v-structure is called ***very thin***, if and only if, only one of its operations is a hope and for this hope, all results are singletons except one which has result a set. Thus, we obtain very thin H_v-structures if we enlarge only one result of any structure.

We referred only to the above two large classes just to show how we can find applications from any applied science, the social ones included, to the H_v-structure theory. Therefore, H_v-structures are offered as models in several sciences.

3 Questionnaires

In every empirical research three main stages could be normally identified: *design, implementation* and *processing of the results.* Main tools in an empirical research include *the questionnaire* where Likert scales are normally and widely used. Likert scales are often used to measure respondents' attitudes by asking the extent to which they agree or disagree with a question or a statement. Likert scales may seem easy to analyze but there are some important issues a data analyst should consider. More specifically, there are certain shortcomings usually identified in this type of scales and they include the range of the scale which each time is upon the researcher to decide as it is not standard how many different subdivisions, or grades, should be used. Moreover, this is not an easy job to accomplish as it is quite different to have 3 or 4 or 5 subdivisions since there are certain problems to overcome in each case. Such problems include lack of a medium choice in a 4-grade scale. Another shortcoming of typical Lickert scales is the difficulty of verbally refining the difference between different subdivisions and make them clear to the participants, especially to less sophisticated ones. This is not an easy process as many researchers report that it takes their subjects longer to comprehend what each subdivision represents on their scale rather than accomplish the actual test. Such a problem is of course not really a matter of language but it involves a number of different factors such as social and psychological. Finally, in the stage of processing the results, the researchers will have only one possibility of working them out, the one they decided to establish when initially designing the experiment. Such a decision, though, might deprive the researchers of the possibility to explore other parameters which might crop up in the process, or even try different subdivisions for either a more accurate calculation or to make their results comparable with another researcher's, who has used different scale.

In order to facilitate the whole process, we introduced an innovation which combines social sciences with Fuzzy Set Theory (Zadeh 1965). This innovation actually suggests to substitute the discrete Likert scales with the continuous V&V bar, minimizing the time and effort of filling in and processing of the questionnaires. Such radical reduction of time is the result of transferring the transformation of the continuous to discrete, from the informant to the researcher.

4 The Bar

During last decades hyperstructures seem to have a variety of applications not only in other branches of mathematics but also in many other sciences

including the social ones. A new application in questionnaires, which combines hyperstructure theory and fuzzy theory, is to replace the scale of Likert by the Vougiouklis & Vougiouklis bar, V&V bar for short.

The suggestion is the following (Kambaki-Vougioukli, Vougiouklis 2008).

Definition. *In every question substitute the Likert scale with 'the bar' whose poles are defined with '0' on the left end, and '1' on the right end:*

The subjects/participants are asked, instead of deciding and checking a specific grade on the scale, to cut the bar at any point they feel expresses their answer to the specific question.

The final suggested length of the bar, according to the Golden Ratio, is 6.2 cm (Vougiouklis, Kambakis-Vougiouklis 2011; 2013; 2015).

Likert Scale disadvantages. There are certain shortcomings usually identified in this type of scales and they include the range of the scale: each time is upon the researcher to decide how many different subdivisions or grades should be used, as it is not standard. Moreover, this is not an easy job to accomplish, as it is quite different to have even or odd number of subdivisions, because of the lack of a medium choice in an even-grade scale. The most serious problems though constitute the difficulty of the limit discrimination of the actual partition. This is another shortcoming of typical Likert scales for the researcher: the difficulty of verbally refining the difference between different grades of a scale and makes them clear to the participants.

Advantages of the Bar. There are identified certain advantages concerning the use of the bar compared to that of a scale during both stages of filling as well as processing a questionnaire. The participants do not have to try to identify the difference between the subdivisions and do not need any special training in order to be able to cut the bar appropriately, i.e. understanding differences that concern linguistic refinement. By contrast, they can do it intuitively avoiding any verbal processing. Yet, what is the main advantage of the bar is that it provides the potential for different types of processing, Likert scales cannot. Therefore, it gives the initiative to the researcher to explore if the given answers follow a special kind of distribution. The researcher can divide the bar into equal steps or the Gauss distribution or parabola, by putting in the same class all segments that belong to the equal step or to equal-area spaces according to Gauss distribution or some kinds of parabola (Kambakis-Vougiouklis, Karakos, Lygeros, Vougiouklis 2011; Kambakis-Vougiouklis, Nikolaidou, Vougiouklis 2017). They lead to special H_v-structures using theta-hopes. Thus the researcher can correct any kind tendency, for more accurate results. A possibility of choosing amongst alternatives is offered by using fuzzy logic in the same way

as it has already been done combining mathematical models with multivalued operation. The bar gives the researchers the possibility to 'escalate' the answers without having to decide in advance how many different grades would be used.

The only disadvantage of the bar is to transfer the data collection to a computer for elaboration. However, there is now a program of filling in a questionnaire on a computer such that the results automatically can be transferred for research elaboration (Nikolaidou, Vougiouklis 2012). This application overcomes the problem of inputting data from questionnaires to processing and eliminates time of data collection, transferring data directly for any kind of elaboration. The application has been implemented using Visual Basic and the data is being saved on a Microsoft Access Database. The application is based on "events" and an OleDbConnection is used to connect the program with the database. Filling-in such questionnaire can be easily achieved by using this application, as it is based on a very simple user interface. The participants asked to 'click' on the bar, to indicate the point that satisfies their answer on the question made. The user has the opportunity, to change his answer by 'clicking' on another point any time before final submit. The results are being saved on a simple database (Microsoft Access Database) indicating the exact point each participant has "cut" the bar.

The main advantage is the fact that it is much quicker to fill in and much easier to explain the procedure to participants.

A research example of the above is the following:

An experiment took place in three schools in Komotini, Greece. 400 students, 14–15 years of age, participated. The purpose of the survey was to investigate their employment of learning strategies while learning a foreign language. The tool used was the 50-item Strategy Inventory Language Learning, questionnaire, widely known as SIILL. There were six categories of strategies, namely mnemonic, cognitive, compensatory, metacognitive, affective and social and all participants had to specify how often they use each strategy and how confident they feel that the application of each strategy facilitates their learning. By contrast to widely used surveys in Greek schools where questionnaires are mainly filled in on paper, the students were happy to work on the computer and although they actually had to answer 100 questions, or twice the same question, they finished the test in 11–12 min than the 18–20 min normally demanded for the 50-item paper version. That was 30% of the of the time used on any Lickert scale. The result was unexpected, and we verified it in a series of successive researches. Thus, we claim that the only one possible explanation is the following:

The time to cut the bar is 30% of the time needed to make a decision on a Lickert scale. That means that in questionnaires *it is faster and easier for researchers to transfer the continuous to discrete than the participants.*

5 Conclusion

The ‘continuous’ and the ‘discrete’ in nature and in science live and fight forever. The questionnaires and the Lickert scales are necessary tools in research in social sciences. The Vougiouklis & Vougiouklis bar is a new tool, alternatively used to Lickert scales. This replacement connects the questionnaires, in the stage of processing the results, with the hyperstructures and fuzzy set theory. We believe that this replacement gives some answers on the fight between continuous and discrete. The V&V bar relieves participants from the difficult process of transforming the continuous into discrete, leaving it to the researchers. Due to this freedom, the time to complete the questionnaires is minimized.

To recap, the V&V bar leaves to the participants the ‘continuous’ which is achieved easily, clearly, naturally and fast, while it gives the researchers the privilege to be the exclusive elaborators of the ‘discrete’ and transformers of the ‘continuous’ into ‘discrete’. Apparently, sometimes the easy job is not the one which appears to be.

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