Science & Philosophy **ISSN**: 2282-7757
Vol. 5(2), 2017, pp. 25-38 **eISSN**: 2282-7765

Mathematical Models for the Comparison of Teaching Strategies in Primary School

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[3]**doi**:10.23756/sp.v5i2.392

Abstract
Starting from the considerations on modern school of some important scholars, which highlight the complexity of the school system, an analytical path is outlined to identify the best strategies by means of a mathematical model. The method followed is the analytical hierarchical one of Saaty that starts from the investigation of the various objectives, criteria and strategies, and indicates procedures to assign qualitative judgments and to transform them into numerical scores. In particular the AHP procedure is applied to find the degree of effectiveness of various strategies for teaching English, in relation to possible contexts that may arise.
Keywords: educational strategies in modern school, AHP procedure, teaching English

1. Teaching in Primary School: Problems, Complexity and Renewal

The role of the school today is implicitly described in the National Guidelines for Curriculum that state: "... *Doing school today means bringing together the complexity of radically innovative ways of learning with a daily work of guidance, attentive to method, new media and multidimensional research ...*"

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[3] © Antonio Maturo and Maria Gabriella Zappacosta. Received: 25-11-2017. Accepted: 27-12-2017. Published: 31-12-2017.

A significant contribution to clarifying the problems of the school system and the expectations of the operators and users was given by the pedagogue Cesare Scurati (2011: 5-6), in which, among other things, the author states: "... *Every time must find the appropriate terms and languages to understand the meaning of the school in the concrete of its components and manifestations* ..." and "... *School is a place wanted by the adult world that today is the target of meeting between the past and the future in the constant search for recomposition and reflection* ...".

In the light of these suggestions, we can point out that the school must ensure school learning that is:

- relevant for all pupils,
- emblematic from the point of view of the indispensable disciplinary knowledge,
- productive, as it will have to be able to face challenges, unprecedented and complex situations because of the profound changes and upheavals that continue to invest in all educational contexts.

In a relatively small period, we have been able to see how the new technologies and the phenomenon of globalization have profoundly changed the human condition, creating a constantly evolving society. Today, boys are increasingly globalized, unequal, and even more isolated in the universe of relationships, both linked to daily life and their contexts, and to the broader ones at national and transnational level. Once the student learned most of his knowledge at school. Today, however, the younger generation receives a lot of information from various individuals, media and educational agencies external to the school. We also note that within a few years, at a rate that we could call *exponential*, content and forms of knowledge have changed substantially, and even more such disturbances have affected the ways of their organization, their production and their transmission. At present, children can experience extracurricular activities overflowing with information, and increased by meeting with a variety of different cultures. At the same time, however, we must point out that the process is fragmented and obscure, with no interpretive filters or educational perspectives that can assemble their varied experiences and the development of their personalities. Therefore, in the face of such a situation, the school cannot afford to abandon its important educational tasks, narrowing its role to the simple transmission of some techniques and some basic knowledge.

In our view, in the light of what has been pointed out, the school's mission must become even more meaningful: its primary task is to substantiate the many educational and extracurricular experiences of the pupils, to heal the crumbling of information to recompose the development of their personal training.

The school must be able to make significant correlations between information and knowledge, to emerge as a basic constructor of essential conceptual and cultural tools, to give meaning to the plurality of information and knowledge that sometimes appear confused and tangled. Finally, we must be inspired by what Italo Fiorin calls "*a didactic of Integrality*" (Fiorin, 2014; Fiorin et al., 2013), while "*... even today is widely spread a didactic of sectorality ...*".

2 Objectives, sustainable alternatives and elements of uncertainty

Using a definition of the French philosopher and psychoanalyst, of Greek origin, Cornelius Castoriadis (1998): "*... We live in the times of ossimors ...*" in which we are urged by interests or ideologies, we are almost forced to visualize binary contexts, to take into account theories that simplify and that are at the limit of the contradictory. Instead, reviewing the pedagogical intuition of Popper's epistemology in terms of conjectures and refutations, we should treasure his teaching placed on the idea that the person who searches for confirmation can find it. The important thing is, however, to "stumble" in the right mistakes, namely those mistakes that urge us to seek the causes of difficulties, to grasp unexpected and singular relationships, traces that detect hidden truths (Popper, 1976; Sciarra, 2006).

In our opinion, “*thought*” should be interpreted as "*metis*", not "*logos*" or "*ratio*". So, it is intuition, perspicacity, wit, ready to relate to the uncertainties and the unpredictability of the world. Today, however, it is important to rely on a "*logic of discovery*", as Cellucci (2005) points out, not based on a closed and certain axiomatic method, but on an analytical method that does not give certainty, but it is able to detect possible irregularities, in order to make any adjustments. After discovering the ineliminability of uncertainty in knowledge, a "*logic of discovery*" aims to "*teach to be confident of its own certainty within a context of reference that must necessarily be open*" (Cellucci, 2005). In addition, Edgar Morin (2000, 2001) suggests that it is necessary to reform not only the organization of knowledge, which must open to doubt, to live with uncertainty; but it is also necessary to reform the same methods of knowledge.

The author states that the IWBs are not enough to renew the teaching processes, but it is appropriate to rewrite the cognitive project, in order to find effective paths able to generate amazement and enchantment. It seems rather more important to use informatic tools for the ex post evaluation of excellences (see e.g. Ceccatelli et al., 2013a), not used for a selection, but rather as an aid to outline guidelines for the improvement of learning (Ceccatelli et al., 2013b).

In addition, a special focus should be given to the evaluation of the social aspects of teaching (Svatoňová, Hošková-Mayerová, 2017; Delli Rocili, Maturo, 2017; Hošková-Mayerová et al., 2017).

Moving now our discussion on the learning of scholastic disciplines, such as Mathematics, Italian and English, we must, in our opinion, take advantage of the stresses just highlighted that we get, as we have seen, from multiple contexts, in order to outline educational paths aimed at stimulating the minds of our young interlocutors in the best possible way.

Ambel (2013) notes, in this regard, that the idea of structuring a school that allows the acquisition of skills in a more advanced and complex perspective, through the activation of a deeply innovative didactics in the curricular choices to be implemented, needs to be strengthened. In order to design effective training paths, the teacher will therefore have to proceed with a review of his own discipline, to a timely reflection on his epistemological status, in order to identify the essential knowledge and the supporting nuclei. Through laboratory didactics, he will stimulate the students to remove and overcome the obstacles they encounter in learning, he will lighten the disciplinary contents, which will become, then, the founding instrument for the acquisition of logical-linguistic skills and autonomy in the study.

In the laboratory context, particular importance must be given to the logical, critical and interdisciplinary aspects. Specific didactic paths for the interdisciplinary teaching of mathematics, probability and statistics are presented in (Maturo, Delli Rocili, 2015; Maturo, 2015).

Ultimately, in our view, what is most relevant is to support the integral development of the pupil, not to encourage him to accumulate knowledge and learning, but to help him mainly along the whole spectrum that goes from the beginning of childhood school to conclusion of the first cycle, in order to develop those that are defined in the "*Indications for the curriculum*" *key competences* (or *citizenship*). In the light of what has emerged from our observations, we will now delineate the path we have identified, in order to verify through the support of the mathematical models related to the hierarchical analytical method of Saaty, which teaching strategy is most profitable for the purposes of learning the English language.

For the purpose of our experimentation, we involved the pupils of the state primary school "Gianni Rodari" of the “Istituto Comprensivo Pescara 1” belonging to two fifth classes.

The primary purpose that we intended to pursue was to act to enhance the A1 level of contact, emphasizing the communicative aspect and also taking care of the part related to the lexicon, always referred to the linguistic-communicative situations that we faced concretely in the classroom.

Using the terminology and international approach of the hierarchical analytical process of Saaty (1980, 2008), which will be described in the next paragraphs, we have identified the following *general objective* (GO): GO = "To encourage students to have a positive and open attitude towards a different linguistic code, aimed at learning English in real communication situations."

The general objective can be explained by various specific objectives. By way of example, focusing mainly on the methodological aspect, we have focused our attention on four specific objectives that seemed to us more significant.

- O1 = "Use English to interact in the classroom and communicate in group situations";
- O2 = "Knowing how to use information through new technologies and collaborate with classmates";
- O3 = "To learn the fundamental linguistic structures with the contribution of music using pieces belonging to various musical genres, to acquire greater security, linguistic mastery and to improve the pronuncial";
- O4 = "To enrich the vocabulary through the C.L.I.L. methodology, Content and Language Integrated Learning".

To achieve these objectives, it is possible to follow various teaching strategies (or alternatives). We have focused our attention above all on four alternatives that have appeared to us most relevant.

- A1 = "Teaching that favors the use of new technologies";
- A2 = "Frontal teaching with the use of routine tools";
- A3 = "Interdisciplinary teaching (C.L.I.L.)";
- A4 = "Teaching of English through music".

3 The mathematical model for evaluating alternatives

Let **A** = {A1, A2, ..., Am} be the set of the *alternatives*, i.e. the possible educational strategies. Moreover, let **O** = {O1, O2, ..., On} be the set of rhe *objectives* that we want to achieve. In the first phase of the decision-making process, a commission, consisting of a set of decision-makers **D** = {D1, D2, ..., Dk}, must establish a procedure to assign to each pair (alternative Ai, objective Oj) a score pij that measures the degree in which the alternative Ai satisfies the objective Oj. Assume that pij $\in$[0, 1], where pij = 0 if the objective Oj is not at all satisfied by Ai and pij = 1 if the objective Oj is completely satisfied. At the end of the procedure we obtain a matrix **P** = [pij] of the scores which is the starting point of the logical-mathematical elaborations that lead to the choice of the alternative, or at least to their ordering, possibly even partial (cf. Maturo, Ventre, 2009a, 2009b).

There may be constraints. For example, it may be necessary to establish for each objective Oj a threshold $\alpha j > 0$, with the constraint $pij \geq \alpha j$, for each i. Furthermore, consideration should be given to *mixed strategies,* i.e. convex linear combinations of alternatives Ai. A mixed strategy has the form A(h1, h2, ..., hm) = h1 A1 + h2 A2 + ... + hm Am, with h1, h2, ..., hm real numbers not negative and such that h1 + h2 + ... + hm = 1.

If we consider also the mixed strategies, then the alternatives Ai are called *pure strategies*. The need to consider mixed strategies also arises in particular if there are "at risk" alternatives, i.e. alternatives that have high scores for certain objectives and low for others (possibly below the threshold). The number hi can represent the fraction of time in which the teaching strategy Ai is adopted. In the case of uncertainty in the assessment of the scores the numbers pij can be replaced by triangular fuzzy numbers p*ij = (aij, cij, bij) with $0 \leq aij \leq cij \leq bij$). For example, if we want to take into account the diversity of judgments of the decision makers Dr, aij can be the minimum of the scores attributed by the decision makers to the couple (Ai, Oj), bij the maximum and cij an appropriately chosen average, for example the arithmetic average or the median.

A preliminary approach to the construction of the fuzzy triangular numbers p*ij is the search for the consensus among the decision makers, in order to arrive at judgments and scores that are not excessively discordant and therefore to fuzzy numbers with not excessive spreads. Studies and algorithms for achieving consensus have been elaborated in (Maturo, Ventre, 2017; Olivieri et al., 2016).

4 The hierarchical analytical method of Saaty for the attribution of weights and scores

Let us recall that (Knuth, 1973) a *directed graph* or *digraph* is a pair G = (V, A), where V is the set of vertices and A the set of arcs. A vertex is indicated with a Latin letter and an arc is an ordered couple (u, v) of vertices, where u is the initial vertex and v the final vertex. An ordered n-tuple of vertices (v1, v2, ..., vn), n > 1, is called *path* with length n-1, formed by the arcs (vi, vi + 1), i = 1, 2, …, n-1.

The hierarchical analytical procedure (AHP) of (Saaty, 1980, 2008) is based on the representation of a decision problem with a directed graph G = (V, A) satisfying the following 5 properties:

- the vertices are distributed in a fixed number n > 2 of levels, numbered from 1 to n;

- there is only one vertex of level 1, called *root*;

- for every vertex v different from the root there is at least one path having the root as the initial vertex and v as the final vertex;

- every vertex u of level $i < n$ is an initial vertex of at least one arc and there are no arcs with an initial vertex of level n;

- if an arc has the initial vertex of level i then it has the final vertex of level $i + 1$.

In this paper we assume $n = 4$. The level 1 vertex is called the *general objective*, indicated with GO. Level 2 vertices are called *specific objectives*, or simply *objectives*. Level 3 vertices are called *criteria* and finally level 4 vertices are the *pure alternatives* or *strategies* of the decision process.

A decision maker D (or a commission) assigns a score to each arc following the AHP procedure proposed in (Saaty, 1980, 2008) and applied in various papers, for example in (Maturo, Ventre, 2009a, 2009b).

The scores are non-negative real numbers and such that the sum of the scores of the arcs coming out of the same vertex u is equal to 1. The score assigned to an arc (u, v) indicates the extent to which the final vertex v (objective, criterion or alternative) meets the initial vertex u (general objective, objective, criterion). The *score of a path* is the product of the scores of the arcs that form the path.

For every vertex v different from GO the score p(v) of v is the sum of the scores of all the paths that start from GO and arrive in v. Starting from these definitions it is verified that, for every level i, the sum of the points of the vertices of level i is equal to 1. The scoring procedure is based on pairwise comparison. Let $x1, x2, ..., xp$ be the final vertices of the arcs coming out of an initial vertex u. If a decision maker considers xr preferable or indifferent to xs, then he must estimate the importance of xr with respect to xs using one of the following qualitative judgments: *indifference*, *weak preference*, *preference*, *strong preference*, *absolute preference*. Qualitative judgments are expressed as numerical values according to the following Saaty scale: indifference = 1, weak preference = 3, preference = 5, strong preference = 7, absolute preference = 9.

If we assign to the object xr one of the previous numbers when it is compared to the object xs, then xs assumes the reciprocal value when it is compared to xr. Then we obtain a pairwise comparison matrix **A** = (ars) with p rows and p columns, called matrix associated to the p-tuple (x1, x2, ..., xp), in which ars is the number assigned to xr when it is compared with xs.

Then the main eigenvalue $\lambda 1$ of the matrix A is calculated and, among the eigenvectors associated to $\lambda 1$, the one is chosen (called *normalized eigenvector*, which is proved to be unique) with all the components w1, w2, ..., wp not negative and with sum equal to 1. For each i, the number wi is the score assigned to the arc (u, xi) from the AHP procedure.

Before finally accepting the scores wi, one must check the consistency of the judgments expressed by the decision maker. The evaluation of a decision-maker may be inconsistent due to the lack of transitivity in the ordering of {x1, x2, ..., xp} by attributing judgments or due to excessive differences in quotients between the corresponding elements of two rows or two columns of matrix **A**. Saaty suggests testing the consistency with the number CI = (λ1-p)/(p-1), called the *coherence index*. If CI < 0.1, then coherence is certainly acceptable and we say that we have *a strong coherence*.

The experiments conducted in the educational field have led us to accept as consistent also assignments of scores in which 0.1 ≤ CI < 0.2. In the present paper, based on the experiments carried out, we also considered to accept situations, defined as *weak coherence*, in which 0.2 ≤ CI < 0.3. If the coherence index is considered too high, then the decision maker is invited to update his assessments.

5 Processing of scores

For simplicity of writing, we use, for matrices, the notation of the software "Mathematica", in which a matrix is seen as a vector of row vectors.

5.1 Calculation of the weights of the specific objectives

Starting from interviews made to experts, the following matrix of pairwise comparison between the specific objectives with respect to the general objective has been obtained:

GO = {{1,3,1/9,1/7}, {1/3,1,1/9,1/7}, {9,9,1,5}, {7,7,1/5,1}}.

We verified that GO has the main eigenvalue λ(GO) = 4.39 and coherence index CI(GO) = 0.13 < 0.2. Therefore, GO can be considered coherent. The normalized eigenvector associated with λ(GO) is:

Vet(GO) = {0.065, 0.038, 0.645, 0.252}.

The components of Vet(GO) are the weights of the objectives O1, O2, O3, O4, respectively.

5.2 Calculation of the criteria weights

Weights of the criteria with respect to O1

The matrix of the pairwise comparison between the criteria with respect to the objective O1 is:

O1 = {{1,1,3,5,2,2}, {1,1,3,2,3,5}, {1/3,1,1,3,3,7}, {1/5,1/3,1/3,1,3,1}, {1/2,1/3,1/3,1/3,1,1/2}, {1/2,1/5,1/7,1,2,1}}

The matrix O1 has the main eigenvalue $\lambda(O1) = 6.91$ and coherence index $CI(O1) = 0.18 < 0.2$. Then O1 can be considered coherent. The normalized eigenvector associated with $\lambda(O1)$ is:

Vet(O1) = {0.276, 0.282, 0.224, 0.082, 0.063, 0.073}.

The components of Vet(O1) are the weights of the criteria C1, C2, …, C6 with respect to the objective O1, respectively.

Weights of the criteria with respect to O2

The matrix of the pairwise comparison between the criteria with respect to the objective O2 is:

O2 = {{1,3,3,1,3,1}, {1/3,1,3,3,1,1}, {1/3,1/3,1,2,3,3}, {1,1,1/2,1,1/3,3}, {1/3,1/3,1/3,3,1,3}, {1,1/2,1/3,1/3,1/3,1}}

We have $\lambda(O2) = 7.31$; $CI(O2) = 0.26 < 0.3$. The matrix O2 is weak coherent and then we can proceed, with an acceptable margin of error, to the calculation of the criteria weights with respect to the objective O2. We obtain:

Vet(O2) = {0.313, 0.196, 0.160, 0.095, 0.153, 0.082}.

Weights of the criteria with respect to O3

The matrix of the pairwise comparison between the criteria with respect to the objective O3 is:

O3 = {{1,1/5,1,3,5,3}, {5,1,7,5,5,3}, {1,1/5,1,3,3,3}, {1/3,1/5,1/5,1,3,1}, {1/5,1,1/7,1/3,1,1}, {1/3,1/5,1/3,1,1,1}}

We have $\lambda(O3) = 7.44$ and $CI(O3) = 0.29 < 0.3$. The matrix O3 is weak coherent and we have:

Vet(O3) = {0.172, 0.453, 0.147, 0.079, 0.092, 0.057}

Weights of the criteria with respect to O4

The matrix of the pairwise comparison between the criteria with respect to the objective O4 is:

O4 = {{1,1,3,3,3,2}, {1,1,2,3,1,3}, {1/3,1/3,1,3,2,3}, {1/3,1/3,1/3,1,3,1}, {1/3,1/3,1/2,1/3,1,1}, {1/2,1/3,1/3,1,1,1}}

We have $\lambda(O4) = 6.21$ and $CI(O4) = 0.04 < 0.1$. The matrix O4 is strong coherent. Moreover

Vet(O4) = {0.292, 0.254, 0.179, 0.108, 0.076, 0.091}.

Absolute weights of the criteria

The absolute weights of the criteria, i.e. the weights of the criteria with respect to the general objective, are obtained by the product rows by columns of the vector Vet(OG) and the matrix M(O) = {Vet(O1), Vet(O2), Vet(O3), Vet(O4)} with 4 rows and 6 columns having as i-tuple row the vector of the criteria weights with respect to the objective Oi. Let Pes(C) be the row vector of the wheights of criteria. We have:

Pes(C) = Vet(GO) M(O) = {0.214, 0.382, 0.160, 0.087, 0.088, 0.068}.

5.3 Calculation of the scores of the strategies

Scores of strategies with respect to the criterion C1

The pairwise comparison matrix is:

C1 = {{1,3,1/3,1/7}, {1/3,1,1/5,1/9}, {3,5,1,1/3}, {7,9,3,1}}.

We have λ(C1) = 4.09 and CI(C1) = 0.03 < 0.1. The matrix C1 is strong coherent and Vet(C1) = {0.101, 0.049, 0.243, 0.607}.

Scores of strategies with respect to the criterion C2

The pairwise comparison matrix is:

C2 = {{1,1/3,1/3,1/5}, {3,1,1,1/3}, {3,1,1,1/3}, {5,3,3,1}}

We have λ(C2) = 4.04 and CI(C2) = 0.01 < 0.1. The matrix C2 is strong coherent and Vet(C2) = {0.078, 0.200, 0.200, 0.522}

Scores of strategies with respect to the criterion C3

The pairwise comparison matrix is:

C3 = {{1,3,1/3,1/5}, {1/3,1,1/3,1/5}, {3,3,1,1/3}, {5,5,3,1}}

We have λ(C3) = 4.20 and CI(C3) = 0.07 < 0.1. The matrix C3 is strong coherent and Vet(C3) = {0.129, 0.074, 0.248, 0.549}.

Scores of strategies with respect to the criterion C4

The pairwise comparison matrix is:

C4 = {{1,1/3,1/5,1/5}, {3,1,1/5,1/5}, {5,3,1,1/3}, {5,5,3,1}}

We have and λ(C4) = 4.12 and CI(C4) = 0.04 < 0.1. The matrix C4 is strong coherent and Vet(C4) = {0.065, 0.115, 0.272, 0.548}

Scores of strategies with respect to the criterion C5

The pairwise comparison matrix is:

C5 = {{1,1/2,1/3,1/3}, {2,1,1/2,1}, {3,2,1,1/2}, {3,2,1,1}}

We have λ(C5) = 4.12 and CI(C5) = 0.04 < 0.1. The matrix C5 is strong coherent and Vet(C5) = {0.108, 0.232, 0.309, 0.351}.

Scores of strategies with respect to the criterion C6

The pairwise comparison matrix is:

C6 = {{1,5,1/3,1/3}, {1/5,1,1/5,1/7}, {3,5,1,1/3}, {3,7,3,1}}

We have λ(C6) = 4.23 and IC(C6) = 0.08 < 0.1. The matrix C6 is strong coherent and Vet(C6) = {0.159, 0.050, 0.278, 0.513}.

Absolute scores of strategies

Let N(C) be the matrix with 6 rows and 4 columns with the i-tuple row equal to the vector of the scores of the strategies with respect to the criterion Ci, i.e. N(C) = {Vet(C1), Vet(C2), Vet(C3), Vet(C4), Vet(C5), Vet(C6)}.

The absolute scores of the strategies, i.e. the scores of the strategies with respect to the general objective are obtained by making the product rows by columns of the vector Pe(C) of the criteria weights and the matrix N(C). Let us denote with Pun(S) the vector row of the absolute scores of the strategies. We obtain:

Pun(S) = Pes(C) N(C) = {0.098, 0.132, 0.238, 0.532}.

6 Conclusions and perspectives of research

The hierarchical analytical procedure of Saaty leads to clearly prefer the strategy A4 with a score of 0.532. The strategy A3 follows with the score 0.238. The strategies A2 and A1 appear to be less effective, with scores of 0.132 and 0.098 respectively.

It should be noted, however, that these scores depend on the assessments, information, experiences of the decision makers who have attributed the weights to the various arcs of the Saaty digraph that link the general objective with the specific ones and the latter with the criteria. They also depend on the scores attributed to the strategies with respect to each criterion.

The scores of the strategies are therefore consistent with the objectives and opinions of the decision makers, but could change with decision makers who have different opinions.

A more detailed analysis could be done considering the constraints. For example, it may be necessary to establish for each objective Oj a threshold $\alpha j>0$, with the constraint $pij \geq \alpha j$, for each i.

In this case it may also be important to consider mixed strategies. A mixed strategy A(h1, h2, h3, h4) = h1 A1 + h2 A2 + h3 A3 + h4 A4 has the score p(h1, h2, h3, h4) = h1 0.098 + h2 0.132 + h3 0.238 + h4 0.532, below the score of strategy A4, but may have the advantage of meeting the various objectives in a more balanced manner. In particular, in presence of constraints, the optimal mixed strategy may be that which maximizes the score p(h1, h2, h3, h4) with the various thresholds, positivity and convexity constraints, or others that are considered opportune.

An alternative interpretation of the achieved results could be to follow each strategy according to a percentage of time equal to the score obtained. For example, a mixed strategy can be followed in which for 53.2% of the time the adopted strategy is A4, for 23.8% is A3, for 13.2% is A2 and finally for 9.8% is A1.

The evaluation of the advantages and disadvantages in the diversification of the strategies could be experimented in class, evaluating the reaction of the students. Probably a mixed strategy, focusing on diversity, can capture the attention of a greater number of children or at least not penalize those with attitudes and mentalities not aligned with the majority or the standards expected by decision makers.

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