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Properties of an anti-vague filter in *BL*- algebras

S. Yahya Mohamed*
P. Umamaheswari†

Abstract

In this paper, the concept of an anti-vague filter of a *BL*-algebra is introduced with suitable illustration, and also obtained some related properties. Further, we have investigated some more equivalent conditions of anti-vague filter.

Keywords: *BL*-algebra; filter; implicative filter; vague set; vague filter; anti-vague filter

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*Assistant Professors, PG and Research Department of Mathematics, Government Arts College, Tiruchirappalli-620 022. Affiliated to Bharathidasan University, Trichirappalli, Tamilnadu, India; yahya_md@yahoo.com

†Assistant Professors, PG and Research Department of Mathematics, Government Arts College, Tiruchirappalli-620 022. Affiliated to Bharathidasan University, Trichirappalli, Tamilnadu, India; umagactrichy@gmail.com

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1. Introduction

Håjek [5] introduced the idea of *BL*-algebras as the algebraic structure for his Basic Logic. The interval [0, 1] endowed with the structure induced by a continuous t- norm is a well-known example of *BL*- algebra. The MV-algebras, on the other hand, are one of the most well-known groups of BL-algebras, having been introduced by Chang [2] in 1958. In 1965, Zadeh [12] introduced the concept of a fuzzy set. The flaw in fuzzy sets is that they only have one feature, which means they cannot convey supporting and opposing data. Gau and Buehrer [4] introduced the principle of vague set in 1993 as a result of this. The authors [7, 8, 9, 10] discussed the vague filter, implicative filter, prime, and Boolean implicative filters of *BL*- algebras, as well as some of their properties.

The frame work of this study is constructed as follow: some basic observations connected to anti-vague filter are provided in "Preliminaries". "Anti-vague filter" presents the new notions of anti-vague filter in *BL*-algebra and investigated some related properties, also derived some equivalent conditions for an anti-vague filter to be a vague filter. Finally, the conclusion is presented in "Conclusion".

2. Preliminaries

In this section, we will go through some basic *BL*-algebra, filter, and vague set concepts, as well as their properties, which will help in the development of the main results.

Definition 2.1[5] A *BL*-algebra is an algebra $(A, \vee, \wedge, *, \rightarrow, 0, 1)$ of type (2, 2, 2, 2, 0, 0) such that

(i) $(A, \vee, \wedge, 0, 1)$ is a bounded lattice,
(ii) $(A, *, 1)$is a commutative monoid,
(iii) $*$ and $\rightarrow$ form an adjoint pair, that is, $z \leq x \rightarrow y$ if and only if $x * z \leq y$ for all $x, y, z \in A$,
(iv) $x \wedge y = x * (x \rightarrow y)$,
(v) $(x \rightarrow y) \vee (y \rightarrow x) = 1$.

Proposition 2.2[6] In a *BL*- algebra *A*, the following properties are hold for all $x, y, z \in A$,

(i) $y \rightarrow (x \rightarrow z) = x \rightarrow (y \rightarrow z) = (x * y) \rightarrow z$,
(ii) $1 \rightarrow x = x$,
(iii) $x \leq y$ if and only if $x \rightarrow y = 1$,
(iv) $x \vee y = \big((x \rightarrow y) \rightarrow y\big) \wedge \big((y \rightarrow x) \rightarrow x\big)$,
(v) $x \leq y$ implies $y \rightarrow z \leq x \rightarrow z$,

(vi) $x \leq y$ implies $z \to x \leq z \to y$,
(vii) $x \to y \leq (z \to x) \to (z \to y)$,
(viii) $x \to y \leq (y \to z) \to (x \to z)$,
(ix) $x \leq (x \to y) \to y$,
(x) $x * (x \to y) = x \wedge y$,
(xi) $x * y \leq x \wedge y$
(xii) $x \to y \leq (x * z) \to (y * z)$,
(xiii) $x * (y \to z) \leq y \to (x * z)$,
(xiv) $(x \to y) * (y \to z) \leq x \to z$,
(xv) $(x * x^{-}) = 0$.

Note. In the sequel, we shall use A to denote as *BL*- algebras and the operation $\vee, \wedge, *$ have priority towards the operations " $\to$ ".
Note. In a *BL*- algebra A, we can define $x^{-} = x \to 0$ for all $x \in A$.

Definition 2.3[13] A filter of a *BL*- algebra A is a non-empty subset F of A such that for all $x, y \in A$,

(i) If $x, y \in F$, then $x * y \in F$,
(ii) If $x \in F$ and $x \leq y$, then $y \in F$.

Proposition 2.4[13] Let F be a non-empty subset of a *BL*- algebra *A*. Then, F is a filter of A if and only if the following conditions are hold

(i) $1 \in F$,
(ii) $x, x \to y \in F$ implies $y \in F$.

A filter *F* of a *BL*-algebra *A* is proper if $F \neq A$.

Definition 2.5[1, 3, 4] A vague set S in the universe of discourse X is characterized by two membership functions given by

(i) A truth membership function $t_S: X \to [0, 1]$,
(ii) A false membership function $f_S: X \to [0, 1]$.

Where $t_S(x)$ is lower bound of the grade of membership of *x* derived from the 'evidence for *x*', and $f_S(x)$ is a lower bound of the negation of *x* derived from the 'evidence against *x*' and $t_S(x)+f_S(x) \leq 1$. Thus the grade of membership of *x* in the vague set *S* is bounded by a subinterval $[t_S(x), 1 - f_S(x)]$ of [0, 1]. The vague set S is written as $S = \{(x, [t_S(x), f_S(x)])/x \in X\}$, where the interval $[t_S(x), 1 - f_S(x)]$ is called the value of x in the vague set S and denoted by $V_S(x)$.

Definition 2.6[4] A vague set S of a set X is called

(i) the zero vague set of X if $t_S(x) = 0$ and $f_S(x) = 1$ for all $x \in X$,
(ii) the unit vague set of X if $t_S(x) = 1$ and $f_S(x) = 0$ for all $x \in X$,

(iii) the α- vague set of X if $t_S(x) = \alpha$ and $f_S(x) = 1 - \alpha$ for all $x \in X$, where $\alpha \in (0, 1)$.

Definition 2.7[4] Let S be a vague set of X with truth membership function t_S and the false membership function f_S. For all $\alpha, \beta \in [0,\ 1]$, the $(\alpha,\ \beta)$-cut of the vague set X is crisp subset $S_{(\alpha,\beta)}$of the set X by $S_{(\alpha,\beta)} = \{V(x) \geq [\alpha, \beta]/x \in X\}$. Obviously, $S_{(0,0)} = X$.

Definition 2.8[4] Let $D[0, 1]$ denote the family of all closed subintervals of [0, 1]. Now, we define refined maximum (*rmax*) and "$\geq$" on elements $D_1 = [a_1, b_1]$ and $D_2[a_2,\ b_2]$ of $D[0, 1]$ as $rmax\ (D_1, D_2) = [\max\{a_1, a_2\}, \max\{b_1, b_2\}]$. Similarly, we can define $\leq, =$ and *rmin*.

3. Anti-Vague Filter

In this section, we introduce the notion of an anti-vague filter of *BL*-algebra with illustration. Moreover, we discuss some related properties.

Definition 3.1 Let S be vague set of a *BL*-algebra A is called an anti vague filter of A if it satisfies the following axioms

(i) $V_S(1) \leq V_S(x)$,

(ii) $V_S(y) \leq rmax\{V_S(x \to y), V_S(x)\}$ for all $x, y \in A$.

Proposition 3.2 Let S be vague set of *BL*-algebra A. S is an anti vague filter of A if and only if the following hold if for all $x, y \in A$,

(i) $t_S(1) \leq t_S(x)$ and $1 - f_S(1) \leq 1 - f_S(x)$,

(ii) $t_S(y) \leq \max\{t_S(x \to y), t_S(x)\}$ and
$1 - f_S(y) \leq \max\{1 - f_S(x \to y), 1 - f_S(y)\}$.

Proof: Let S be an anti-vague filter of A. Then from (i) of definition 3.1 and the definition of V_S, we have (i) straight forward. From (ii) of definition 3.1and the definition of V_S, (ii) is obvious. ■

The following is the example of definition 3.1 and proposition 3.2.

Example 3.3 Let $A = \{0, a, b, 1\}$. The binary operations $' * '$ and $' \to '$ give by the following tables 3.1 and 3.2:

Table3.1: ‘ $*$ ′Operator

$*$	0	a	b	1
0	0	0	0	0
a	0	0	a	b
b	0	a	b	b
1	0	a	b	1

Table 3.2: ‘→’ Operator

$\rightarrow$	0	a	b	1
0	1	1	1	1
a	a	1	1	1
b	0	a	1	1
1	0	a	b	1

Then $(A, \vee, \wedge, *, \rightarrow, 0, 1)$ is a BL- algebra. Define a vague set S of A as follows:

$$S = \{(1, [0.2, \ 0.7]), (a, [0.3, \ 0.5]), (b, [0.3, \ 0.5]), (0, [0.2, \ 0.7])\}.$$

It is easily verified that S is an anti-vague filter of A and satisfy the conditions (i) and (ii) of proposition 3.2.

Proposition 3.4 Every anti-vague filter S of BL- algebra A is order preserving.

Proof: Let S be an anti-vague filter of BL-algebra A.

Then, we prove that if $x \leq y$, then $V_S(x) \geq V_S(y)$ for all $x, y \in A$.

From (ii) of the proposition 3.2, we have,

$$t_S(y) \leq \max\{t_S(x \rightarrow y), t_S(x)\}$$

$= \max\{t_S(1), t_S(x)\},$

[From (iii) of proposition 2.2]

Also, we have $1 - f_S(y) \leq \max\{1 - f_S(x \to y), 1 - f_S(y)\}$.

From (i) of the proposition 3.2, we have $t_S(1) \leq t_S(x)$ and $1 - f_S(1) \leq 1 - f_S(x)$. Thus, $t_S(y) \leq \max\{t_S(x), 1 - f_S(y)\}$

$\leq 1 - f_S(y)$, and so

$$\begin{aligned} V_S(y) &= [t_S(y), 1 - f_S(y)] \\ &\leq [t_S(x), 1 - f_S(x)] \\ &= V_S(x). \end{aligned}$$

Hence $V_S(x) \geq V_S(y)$. ■

Proposition 3.5 Let S be a vague set of *BL*- algebra A, S be an anti-vague filter of A if and only if $x \to (y \to z) = 1$ implies $V_S(z) \leq rmax\{V_S(x), V_S(y)\}$ for all $x, y, z \in A$.

Proof: Let S be an anti-vague filter of *BL*-algebra A.

Then, from (ii) of the definition 3.1, we have

$V_S(z) \geq rmax\{V_S(z \to y), V_S(y)\}$ for all $x, y, z \in A$.

Now, $V_S(z \to y) \leq rmax\{V_S(x \to (y \to z), V_S(x)\}$.

If $x \to (y \to z) = 1$, then we have

$V_S(z \to y) \leq rmax\{V_S(1), V_S(x)\} = V_S(x)$.

So, $V_S(z) \leq rmax\{V_S(x), V_S(y)\}$.

Conversely, since $x \to (x \to 1) = 1$ for all $x \in A$.

Then $V_S(1) \leq rmax\{V_S(x), V_S(x)\}$

$= V_S(x)$.

On the other hand, from $(x \to y) \to (x \to y) = 1$.

It follows that $V_S(y) \leq rmax\{V_S(x \to y), V_S(x)\}$.

From the definition 3. 1, S is the anti vague filter of A. ■

From (i) of the proposition 2.2, and the proposition 3.5, we have the following.

Corollary 3.6 Let S be vague set of BL- algebra A, S be an anti vague filter of A if and only if $x * y \leq z$ or $y * x \leq z$ implies $V_S(z) \leq rmax\{V_S(x), V_S(y)\}$ for all $x, y, z \in A$.

Proposition 3.7 Let S be a vague set of BL- algebra A, S be an anti vague filter of A if and only if (i) $x \leq y$, then $V_S(x) \geq V_S(y)$,

(ii) $V_S(x * y) \leq rmax\{V_S(x), V_S(y)\}$ for all $x, y, \in A$.

Proof: Let S be an anti vague filter of BL- algebra A. Then, from the proposition 3.4, we have $x \leq y$, $V_S(x) \geq V_S(y)$.

Since $x * y \leq x * y$ and corollary 3.6, we have $V_S(x * y) \leq rmax\{V_S(x), V_S(y)\}$.

Conversely, let S be a vague set and satisfies (i) and (ii). For all $x, y, z \in A$, if $x * y \leq z$, then from (i) and (ii), we get $V_S(z) \leq rmax\{V_S(x), V_S(y)\}$.

From corollary 3.6, we have S is an anti vague filter. ■

Proposition 3.8 Let S be a vague set of BL- algebra$A$. Let S be an anti vague filter of A. The following holds for all $x, y, z \in A$,

(i) If $V_S(x \to y) = V_S(1)$, then $V_S(x) \geq V_S(y)$,

(ii) $V_S(x \vee y) = rmax\,\{V_S(x), V_S(y)\}$,

(iii) $V_S(x * y) = rmax\,\{V_S(x), V_S(y)\}$,

(iv) $V_S(1) = rmax\{V_S(x), V_S(x^-)\}$,

(v) $V_S(x \to z) \leq rmax\,\{V_S(x \to y), V_S(y \to z)\}$,

(vi) $V_S(x \to y) \geq V_S(x * z \to y * z)$,

(vii) $V_S(x \to y) \geq V_S((y \to z) \to (x \to z))$,

(viii) $V_S(x \to y) \geq V_S\big((z \to x) \to (z \to y)\big)$.

Proof: (i) Let S be an anti vague filter of BL- algebra A.

Then, from the definition 3.1, and since $V_S(x \to y) = V_S(1)$.

We have $V_S(y) \leq rmax\{V_S(x), V_S(x \to y)\}$

$$= rmax\{V_S(x), V_S(1)\}$$
$$= V_S(x).$$

Thus, $V_S(x) \geq V_S(y)$.

(ii) Since $x \vee y \geq x$ and $x \vee y \geq y$.

From the proposition 3.4, we get $V_S(x \vee y) \leq rmax\{V_S(x), V_S(y)\}$.

From the definition 3.1 we have

$$V_S(x \vee y) \leq rmax\{V_S(x \rightarrow (x \vee y)), V_S(x)\}$$
$$= rmax\{V_S((x \rightarrow x) \vee (x \rightarrow y)), V_S(x)\}$$
$$= rmax\{V_S(x \rightarrow y), V_S(x)\}$$
$$\leq rmax\{rmax\{V_S\big(y \rightarrow (x \rightarrow y)\big), V_S(y)\}, V_S(x)\}$$
$$= rmax\{rmax\{V_S(1), V_S(y)\}, V_S(x)\}$$
$$= rmax\{V_S(y), V_S(x)\}$$
$$= rmax\{V_S(x), V_S(y)\}$$

Hence, $V_S(x \vee y) = rmax\ \{V_S(x), V_S(y)\}$.

(iii) From (ii) of proposition 3.7, we have

$V_S(x * y) \leq rmax\{V_S(x), V_S(y)\}$.

Since $x * y \geq x \vee y$, proposition 3.4, and (ii), we have

$$V_S(x * y) \geq V_S(x \vee y)$$
$$= rmax\{V_S(x), V_S(y)\}.$$

Thus, $V_S(x * y) = rmax\ \{V_S(x), V_S(y)\}$.

(iv) From (iii), we have $rmax\ \{V_S(x), V_S(x^-)\} = V_S(x * x^-) = V_S(1)$.

Therefore, $V_S(1) = rmax\ \{V_S(x), V_S(x^-)\}$.

(v) From (iii) and proposition 3.4, since $(x \rightarrow y) * (y \rightarrow z) \leq x \rightarrow z$,

we get $V_S((x \rightarrow y) * (y \rightarrow z)) \geq V_S((x \rightarrow z)$,

$rmax\{V_S\big((x \rightarrow y), V_S(y \rightarrow z)\big)\} \geq V_S((x \rightarrow z)$.

Therefore, we have $V_S(x \rightarrow z) \leq rmax\ \{V_S(x \rightarrow y), V_S(y \rightarrow z)\}$.

From the proposition 2.2 and (i) of proposition 3.7 we can prove (vi), (vii) and (viii) easily. ■

Proposition 3.9 Let S be a vague set of BL- algebra A, S be an anti vague filter of A if and only if (i) $V_S(1) \leq V_S(x)$,

(ii) $V_S\left(\left(x \to (y \to z)\right) \to z\right) \leq rmax\{V_S(x), V_S(y)\}$

for all $x, y, \in A$.

Proof: Let S be an anti vague filter of A. By the definition 3.1, (i) is straight forward.

Since, $V_S\left(\left(x \to (y \to z)\right) \to z\right) \leq rmax\left\{V_S\left(\left(x \to (y \to z)\right) \to (y \to z)\right), V_S(y)\right\}$. (3.1)

Now, we have $(x \to (y \to z)) \to (y \to z) = x \vee (y \to z) \geq x$.

$V_S((x \to (y \to z)) \to (y \to z)) \leq V_S(x)$. [from the proposition 3.4] (3.2)

Using (3.2) in (3.1), we have $V_S((x \to (y \to z)) \to z) \leq rmax\{V_S(x), V_S(y)\}$.

Conversely, suppose (i) and (ii) hold.

Since $V_S(y) = V_S(1 \to y)$

$= V_S(((x \to y) \to (x \to y) \to y)$

$\leq rmax\{V_S(x \to y), V_S(y)$.

From (i), S is an anti vague filter of A. ■

Proposition 3.10 Intersection of two anti vague filters of A is also an anti vague filter of A.

Proof: Let U and W be two anti vague filters of A.

To Prove: $U \cap W$ is an anti vague filter of A.

For all $x, y, z \in A$ such that $z \leq x \to y$, then $z \to (x \to y) = 1$.

Since, U, W are two anti vague filters A, we have $V_U(y) \leq rmax\{V_U(z), V_U(x)\}$ and $V_W(y) \leq rmax\{V_W(z), V_W(x)\}$.

That is, $t_U(y) \leq \max\{t_U(z), t_U(x)\}$ and

$1 - f_U(y) \leq \max\{1 - f_U(z), 1 - f_U(x)\}$, $t_W(y) \leq \max\{t_W(z), t_W(x)\}$ and

$1 - f_W(y) \leq \max\{1 - f_W(z), 1 - f_W(x)\}$.

Since, $t_{U\cap W}(y) = \min\{t_U(y), t_W(y)\}$

$\leq \max\{\max\{t_U(z), t_U(x)\}, \max\{t_W(z), t_W(x)\}\}$

$= \max\{\max\{t_U(z), t_W(z)\}, \max\{t_U(x), t_W(x)\}\}$

$= \max\{\max\{t_{U\cap W}(z), t_{U\cap W}(x)\}\}$

and $1 - f_{U\cap W}(y) = \max\{1 - f_U(y), 1 - f_W(y)\}$

$\leq \max\{\max\{1 - f_U(z), 1 - f_U(x)\}, \max\{1 - f_W(z), 1 - f_W(x)\}\}$

$= \max\{\max\{1 - f_U(z), 1 - f_W(z)\}, \max\{1 - f_U(x), 1 - f_W(x)\}\}$

$= \max\{\max\{1 - f_{U\cap W}(z), 1 - f_{U\cap W}(x)\}\}$.

Hence, $V_{U\cap W}(y) = [t_{U\cap W}(y),\ 1 - f_{U\cap W}(y)] \leq rmax\{V_{U\cap W}(z), V_{U\cap W}(x)\}$.

Thus $U \cap W$ is an anti vague filter of A. ■

Corollary 3.11 Let R_j be a family of anti vague filters of A, where $j \in I$, I is a index set, then $\bigcap_{j\in I} R_j$ is an anti vague filter of A.

Note: Union two anti vague filters of *BL*- algebra A need not be an anti vague filter of A.

Proposition 3.12 A ρ- vague set and zero vague set of a *BL*-algebra A are anti vague filters of A.

Proof: Let S be a ρ-vague set of *BL*-algebra A, and S be an anti vague filter of A.

Then, from the proposition 3.4, we have if $x \leq y$, then $V_S(x) \geq V_S(y)$ for all $x, y, \in A$.

To prove: $V_S(x * y) \leq rmax\{V_S(x), V_S(y)\}$ for all $x, y, \in A$.

Now, $t_S(x * y) = \rho = \max\{\rho,\ \rho\} = \max\{t_S(x), t_S(y)\}$ (3.3)

and $1 - f_S(x * y) = \rho = \max\{\rho,\ \rho\} = \max\{1 - f_S(x), 1 - f_S(y)\}$ for all $x, y, \in A$ (3.4)

From (3.3) and (3.4), we have $V_S(x * y) \leq rmax\{V_S(x), V_S(y)\}$.

Thus, ρ- vague set is an anti vague filter of A.

Similarly, we prove zero vague set is an anti vague of A. ■

Theorem 3.13 Let S be a vague set of BL-algebra A, S be an anti vague filter of A if and only if the set $S_{(\rho,\sigma)}$ is either empty or a filter of A for all $\rho, \sigma \in [0,\ 1]$, where $\rho \leq \sigma$.

Proof: Let S be an anti vague filter of BL-algebra A and $S_{(\rho,\sigma)} \neq \emptyset$ for all $\rho, \sigma \in [0,\ 1]$.

To prove: $S_{(\rho,\sigma)}$ is a filter of A.

If $x \leq y$ and $x \in S_{(\rho,\sigma)}$. From the proposition 3.12, we have $V_S(y) \leq V_S(x) \leq [\rho, \sigma]$ for all $x, y \in A$.

Thus, $y \in S_{(\rho,\sigma)}$.

If $x, y \in S_{(\rho,\sigma)}$, then $V_S(x)$ and $V_S(y) \leq [\rho, \sigma]$.

From (ii) of the proposition 3.7, we have $V_S(x * y) \leq rmax\{V_S(x), V_S(y)\} \leq [\rho,\ \sigma]$.

Thus, $x * y \in S_{(\rho,\sigma)}$. Hence $S_{(\rho,\sigma)}$ is a filter of A.

Conversely, if for all $\rho, \sigma \in [0,\ 1]$, the set $S_{(\rho,\sigma)}$ is either empty or a filter of A.

Let $t_S(x) = \rho_1$, $t_S(y) = \rho_2$, $1 - f_S(x) = \sigma_1$ and $1 - f_S(y) = \sigma_2$.

Put $\rho = \max\{\rho_1, \rho_2\}$ and $\sigma = \max\{1 - \sigma_1, 1 - \sigma_2\}$.

Then, $t_S(x)$, $t_S(y) \leq \rho$ and $1 - f_S(x), 1 - f_S(y) \leq \sigma$.

Thus, $V_S(x)$ and $V_S(y) \leq [\rho, \sigma]$, that is $x, y \in S_{(\rho,\sigma)}$.

Thus, $S_{(\rho,\sigma)} \neq \emptyset$.

Hence, by the assumption $S_{(\rho,\sigma)}$ is a filter of A.

To prove: S is an anti vague filter of A.

If $x \leq y$, $t_S(x) = \rho$ and $1 - f_S(x) = \sigma$.

Then $x \in S_{(\rho,\sigma)}$.

Since, $S_{(\rho,\sigma)}$ is a filter, $y \in S_{(\rho,\sigma)}$, that is, $V_S(y) \leq [\rho, \sigma]$. (3.5)

Since, $S_{(\rho,\sigma)}$ is filter of A, $x * y \in S_{(\rho,\sigma)}$.

That is, $\vartheta_S(x * y) \leq [\rho, \sigma]$ for all $x, y \in A$

$$= [\max\{\rho_1, \rho_2\}, \max\{1 - \sigma_1, 1 - \sigma_2\}]$$
$$= rmax\{[t_S(x), 1 - f_S(x)],\ [t_S(y), 1 - f_S(y)]$$
$$= rmax\{V_S(x), V_S(y)\} \text{ for all } x, y \in A. \quad (3.6)$$

From (3.5) and (3.6), S is an anti vague filter of A. ■

Note. The filter $S_{(\rho,\sigma)}$ is called a vague-cut filter of *BL*- algebra A.

Proposition 3.14 Let S be an anti vague filter of *BL*-algebra A. Then S_ρ is either empty or a filter of A for all $\rho \in [0, 1]$.

Proof: Let S be an anti vague filter of *BL*-algebra $\mathcal{B}$. Then from the theorem 3.13, the proof is obvious. ■

4. Conclusion

In the present paper the notion of an anti-vague filter in *BL*- algebra with suitable examples are studied. Also investigated some related properties with the help of more implication of an anti-vague filter of *BL*-algebra.

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