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On δ-preregular e*-open sets in topological spaces

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Abstract

In this paper, we introduce a new class of sets called, δ-preregular e^*-open sets and investigate their properties and characterizations. By using δ-preregular e^*-open sets, we obtain decompositions of complete continuity and decompositions of perfect continuity.

Keywords: δ-preopen;e^*-open;e^*-closed;δpe^*-open;δpe^*-closed;δpe^*-continuity.

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1 Introduction

The study of δ-open sets was initiated by Veličko[Velicko, 1968] in 1968. Following this Raychaudhuri and Mukherjee[Raychaudhuri and Mukherjee, 1993] established the concept of δ-preopen sets. Later, Ekici[Ekici, 2009] introduced the concept of e^*-open sets as a generalization of e-open sets. The aim of this paper is to introduce and study a new class of sets called, δ-preregular e^*-open sets using δ-preinterior and e^*-closure operators. The notion of δpe*-continuity is also introduced which is stronger than δ-precontinuity.Finally, we obtain decompositions of complete continuity and decompositions of perfect continuity.

Throughout this paper, (U,τ) and (V,η)(or simply U and V) represent topological spaces on which no separation axioms are assumed unless explicitly stated and f:(U,τ)$\rightarrow$(V,η) or simply f:U $\rightarrow$ V denotes a function f of a topological space U into a topological space V. Let N $\subseteq$ U, then
cl(N) = $\cap\{$F: N $\subseteq$ F and $F^c \in \tau\}$ is the closure of N and
int(N) = $\cup\{$O: O $\subseteq$ N and O $\in \tau\}$ is the interior of N.

2 Preliminaries

Definition 2.1. *A set M $\subseteq$ U is called δ-closed[Velicko, 1968] if M = δ-cl(M) where δ-cl(M)=$\{p\in U{:}int(cl(G))\cap M\neq\phi, G\in\tau$ and $p\in G\}$.*

Definition 2.2. *A set M $\subseteq$ U is called*
(1) e-open[Ekici, 2008c] if M $\subseteq$ cl(δ-int(M))$\cup$int(δ-cl(M)) and e-closed if cl(δ-int(M))$\cap$int(δ-cl(M))$\subseteq$M.
(2) a-open[Ekici, 2008d] if M $\subseteq$ int(cl(δ-int(M))) and a-closed if cl(int(δ-cl(M)))$\subseteq$M.
(3)e^*-open[Ekici, 2009] if M $\subseteq$ cl(int(δ-cl(M))) and e^*-closed if int(cl(δ-int(M)))$\subseteq$M .
(4)δ-semiopen[Park et al., 1997] if M$\subseteq$cl(δ-int(M))) and δ-semiclosed if int(δ-cl(M))$\subseteq$M).
(5)δ-preopen[Raychaudhuri and Mukherjee, 1993] if M$\subseteq$int(δ-cl(M)) and δ-preclosed if cl(δ-int(M))$\subseteq$M.
(6)regular-open[Stone, 1937] if M = int(cl(M)) and regular-closed if M=cl(int(M)).

Definition 2.3. *[Ekici, 2008b] A subet M of a space U is said to be a δ-dense set if δ-cl(M)=U.*

The class of open(resp,closed, regular open,δ-preopen, δ-semiopen, e^*-open and clopen) sets of (U,τ) is denoted by O(U) (resp,C(U), RO(U), δPO(U), δSO(U), e^*O(U) and CO(U)).

Theorem 2.1. *[Raychaudhuri and Mukherjee, 1993] Let M be a subset of a space (U,τ), then δ-pcl(M)=M$\cup$cl(δ-int(M)) and δ-pint(M)=M$\cap$int(δ-cl(M)).*

Theorem 2.2. *[Ekici, 2009]Let M be a subset of a space (U,τ),then:*
(i) e^*-cl(M) = M$\cup$ int(cl(δ-int(M)) and e^*-int(M) = M$\cap$cl(int(δ-cl(M))
(ii) int(cl(δ-int(M))=e^*-cl(δ-int(M))=δ-int(e^*-cl(M)).

Theorem 2.3. *Let M be a subset of a space (U,τ),then:*
(i)δ-pint(e^*-cl(M))=e^*-cl(M)$\cap$ int(δ-cl(M)).
(ii)δ-pint(e^*-cl(M))=δ-pint(M)$\cup$ int(cl(δ-int(M)).
(iii)δ-pint(e^*-cl(M))=δ-pint(M)$\cup$ e^*-cl(δ-int(M))
(iv)δ-pint(e^*-cl(M))=δ-pint(M)$\cup$ δ-int(e^*-cl(M)).
(v) δ-pint(e^*-cl(M)) = (M$\cap$int(δ-cl(M))$\cup$ int(cl(δ-int(M))

Lemma 2.1. *[Benchalli et al., 2017]For a subset M of a space (U,τ),the following are equivalent:*
(a)M is clopen;
(b)M is δ-open and δ-closed;
(c)M is regular-open and regular-closed.

Definition 2.4. *[Kohli and Singh, 2009] A space (U,τ) is called δ-partition if δO(U)=C(U).*

Definition 2.5. *[Caldas and Jafari, 2016] A space (U,τ) is a δ-door space if every subset of U is δ-open or δ-closed.*

Theorem 2.4. *[Caldas and Jafari, 2016] If (U,τ) is a δ-door space, then every δ-preopen set in (U,τ) is δ-open.*

3 δ-preregular e^*-open sets in topological spaces

Definition 3.1. *A subset N of a space (U,τ) is said to be δ-preregular e^*-open(briefly δpe^*-open) if N =δ-pint(e^*-cl(N)). The complement of a δ-preregular e^*-open is called a δ-preregular e^*-closed(briefly δpe^*-closed) set.*
Clearly, N is δpe^*-closed if and only if N = δ-pcl(e^*-int(N))

The class of δpe^*-open (resp,δpe^*-closed) sets of (U,τ) will be denoted by δPE^*O(U)(resp,δPE^*C(U)).

Theorem 3.1. *Let (U,τ) be a topological space and M, N $\subseteq$ U. Then the following hold:*
(i) If M $\subseteq$ N, then δ-pint(e^*-cl(M) $\subseteq$ δ-pint(e^*-cl(N)).
(ii) If M $\in\delta$PO(U), then M $\subseteq$ δ-pint(e^*-cl(M)).
(iii) If M $\in e^*$C(U), then e^*-cl(δ-pint(M)) $\subseteq$ M.

(iv) δ-pint(e^*-cl(N)) is δpe^*-open
(v) If M $\in e^*$C(U), then δ-pint(M) is δpe^*-open..
Proof:*(i)Obvious.*
(ii) Let M $\in \delta$PO(U). As M $\subseteq e^*$-cl(M),then M $\subseteq \delta$-pint(e^*-pcl(M).
(iii) Let M $\in e^*$C(U). Since δ-pint(M) $\subseteq$ M, then e^*-cl(δ-pint(M)) $\subseteq$ M.
(iv) We have
δ-pint(e^*-cl(δ-pint(e^*-cl(M)) $\subseteq \delta$-pint(e^*-cl(e^*-cl(M)) = δ-pint(e^*-cl(M) and
δ-pint(e^*-cl(δ-pint(e^*-cl(M))) $\supseteq \delta$-pint(δ-pint(e^*-cl(M)) = δ-pint(e^*-cl(M).
Hence δ-pint(e^*-cl(δ-pint(e^*-cl(M))) = δ-pint(e^*-cl(M).
(v) Suppose that M $\in e^*$C(U). By (i),
δ-pint(e^*-cl(δ-pint(M))$\subseteq \delta$-pint(e^*-cl(M)=δ-pint(M).
On the other hand, we have
δ-pint(M) $\subseteq e^*$-cl(δ-pint(M) so that
δ-pint(M) $\subseteq \delta$-pint(e^*-cl(δ-pint(M)).
Therefore δ-pint(e^*-cl(δ-pint(M))=δ-pint(M).
This shows that δ-pint(M) is δpe^*-open.

Theorem 3.2. *(i)Every δpe^*-open set is δ-preopen(hence e-open,e^*-open).*
(ii)Every δpe^*-open set is e^*-closed..
Proof: *(i)Let M be δpe^*-open,then by Theorem 2.3(i),*
δ-pint(e^*-cl(M))=e^*-cl(M)$\cap$int(δ-cl(M).
Therefore, M $\subseteq$int(δ-cl(M), M is δ-preopen.
(ii)Let N be δpe^*-open.By Theorem 2.3(ii), N =δ-pint(N)$\cup$int(cl(δ-int(N))).
Therefore,int(cl(δ-int(N)))$\subseteq$N. Thus N is e^*-closed.

Remark 3.1. *By the following example,we show that every δ-preopen(resp,e^*-closed) set need not be a δpe^*-open set*

Example 3.1. *Let U = $\{a,b,c,d\}$ and τ = $\{U, \phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$. Then $\{a,b,c\}$ is a δ-preopen set but $\{a,b,c\} \notin \delta PE^*$O(U) and $\{d\}$ is e^*-closed but $\{d\} \notin \delta PE^*$O(U)it is not δpe^*-open*

Corolary 3.1. *For a topological space (U,τ), we have*
δ-PO(U)$\cap\delta$-PC(U) $\subseteq \delta PE^*$O(U)$\subseteq e^*$O(U)$\cap e^*$C(U).
Proof: *Obvious.*
The converse inclusions in the above corollary need not be true as seen from the following example

Example 3.2. *Let (U,τ) as in Example 3.1,then $\{b\}$ is δpe^*-open but it is not δ-preclopen. Moreover, $\{a,d\}$ is e^*-clopen but not δpe^*-open*

Remark 3.2. *The notions of δpe^*-open sets and δ-open sets (hence a-open sets, δ-semiopen sets, δ^*-sets) are independent of each other.*

Example 3.3. *Consider (U,τ) as in Example 3.1.The set $\{a\}$ is δpe^*-open but it is not δ^*-set. Moreover, $\{a,b,c\}$ is δ-open but not δpe^*-open*

Theorem 3.3. *In a δ-partition space (U,τ), a subset M of U is δpe^*-open if and only if it is δ-preopen.*
Proof: *Necessity:It follows from Theorem 3.2(i) .*
Sufficiency:Let N be δ-preopen. Since (U,τ) is δ-partition and by Theorem 2.3(ii), we have δ-pint(e^*-cl(M)) = δ-pint(M)$\cup$ int(cl(δ-int(M))

$$\begin{aligned} &= M \cup int(cl(cl(M)) \\ &= M \cup int(cl(M) \\ &= M \cup \delta\text{-}int(cl(M) \\ &= M \cup \delta\text{-}int(\delta\text{-}int(M) \\ &= M \cup \delta\text{-}int(M) = M \end{aligned}$$

Therefore, δ-pint(e^*-cl(M)) = M.Hence M is δpe^*-open.

Theorem 3.4. *A subset $N \subseteq U$ is δpe^*-open if and only if N is e^*-closed and δ-preopen.*
Proof: *Necessity:It follows from Theorem 3.2.*
Sufficiency:Let N be both e^*-closed and δ-preopen. Then N = e^*-cl(N) and N = δ-pint(N). Therefore, δ-pint(e^*-cl(N)) = δ-pint(N) = N. Hence N is δpe^*-open.

Remark 3.3. *The class of δpe^*-open sets is not closed under finite union as well as finite intersection. It will be shown in the following example.*

Example 3.4. *Consider (U,τ) as in Example 3.1. Let A = $\{a,c\}$ and B = $\{b,c\}$,the A and B are δpe^*-open sets but $A \cup B = \{a,b,c\} \notin \delta PE^*O(U)$.*
Moreover,C = $\{a,b,d\}$ and D = $\{b,c,d\}$ are δpe^*-open sets but
$C \cap D = \{b,d\} \notin \delta PE^*O(U)$.

Theorem 3.5. *For a subset M of a space (U,τ),the following are equivalent:*
(i) M is δpe^*-open.
(ii) M = e^*-cl(M)$\cap$ int(δ-cl(M)).
(iii) M = δ-pint(M)$\cup$ int(cl(δ-int(M)).
(iv)M = δ-pint(M)$\cup$ e^*-cl(δ-int(M))
(v) M = δ-pint(M)$\cup$ δ-int(e^*-cl(M)).
(vi) M = (M$\cap$int(δ-cl(M))$\cup$ int(cl(δ-int(M)).
Proof:*It follows from Theorem 2.3*

Theorem 3.6. *In any space (U,τ) , the empty set is the only subset which is nowhere δ-dense and δpe^*-open.*
Proof: *Suppose M is nowhere δ-dense and δpe^*-open. Then by Theorem 2.3(i), M = δ-pint(e^*-cl(M)) =e^*-cl(M)$\cap$ int(δ-cl(M)= e^*-cl(M)$\cap\phi$ = ϕ.*

Lemma 3.1. *If (U,τ) is a δ-door space, then any finite intersection of δ-preopen sets is δ-preopen.*
Proof:*Obvious since δO(X) is closed under finite intersection.*

Theorem 3.7. *If (U,τ) is a δ-door space, then any finite intersection of δpe^*-open sets is δpe^*-open.*
Proof:*Let $\{A_i$:i=1,2,...,n$\}$ be a finite family of δpe^*-open. Since the space (U,τ) is δ-door, then by Lemma 3.1, we have $\bigcap\limits_{i=n}^{n} A_i \in \delta PO(U)$.*
By Theorem 3.1(ii), $\bigcap\limits_{i=n}^{n} A_i \subseteq$ δ-pint(e^*-cl($\bigcap\limits_{i=n}^{n} A_i$).
For each i, we have $\bigcap\limits_{i=n}^{n} A_i \subseteq A_i$ and thus δ-pint(e^*-cl($\bigcap\limits_{i=n}^{n} A_i$)$\subseteq$ δ-pint(e^*-cl(A_i) = A_i. Therefore, δ-pint(e^*-cl($\bigcap\limits_{i=n}^{n} A_i$)$\subseteq \bigcap\limits_{i=n}^{n} A_i$.

Lemma 3.2. *If a subset M of a space (U,τ) is regular open,then M = int(cl(M)=int(δ-cl(M)).*

Theorem 3.8. *Every regular open set is δpe^*-open.*
Proof: *Let M be regular open. Then M=int(cl(M))=int(δ-cl(M)). By Theorem 2.6(i), δ-pint(e^*-cl(M)) = e^*-cl(M)$\cap$int(δ-cl(M))=e^*-cl(M)$\cap$M=M. This shows that M is δpe^*-open.*

Definition 3.2. *A subset M of a space (U,τ) is called δ^*-set if int(δ-cl(M))$\subseteq$cl(δ-int(M))*

Theorem 3.9. *(i) Every δ-semiopen set is δ^*-set.*
(ii)Every δ-semiclosed set is δ^*-set.
Proof:*Clear*

Definition 3.3. *A subset M of a space (U,τ) is called*
b^*-open if M = cl(δ-int(M))$\cup$ int(δ-cl(M)).
b^*-closed if M = cl(δ-int(M))$\cap$ int(δ-cl(M))

Theorem 3.10. *A subset M of a space (U,τ) is regular open if and only if it is b^*-closed.*
Proof:*Let M be regular open. Then by Lemma 3.2, M = int(cl(M)=int(δ-cl(M)). Since every regular open set is δ-open, we have cl(δ-int(M))$\cap$ int(δ-cl(M)) = cl(M)$\cap$ M = M. Hence A is b^*-closed.*
Conversely, let M be b^*-closed.Then int(cl(δ-int(M))$\subseteq$int(δ-cl(δ-int(M))$\subseteq$ cl(δ-int(M))$\cap$ int(δ-cl(M))=M. By Definition 3.3, we have M $\subseteq$ int(δ-cl(M)) $\subseteq$ int(δ-cl(cl(δ-int(M))) = int(cl(cl(δ-int(M))) = int(cl(δ-int(M))).
Therefore, M = int(cl(δ-int(M)). Now, int(cl(M)) = int(cl(int(cl(δ-int(M))) = int(cl(δ-int(M)) = M. Hence M is regular open.

Theorem 3.11. *(i) Every b^*-closed set is δ-preopen.*
(ii)Every b^*-closed set is δ-semiopen.
(iii)Every b^*-closed set is δpe^*-open.
Proof:*(i) and (ii) are obvious*
(iii)Let M be b^*-closed,then we have $M = int(cl(\delta\text{-}int(M))$. Then $\delta\text{-}pint(e^*\text{-}cl(M))$ $= \delta\text{-}pint(M) \cup int(cl(\delta\text{-}int(M)) = \delta\text{-}pint(M) \cup M = M$.Hence M is δpe^*-open

Remark 3.4. *The above discussions can be summarized in the following diagram:*
DIAGRAM

$$\begin{tikzcd}
\text{regular open} \arrow[r] \arrow[d, leftrightarrow] & \delta\text{-open} \arrow[r] & a\text{-open} \arrow[r] & \delta\text{-semiopen} \arrow[r] \arrow[d] & \delta^*\text{-set} \arrow[d] \\
b^*\text{-closed} \arrow[r] & \delta pe^*\text{-open} \arrow[r] & \delta\text{-preopen} \arrow[r] & e\text{-open} \arrow[r] & e^*\text{-open}
\end{tikzcd}$$

Theorem 3.12. *For a subset M of a space (U,τ), the following are equivalent:*
(i) M is regular open;
(ii) M is δpe^*-open and δ-open;
(iii) M is δpe^*-open and a-open;
(iv) M is δpe^*-open and δ-semiopen;
(v) M is δpe^*-open and δ^*-set.
Proof: *(i) $\longrightarrow$(ii)$\longrightarrow$(iii)$\longrightarrow$(iv)$\longrightarrow$(v):Follows from the above diagram*
(v)$\longrightarrow$(i):Let M be δpe^*-open and δ^*-set.Then $int(\delta\text{-}cl(M))\subseteq cl(\delta\text{-}int(M))$ and
$int(\delta\text{-}cl(M))\subseteq int(cl(\delta\text{-}int(M))\subseteq int(\delta\text{-}cl(\delta\text{-}int(M))\subseteq int(\delta\text{-}cl(M))$.
Therefore we have $int(\delta\text{-}cl(M))=int(cl(\delta\text{-}int(M))$.
Since M is δpe^*-open, $M = \delta\text{-}pint(\delta\text{-}pcl(M))$

$$\begin{aligned}
&=(M\cup int(cl(\delta\text{-}int(M))\cap int(\delta\text{-}cl(M)) \\
&=int(\delta\text{-}cl(M)\cap int(\delta\text{-}cl(M)) \\
&=int(\delta\text{-}cl(M)).
\end{aligned}$$

Therefore $M =int(\delta\text{-}cl(M))=int(cl(M))$ and hence M is regular open.

Theorem 3.13. *For a subset M of a space (U,τ), the following are equivalent:*
(i) M is regular open.
(ii) M is δpe^*-open and δ-semiclosed.
(iii) M is e^*-closed and a-open.
Proof: *(i)$\longrightarrow$(ii):It follows from Theorem 3.8*
(ii)$\longrightarrow$(i):Let M be δpe^*-open and δ-semiclosed. Since every δ-semiclosed set is δ^*-set. Hence by Theorem 3.12(v), M is regular open.
(ii) $\longrightarrow$(iii):Clear
(i)$\longleftrightarrow$(iii):It is shown in Theorem 3 [Ekici, 2008b]

Corolary 3.2. *For a subset M of a space (U,τ), the following are equivalent:*
(i) M is regular open;
(ii) M is δpe^*-open and δ-open;

(iii) M is δpe^*-open and a-open;
(iv) M is δpe^*-open and δ-semiopen;
(v) M is δpe^*-open and δ^*-set;.
(vi) M is δpe^*-open and δ-semiclosed;
(vii) M is e^*-closed and a-open;
(viii) M is b^*-closed.

Theorem 3.14. *For a subset M of a space (U,τ), the following are equivalent:*
(i) M is clopen;
(ii) M is δ-open and δ-closed;
(iii) M is regular open and regular closed;
(iv) M is δpe^*-open and δ-closed.
Proof: *(i) $\longleftrightarrow$(ii)$\longleftrightarrow$(iii):Follows from Lemma 2.1*
(iii)$\longrightarrow$(iv). It follows from Theorem 3.8
(iv)$\longrightarrow$(ii)Let M be δpe^*-open and δ-closed.By Theorem 2.3(i) , we have N = e^*-cl(N) $\cap$ int(δ-cl(N)) = e^*-cl(N) $\cap\delta$-int(δ-cl(N))=δ-pcl(N)$\cap\delta$-int(N)=δ-int(N). Therefore M is δ-open.

4 Decompositions of complete continuity

In this section, the notion of regular δ-preopen continuity is introduced and the decompositions of complete continuity are discussed.

Definition 4.1. *A function f:(U,τ)$\rightarrow$(V,σ) is said to be*
(i) δpe^*-continuous if the inverse image of every open subset of (V,σ) is δpe^*-open set in (U,τ).
(ii)perfectly continuous[Noiri, 1984] (resp,e-continuous[Ekici, 2008c], e^*-continuous[Ekici, 2009], δ-almost continuous[Raychaudhuri and Mukherjee, 1993], δ^*-continuous, contra-super-continuous[Jafari and Noiri, 1999], completely continuous[Arya and Gupta, 1974], RC-continuous[Dontchev and Noiri, 1998], super-continuous[Munshi and Bassan, 1982], contra continuous[Dontchev, 1996], a-continuous[Ekici, 2008d], δ-semicontinuous[Noiri, 2003], contra e^*-continuous[Ekici, 2008a], contra δ-semicontinuous[Ekici, 2004], contra b^*-continuous) if the inverse image of every open subset of (V,σ) is clopen (resp,e-open,e^*-open,δ-preopen,δ^*-set, δ-closed, regular open, regular closed, δ-open, closed, a-open, δ-semiopen, e^*-closed, δ-semiclosed, b^*-closed) set in (U,τ)

By Theorems 3.9 and 3.11, we obtain the following theorem.

Theorem 4.1. *(i) Every contra b^*-continuous set is δ-almost continuous.*
(ii)Every contra b^*-continuous set is δ-semicontinuous

(iii)Every contra b^*-continuous set is δpe^*-continuous.
(iv) Every δ-semicontinuous set is δ^*-continuous.
(v)Every contra δ-semicontinuous is δ^*-continuous.

Remark 4.1. *By Diagram I, we have the following diagram:*

DIAGRAM II

$$\begin{array}{ccccccccc} c.cont. & \longrightarrow & s.cont. & \longrightarrow & a.cont. & \longrightarrow & \delta s.cont. & \longrightarrow & \delta^*.cont. \\ \updownarrow & & & & & & \downarrow & & \downarrow \\ cb^*.cont. & \longrightarrow & \delta pe^*.cont. & \longrightarrow & \delta p.cont. & \longrightarrow & e.cont. & \longrightarrow & e^*.cont. \end{array}$$

where c.cont.=completely continuity, s.cont.=super continuity, a.cont.=a-continuity, δs.cont.=δ-semicontinuity, δ^*.cont.=δ^*-continuity, cb*.cont.=contra b*-continuity, δpe^*.cont.=δ-preregular e^*-continuity, δp.cont.=δ-precontinuity, e.cont.=e-continuity, e*.cont.=e*-continuity

Theorem 4.2. *For a function f:(U,τ)$\rightarrow$(V,η), the following are equivalent:*
(i) f is completely continuous;
(ii)f is δpe^*-continuous and super continuous;
(iii)f is δpe^*-continuous and a-continuous;
(iv) f is contra e*-continuous and a-continuous;
(v)f is δpe^*-continuous and δ-semicontinuous;
(vi)f is δpe^*-continuous and contra δ-semicontinuous;
(vii)f is δpe^*-continuous and δ^*-continuous;
(viii) f is contra b*-continuous.

Remark 4.2. *(i) δpe^*-continuity and super-continuity(hence a-continuity,δ-semicontinuity, δ^{**}-continuity) are independent notions.*
(ii) δpe^*-continuity and contra δ-semicontinuity are independent notions.

Example 4.1. *Let (U,τ) be a space as in Example 3.1 and let η = $\{U, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$*
(i) Define f:(U,τ) $\rightarrow$ (U,η) by f(a) = f(c) = a , f(b) = b and f(d) = d. Clearly f is super-continuous but for $\{a,b\}\in$ O(V), $f^{-1}(\{a,b\})$ = $\{a,b,c\} \notin \delta PE^*O(U)$. Therefore f is not δpe^*-continuous.
Define g:(U,τ) $\rightarrow$ (U,η) by g(a) = b, g(b) = g(c) = g(d) = a.Then g is δpe^*-continuous but for $\{a\} \in$ O(V), $g^{-1}(\{a\})$ = $\{b,c,d\} \notin q^*O(U)$. Therefore g is not q^*-continuous.
(ii)Define f:(U,τ) $\rightarrow$ (U,η) by f(a) = f(c) = f(d) = b and f(b) = a. Clearly f is δ-semiregular-continuous but for $\{b\}\in$ O(V), $f^{-1}(\{b\})$ = $\{a,c,d\} \notin \delta PE^*PO(U)$. Therefore f is not δpe^*-continuous.
Define g:(U,τ) $\rightarrow$ (U,η) by g(a) = g(b)=g(d)=a,g(c) = b.Then g is δpe^*-continuous

but for $\{a\} \in$ O(V), $g^{-1}(\{a\})$ = $\{a,b,d\} \notin \delta$SC(U).Therefore g is not contra δ-semicontinuous.

5 Decompositions of perfectly continuity

In this section, the decompositions of perfectly continuity are obtained.

Theorem 5.1. *For a function f:(U,τ) $\rightarrow$ (U,η), the following are equivalent:*
(i) f is perfectly continuous;
(ii) f is super continuous and contra super continuous;
(iii) f is completely continuous and RC-continuous;
(iv) f is δpe^*-continuous and contra super continuous.
Proof: *It is a direct consequence of Theorem 3.14*

Remark 5.1. *As shown by the following examples,δpe^*-continuity and contra super continuity are independent of each other.*

Example 5.1. *Consider (U,τ) as in Example 3.1 and (U,η) as in Example 4.1. Define f: (U,τ) $\rightarrow$ (U,η) by f(a) = f(c) = f(d) = a and f(b) = c. Then f is contra super continuous but it is not δpe^*-continuous since $\{a\}\in$ O(V), $f^{-1}(\{a\})$ = $\{a,c,d\} \notin$ δPE^*O(U). Define g: (U,τ) $\rightarrow$ (U,η) by g(a) =b, g(b) = g(c) = g(d) = a.Then g is δpe^*-continuous but it is not contra super continuous since $\{a\} \in$ O(V), $g^{-1}(\{a\})$ = $\{b,c,d\} \notin \delta$C(U).*

6 Conclusions:

The notions of sets and functions in topological spaces and fuzzy topological spaces are extensively developed and used in many engineering problems, information systems, particle physics, computational topology and mathematical sciences. By researching generalizations of closed sets, some new continuity have been founded and they turn out to be useful in the study of digital topology. Therefore, δpe*-continuous functions defined by δpe*-open sets will have many possibilities of applications in digital topology and computer graphics.

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