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Relevance of financial information in quick loans negotiation

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Abstract

Nowadays, most loan transactions are contracted by using the exponential discounting as the underlying standard economic model to value this type of financial operations. In a framework of absence of fees to be paid by the borrower, the interest rate of the exponential discount function is, moreover, the true interest rate of the operation. Nevertheless, there exist a set of circumstances which make this identity false. Among others, these characteristics are: the use of linear discount as the underlying discount function, splitting time when using a nominal interest rate, and the existence of fees in a loan at 0% interest rate. All these cases will be analyzed in this paper in the context of the so-called quick loans.

Keywords: Quick loans, linear discounting, exponential discounting, true interest rate.

1. Introduction

Banking loans exhibit a set of characteristics which are usual in all financial operations offered by banks (Brealey and Myers, 2002):

- The constant payment is calculated by using the exponential discounting (Gil Peláez, 1993).
- The interest rate (constant or variable) is nominal. Therefore, splitting the period of interest results in an increase of the so-called true interest rate (Cruz Rambaud and Valls Martínez, 2014).
- There exist some initial and final fees which increase the true cost for the borrower. In some cases, there are also some fees to be paid at the end of each period of interest. Of course, they result in an additional profitability for the lender (Ferruz Agudo, 1994).

In effect, when dealing with banking loans, the constant payment (denoted by a) is usually calculated by using the French method (Valls Martínez and Cruz Rambaud, 2013), viz:

$$a = \frac{C_0 i_{(k)}}{1 - (1 + i_{(k)})^{nk}}\,, \tag{1}$$

where:

- C_0 is the loan principal.
- k is the number of subperiods into which is divided any period of interest.
- $i_{(k)} = \frac{i}{k}$ is the interest rate to be applied to a subperiod of length $\frac{1}{n}$.
- n is the loan duration (in years).

In this context, the true interest of the financial operation arises from the following equation:

$$C_0 = G_0 + a\frac{1 - (1 + i^*_{(k)})^{-nk}}{i^*_{(k)}}\,, \tag{2}$$

from where:

$$i^* = (1 + i_{(k)})^k - 1\,. \tag{3}$$

In this way, the Bank of Spain (Banco de España, 1990) introduced the so-called *Annual Equivalent Interest Rate*[1], denoted by i^*, which includes all amounts paid by the borrower (periodic payments and fees) and received by the lender. Therefore, this parameter measures the true interest supported by the borrower in a financial operation (Van Horne, 1997). If all fees are received by the lender, his true profitability is also i^* (Gil Luezas and Gil Peláez, 1987). The main objective of this paper is the calculation of the annual equivalent rate of several types of loans as a parameter which will allow us to choose a loan among a set of offers.

This parameter is very important because it offers a true measure of the cost (in percentage) of the financial transaction. In effect, the current economic crisis has resulted in a restriction of credits for families and companies. These limitations affected the principal, the duration, and the conditions to be satisfied by the borrower. In this context, the presence of unmet needs by families has favored the increase of the demand, and consequently the supply, of the so-called *quick loans*. One of the main characteristics of this type of loans is that the lender companies take advantage from the necessities of families by requiring excessive conditions such as the use of linear discounting instead of the exponential as the underlying model to amortize the loan, or the exorbitant interest rates applied in these amortizing transactions. This paper analyzes the true cost (resp. profitability) for the borrower (resp. lender) in these loans.

Thus, the paper is organized as follows. After this Introduction, Section 2 presents some preliminary concepts which are necessary to the development of the rest of the paper. Section 3 describes and analyzes those loans whose amortization schedule is built based on the linear discount function. Section 4 describes those loans at 0% interest rate with initial fees to be paid by the borrower. These fees makes that the true interest rate of the transaction is different from zero. Finally, Section 5 summarizes and concludes.

2. Preliminaries

For the sake of clarity, first we are going to introduce some definitions in order to present these previous important concepts.

Definition 1 (Cruz Rambaud and Valls Martínez, 2016). A *loan transaction* is a couple of sets of instalments (with specification of their respective maturities):

$$\wp = \{(C_0, 0)\},$$

[1] In Spain, this parameter has been named *Tasa Anual Equivalente* (acronym: TAE).

and

$$\aleph = \{(a_1,1),(a_2,2),\ldots,(a_n,n)\},$$

where $\wp$ represents the amount (*principal*) paid by a person, called the *lender*, and received by another person, called the *borrower*, and $\aleph$ denotes the series of amounts paid by the borrower so that the final balance is 0. Usually, the equilibrium between these two sets of payments is achieved by projecting all involved maturities up to the time 0 with the exponential discount function, such that

$$C_0 = \sum_{s=1}^{n} a_s (1+i)^{-s}, \tag{4}$$

which is the so-called *equation of financial equivalence at instant* 0 (De Pablo, 2000).

If the loan transaction includes some fees, the equation of financial equivalence (4) does not hold. Therefore, it is convenient to introduce the following definition.

Definition 2 (Valls Martínez and Cruz Rambaud, 2016; Valls Martínez *et al.*, 2017). The *true interest rate* of the loan operation introduced in Definition 1 is the interest rate (denoted by i^*) which restores the equilibrium between the two sets of payments:

$$C_0 = G_0 + \sum_{s=1}^{n} (a_s + g_s)(1+i^*)^{-s} + G_n (1+i^*)^{-n}, \tag{5}$$

where:

- G_0 is the amount of initial fees,
- g_s denotes the periodic fees, and
- G_n represents the total final fees.

3. Amortizing loans with linear discounting

The amortization schedule of a loan by using the linear discounting (Ayres, 1963) as the underlying discount function is the following:

s	I_s	A_s	a_s	C_s
0	-	-	-	C_0
1	$C_0 i$	$\frac{C_0}{n}$	$C_0\left(\frac{1}{n}+i\right)$	$\frac{n-1}{n}C_0$
2	$C_0 i$	$\frac{C_0}{n}$	$C_0\left(\frac{1}{n}+i\right)$	$\frac{n-2}{n}C_0$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$C_0 i$	$\frac{C_0}{n}$	$C_0\left(\frac{1}{n}+i\right)$	0

Table 1. Loan amortization schedule with linear discounting. **Source**: Own elaboration.

Observe that this amortization schedule is a mix of:

- The American amortization method (Valls Martínez and Cruz Rambaud, 2015) since all interest quotas (I_s) are constant and equal to $C_0 i$.
- The amortization method of constant repayment (Bodie *et al.*, 2004) since all repayments (A_s) are constant and equal to $\frac{C_0}{n}$.
- The French amortization method (Biehler, 2008) since all periodic payments (denoted by a_s) are constant and equal to $C_0\left(\frac{1}{n}+i\right)$.

As a consequence, the outstanding principal (C_s) is a decreasing arithmetic progression with difference $\frac{C_0}{n}$. The true interest rate results from the following equation:

$$C_0 = G_0 + \left(\frac{C_0}{n} + C_0 i^*\right) a_{\overline{n|}i^*}, \tag{6}$$

that is to say

$$C_0 = G_0 + \left(\frac{C_0}{n} + C_0 i^* \right) \frac{1-(1+i^*)^{-n}}{i^*}. \qquad (7)$$

Example 1. Let us consider a loan with the following characteristics:

- $C_0 = \$50.000$.
- $G_0 = \$2.000$.
- n = 5 years.
- i = 7%.

The amortization schedule is the following:

s	I_s	A_s	a_s	C_s
0	-	-	-	\$50,000
1	\$3,500	\$10,000	\$10,000	\$40,000
2	\$3,500	\$10,000	\$10,000	\$30,000
3	\$3,500	\$10,000	\$10,000	\$20,000
4	\$3,500	\$10,000	\$10,000	\$10,000
5	\$3,500	\$10,000	\$10,000	\$0

Table 2. Amortization schedule of the loan in Example 1. **Source**: Own elaboration.

The true interest rate arises from the following equation:

$$50{,}000 = 2{,}000 + 13{,}500 \frac{1-(1+i^*)^{-5}}{i^*}, \qquad (8)$$

from where $i^* = 12.56\%$. Observe that, as $a_{\{ni\}}$ is decreasing with respect to i (Brigham and Daves, 2007), one has the following chain of implications:

$$i \uparrow \Rightarrow \frac{n(C_0 - G_0)}{C_0(1+in)} \downarrow \Rightarrow i^* \uparrow.$$

In effect, without considering the initial fees, the true interest rate is $i^* = 10.92\%$, whilst by considering i = 10%, the true interest rate is $i^* = 15.24\%$. Analogously, if n is large enough, we can state that the true interest rate is also increasing. Thus, for a duration of ten years for the previous loan, one has $i^* = 11.03\%$.

It is noteworthy to take into account that a loan amortized by using the linear discount exhibits a true interest rate much higher than i.

4. Amortizing loans with 0% interest rate and initial fees

The amortization schedule of a loan at 0% interest rate is the following:

s	I_s	A_s	a_s	C_s
0	-	-	-	C_0
1	0	$\frac{C_0}{n}$	$\frac{C_0}{n}$	$\frac{n-1}{n}C_0$
2	0	$\frac{C_0}{n}$	$\frac{C_0}{n}$	$\frac{n-2}{n}C_0$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	0	$\frac{C_0}{n}$	$\frac{C_0}{n}$	0

Table 3. Amortization schedule of a loan at 0% interest rate. **Source**: Own elaboration.

Observe that this amortization schedule is a mix of:

- The amortization method of constant repayment since all repayments (A_s) are constant and equal to $\frac{C_0}{n}$.
- The loans with waiting periods (no interests) for all periods of time (Brealey *et al.*, 2006).
- The French amortization method since all periodic payments (a_s) are constant and equal to $C_0\left(\frac{1}{n}+i\right)$.

As a consequence, the outstanding principal (C_s) is decreasing in arithmetic progression with difference $\frac{C_0}{n}$. In case of existence of initial fees, denoted by G_0, the true interest rate results from the following equation:

$$C_0 = G_0 + \frac{C_0}{n} a_{\overline{n}|i^*}, \tag{9}$$

that is to say

$$C_0 = G_0 + \frac{C_0}{n}\frac{1-(1+i^*)^{-n}}{i^*}. \qquad (10)$$

Example 2. Let us consider the loan of Example 1. In this case, the amortization schedule is the following:

s	I_s	A_s	a_s	C_s
0	-	-	-	$50,000
1	$0	$10,000	$10,000	$40,000
2	$0	$10,000	$10,000	$30,000
3	$0	$10,000	$10,000	$20,000
4	$0	$10,000	$10,000	$10,000
5	$0	$10,000	$10,000	$0

Table 4. Amortization schedule of the loan in Example 2. **Source**: Own elaboration.

The true interest rate arises from the following equation:

$$50{,}000 = 2{,}000 + 10{,}000\frac{1-(1+i^*)^{-5}}{i^*}, \qquad (11)$$

from where $i^* = 1.38\%$. Observe that, as a{ni} is decreasing with respect to i, one has the following implications:

$$G_0 \uparrow \Rightarrow C_0 - G_0 \downarrow \Rightarrow i^* \uparrow.$$

5. Conclusion

The annual equivalent interest rate is a noteworthy parameter able to measure the cost (resp. profitability) for the borrower (resp. lender) in a loan transaction. As shown in this paper, it has been obtained as the solution of the equation of financial equivalence which makes equal the true amounts paid and received by the borrower. In general, this parameter is an accurate measure of the cost/profitability of any financial operation. In particular, in the case of quick loans, it allows us to know the exorbitant true interest rates (sometimes near 25% interest rate) applied in these amortizing loans. Therefore, the annual equivalent interest rate quantifies the financial information of all banking products, in particular the quick loans offered by some private companies.

But, in the case of loan transactions, it allows us to choose among several offers of credits becoming an important instrument to negotiate that loan with the smallest true interest rate. Finally, this paper shows that even the loans at 0% interest rate can represent an excessive cost for borrower. Indeed, the annual equivalent interest rate is a relevant instrument for loans negotiation.

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