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H_v-semigroups as noise pollution models in urban areas

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Abstract

Poor urban planning may give rise to noise pollution, since side-by-side industrial and residential buildings can result in noise pollution in the residential area. In this paper we represent the noise pollution with an H_v-semigroup. More specific, we introduce the concept of right reproductive H_v-semigroup which seems to be a useful tool to study the noise pollution problem in urban areas.

Key words: H_v-structures, H_v-group, H_v-semigroup

MSC2010: 20N20.

1 Introduction

The H_v-structures, introduced in the Fourth AHA Congress [13], where the known axioms were replaced by weaker ones. More precisely in axioms like associativity, commutativity and so on, the equality was replaced by the non-empty intersection. In $(H, \cdot)$ we abbreviate by
WASS, the *weak associativity*: $(xy)z \cap x(yz) \neq \varnothing, \ \forall x, y, z \in H$ and by
COW, the *weak commutativity*: $xy \cap yx \neq \varnothing, \ \forall x, y \in H$

Recall some basic definitions [14]:

Definition 1.1 *Let H be a non-empty set and $\cdot : H \times H \to \mathbf{P}(H) - \{\varnothing\}$ be a hyperoperation defined on H. Also, we have abbreviated the "hyperoperation" by "hope"[16]. The $(H, \cdot)$ is called H_v-semigroup if it is WASS and it is called*

H_v-group if it is H_v-semigroup and the reproduction axiom $x \cdot H = H \cdot x = H, \forall x \in H$, is valid. The hyperstructure $(R, +, \cdot)$ is called H_v-ring if both hopes $(+)$ and $(\cdot)$ are WASS, the reproduction axiom is valid for $(+)$ and $(\cdot)$ is weak distributive with respect to $(+)$:

$$x(y+z) \cap (xy+xz) \neq \varnothing, (x+y)z \cap (xz+yz) \neq \varnothing, \forall x, y, z \in R.$$

An H_v-ring $(R, +, \cdot)$ is called dual H_v-ring [7] if the hyperstructure $(R, \cdot, +)$ is H_v-ring, too.

Let $(H, \cdot)$ be a hypergroupoid. An element $e \in H$ is called *right unit* element if $a \in a \cdot e, \forall a \in H$ and is called *left unit* element if $a \in e \cdot a, \forall a \in H$. The element $e \in H$ is called *unit* element if $a \in a \cdot e \cap e \cdot a, \forall a \in H$. An element $e \in H$ is called *right scalar* unit element if $a = a \cdot e, \forall a \in H$ and is called *left scalar* unit element if $a = e \cdot a, \forall a \in H$. It is called *scalar unit* element if $a = a \cdot e = e \cdot a, \forall a \in H$. Let $(H, \cdot)$ be a hypergroupoid endowed with at least one unit element. An element $a' \in H$ is called an inverse element of the element $a \in H$, if there exists a unit element $e \in H$ such that $e \in a' \cdot a \cap a \cdot a'$. The element $a \in H$ is called *right simplifiable* element (resp. *left*) if $\forall x, y \in H$ the following is valid:

$$x \cdot a = y \cdot a \Rightarrow x = y \text{ (resp. } a \cdot x = a \cdot y \Rightarrow x = y).$$

Moreover, let us define here: If $x \in x \cdot y$ (resp. $x \in y \cdot x$) $\forall y \in H$, then, x is called *left absorbing-like* element (resp. *right absorbing-like element*).The n^{th} power of an element h, denoted h^s, is defined to be the union of all expressions of n times of h, in which the parentheses are put in all possible ways. An H_v-group $(H, \cdot)$ is called *cyclic* with finite period with respect to $h \in H$, if there exists a positive integer s, such that $H = h^1 \cup h^2 \cup ... \cup h^s$. The minimum such an s is called period of the generator h. If all generators have the same period, then H is cyclic with period. If there exists $h \in H$ and s positive integer, the minimum one, such that $H = h^s$ then H is called *single-power cyclic* and h is a generator with *single-power period s*. The cyclicity in the infinite case is defined similarly. Thus, for example, the H_v-group $(H, \cdot)$ is called *single-power cyclic with infinite period* with generator h if every element of H belongs to a power of h and there exists $s_0 \geq 1$, such that $\forall s \geq s_0$ we have: $h^1 \cup h^2 \cup ... \cup h^{s-1} \subset h^s$.
An element $a \in H$ is called *idempotent* element if $a^2 = a$.
The main tool to study H_v-groups, is the fundamental relation β^*. The relation β^* is defined in H_v-groups, as the smallest equivalence relation on H,

such that the quotient would be group. It is called the fundamental group and β^* is called the *fundamental equivalence* relation on H. The relation β is defined on an H_v-group in the same way as in a hypergroup.
The basic Theorem is the following [14]: Let $(H, \cdot)$ be an H_v-group and denote by $\mathbf{U}$ the set of all finite products of elements of H. We define the relation β on H by setting xβy iff $\{x, y\} \subset \mathbf{u}$ where $\mathbf{u} \in \mathbf{U}$. Then β^* is the transitive closure of β.
An element $s \in H$ is called *single* if $\beta^*(s) = \{s\}$. The set of all single elements is denoted by S_H and if $S_H \neq \varnothing$ then one can find easily the fundamental classes.
A way to find the fundamental classes is given in [6],[13],[14].

For more definitions, results and applications onH_v-structures, see also books [4],[5],[14].

2 The noise problem

Noise pollution is displeasing human, animal or machine-created sound that disrupts the activity or balance of human or animal life. The source of most outdoor noise worldwide is not only transportation systems (including motor vehicle noise, aircraft noise and rail noise), but, noise caused by people as well (audio entertainment systems, electric megaphones and loud people) [9]. The fact that noise pollution is also a cause of annoyance, is that, a 2005 study by Spanish researchers found that in urban areas households are willing to pay approximately 4 Euros per decibel per year for noise reduction [2]. Poor urban planning may give rise to noise pollution, since side-by-side industrial and residential buildings can result in noise pollution in the residential area.

We set the following problem: *The noise pollution that comes from a certain block of flats in urban areas, obviously annoys not only the block of flats itself but possibly neighboring blocks of flats or buildings, as well.*

If every city is considered as a set Ω with elements the blocks of flats or the buildings, then, the above situation could be described with an algebraic hyperstructure and its properties. In this paper, we present the *right reproductive H_v-semigroup*, as a tool to study the noise pollution problem in urban areas. One can find hyperstructures on related problems in several survey papers like [10], [11],[12] and papers with a wide variety of applications [1], [3], [15].

Definition 2.1 *A hypergroupoid (**H**,*), such that, the weak associativity holds and* $\forall x \in \mathbf{H}$, ***H***x=**H** , is called **right reproductive*** $\mathbf{H_v}$***-semigroup**. A hypergroupoid **H**,*), such that, the weak associativity holds and* $\forall x \in \mathbf{H}$, *x*__H__=__H__ , is called **left reproductive*** $\mathbf{H_v}$***-semigroup***

Now we give the following definition:

Definition 2.2 *Let* $\mathbf{\Omega} \neq \varnothing$ *and* $f : \mathbf{\Omega} \to \mathbf{P}(\mathbf{\Omega})$ *be a map, then we define a hyperoperation* r_L*:* $\mathbf{\Omega} \times \mathbf{\Omega} \to \mathbf{P}(\mathbf{\Omega})$*, on* $\mathbf{\Omega}$ *as follows:* $\forall x, y \in \mathbf{\Omega}$*, we set*

$$xr_Ly = f(x) \cup \{x\}$$

We call the above hyperoperation (r_L) *noise hyperoperation. Remark that the noise hyperoperation, always contains the element* $x \in \mathbf{\Omega}$*. That means that the element* $x \in \mathbf{\Omega}$ *could be considered as the representative of the elements of the set* xr_Ly*. So, we symbolize:*

$$xr_Ly = f(x) \cup \{x\} = [x]$$

If, $\forall x \in \mathbf{\Omega}, x \in f(x)$ *then the hyperoperation is simplified as*

$$xr_Ly = f(x) = [x]$$

Therefore, the noise hyperoperation (r_L) *depends only on the left element. That means that if one composes an element x, on the left, with any other element y, on the right, then the result is always the same set [x].*

Example 2.1 *Consider a set* $\mathbf{\Omega} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$ *and a map* $f : \mathbf{\Omega} \to \mathbf{P}(\mathbf{\Omega})$ *such that:*

$$f(x_1) = \{x_2\}, f(x_2) = \{x_2, x_3\}, f(x_3) = \{x_2\}, f(x_4) = \{x_4\}$$
$$f(x_5) = \{x_5, x_6, x_7\}, f(x_6) = \{x_6, x_7\}, f(x_7) = \{x_5\},$$
$$f(x_8) = \{x_8, x_9\}, f(x_9) = \varnothing.$$

Then, as in the defined above noise hyperoperation:

$$[x_1] = f(x_1) \cup \{x_1\} = \{x_1, x_2\}, [x_2] = f(x_2) = \{x_2, x_3\},$$
$$[x_3] = f(x_3) \cup \{x_3\} = \{x_2, x_3\}, [x_4] = f(x_4) = \{x_4\},$$
$$[x_5] = f(x_5) = \{x_5, x_6, x_7\}, [x_6] = f(x_6) = \{x_6, x_7\}$$
$$[x_7] = f(x_7) \cup \{x_7\} = \{x_5, x_7\}, [x_8] = f(x_8) = \{x_8, x_9\},$$
$$[x_9] = f(x_9) \cup \{x_9\} = \{x_9\}.$$

Then, the "multiplication" table of (r_L) *is given by:*

$\mathbf{r_L}$	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{x_4}$	$\mathbf{x_5}$	$\mathbf{x_6}$	$\mathbf{x_7}$	$\mathbf{x_8}$	$\mathbf{x_9}$
$\mathbf{x_1}$	x_1, x_2	x_1, x_2	x_1, x_2	x_1, x_2	x_1, x_2	x_1, x_2	x_1, x_2	x_1, x_2	x_1, x_2
$\mathbf{x_2}$	x_2, x_3	x_2, x_3	x_2, x_3	x_2, x_3	x_2, x_3	x_2, x_3	x_2, x_2	x_2, x_3	x_2, x_3
$\mathbf{x_3}$	x_2, x_3	x_2, x_3	x_2, x_3	x_2, x_3	x_2, x_3	x_2, x_3	x_2, x_2	x_2, x_3	x_2, x_3
$\mathbf{x_4}$	x_4	x_4	x_4	x_4	x_4	x_4	x_4	x_4	x_4
$\mathbf{x_5}$	x_5, x_6, x_7	x_5, x_6, x_7	x_5, x_6, x_7	x_5, x_6, x_7	x_5, x_6, x_7	x_5, x_6, x_7	x_5, x_6, x_7	x_5, x_6, x_7	x_5, x_6, x_7
$\mathbf{x_6}$	x_6, x_7	x_6, x_7	x_6, x_7	x_6, x_7	x_6, x_7	x_6, x_7	x_6, x_7	x_6, x_7	x_6, x_7
$\mathbf{x_7}$	x_5, x_7	x_5, x_7	x_5, x_7	x_5, x_7	x_5, x_7	x_5, x_7	x_5, x_7	x_5, x_7	x_5, x_7
$\mathbf{x_8}$	x_8, x_9	x_8, x_9	x_8, x_9	x_8, x_9	x_8, x_9	x_8, x_9	x_8, x_9	x_8, x_9	x_8, x_9
$\mathbf{x_9}$	x_9	x_9	x_9	x_9	x_9	x_9	x_9	x_9	x_9

Example 2.2 *Let* $\mathbf{X} \neq \varnothing$ *and* $m : \mathbf{X} \to [0, 1]$ *be a fuzzy subset of* $\mathbf{X}$. *We define the hyperoperation* $(\circ)$ *on* $\boldsymbol{X}$ *as follows:*
$\forall x, y \in \mathbf{X}$, $\circ : \mathbf{X} \times \mathbf{X} \to \mathbf{P}(\mathbf{X})$, *such that*

$$x \circ y = \{z \in \mathbf{X} / m(z) = m(x)\}$$

Then, consider the map $f(x) = \{z \in \mathbf{X} / m(z) = m(x)\}$. *Since* $x \in f(x), \forall x \in \mathbf{X}$, *as above we have the hyperoperation* (r_L) *on* $\boldsymbol{X}$ *as follows :* $\forall x, y \in \mathbf{X}$, $r_L : \mathbf{X} \times \mathbf{X} \to \mathbf{P}(\mathbf{X})$, *such that* $x r_L y = f(x)$.

3 The right reproductive H_v-semigroup

Some properties of $(\mathbf{r_L})$

1. $x r_L \boldsymbol{\Omega} = [x], \forall x \in \boldsymbol{\Omega}$
2. $[x] r_L y = [x] r_L [y] \supset [x], \forall y \in \boldsymbol{\Omega}$
3. $x^2 = x r_L x = [x], \forall x \in \boldsymbol{\Omega}$

Proposition 3.1 *The hypergroupoid* $(\boldsymbol{\Omega}, r_L)$ *is an* H_v*-semigroup.*

Proof. We have to prove that the weak associativity holds. Indeed, $\forall x, y, z \in \boldsymbol{\Omega}$

$$x r_L (y r_L z) = \bigcup_{v \in y r_L z} (x r_L v) = \bigcup_{v \in [y]} (x r_L v) = [x]$$

$$(x r_L y) r_L z = \bigcup_{w \in x r_L y} (w r_L z) = \bigcup_{w \in [x]} (w r_L z) = \bigcup_{w \in [x]} [w] \supset [x]$$

Therefore $(x r_L y) r_L z \supset x r_L (y r_L z)$, so

$(x r_L y) r_L z \cap x r_L (y r_L z) \neq \varnothing, \forall x, y, z \in \boldsymbol{\Omega}$. $\square$

Proposition 3.2 $\forall x \in \Omega, \Omega r_L x = \Omega$ *and* $x r_L \Omega = [x]$.

Proof.

$$\forall x \in \Omega, \Omega r_L x = \bigcup_{\omega \in \Omega} (\omega r_L x) = \bigcup_{\omega \in \Omega} [\omega] = \Omega.$$

On the other hand,

$$\forall x \in \mathbf{\Omega}, x r_L \mathbf{\Omega} = \bigcup_{\omega \in \Omega} (x r_L \omega) = [x].\square$$

By propositions (3.1) and (3.2), we get that:

Proposition 3.3 *The hypergoupoid* $(\mathbf{\Omega}, r_L)$ *is a right reproductive* H_v*-semigroup.*

Remark that the right reproductive H_v-semigroup $(\mathbf{\Omega}, r_L)$ is an H_v-group if, $\forall x \in \mathbf{\Omega}$, we have $x r_L \mathbf{\Omega} = \mathbf{\Omega}$.

Proposition 3.4 *The strong associativity of* (r_L) *is valid iff we have*

$$\bigcup_{w \in [x]} (w r_L z) = [x], \forall x, z \in \mathbf{\Omega}$$

Proof. Let $(x, y, z) \in \mathbf{\Omega^3}$, such that $(x r_L y) r_L z = x r_L (y r_L z)$, then

$$(x r_L y) r_L z = x r_L (y r_L z) \Rightarrow [x] r_L z = x r_L [y] \Rightarrow [x] r_L z = [x] \Rightarrow \bigcup_{w \in [x]} (w r_L z) = [x].$$

Now,let $(x, y, z) \in \mathbf{\Omega^3}$, such that

$$\bigcup_{w \in [x]} (w r_L z) = [x]$$

then,

$$(x r_L y) r_L z = [x] r_L z = \bigcup_{w \in [x]} (w r_L z) = [x]$$

$$x r_L (y r_L z) = x r_L [y] = [x].\square$$

For the hyperoperation (r_L), we shall check conditions such that the strong or the weak commutativity is valid.

Proposition 3.5 . *If* $y \in [x]$ *and* $x \in [y], \forall x, y \in \mathbf{\Omega}$*, then the weak commutativity of* (r_L) *is valid. The strong commutativity of* (r_L) *is valid, iff* $[x] = [y], \forall x, y \in \mathbf{\Omega}$.

Proof. Let $y \in [x]$ and $x \in [y], \forall x, y \in \mathbf{\Omega}$, then

$$y \in [x] \text{ and } x \in [y] \Rightarrow x, y \in [x] \text{ and } x, y \in [y] \Rightarrow [x] \cap [y] \neq \varnothing \Rightarrow$$

$$\Rightarrow (x r_L y) \cap (y r_L x) \neq \varnothing.$$

The proof for the strong commutativity is straightforward.$\square$

Proposition 3.6 *Let* $(\mathbf{\Omega}, +, r_L)$ *be an* H_v*-ring. If* $x r_L \mathbf{\Omega} = \mathbf{\Omega}, \forall x \in \mathbf{\Omega}$ *then the hyperstructure* $(\mathbf{\Omega}, +, r_L)$ *is a dual* H_v*-ring, i.e. both* $(\mathbf{\Omega}, +, r_L)$ *and* $(\mathbf{\Omega}, r_L, +)$ *are* H_v*-rings.*

Proof. From the remark of proposition 3.3, we have that the $(\mathbf{\Omega}, r_L)$ is an H_v-group. For the weak distributivity of $(+)$ with respect to (r_L) we have: $\forall x, y, z \in \mathbf{\Omega}$

$$x + (y r_L z) \supset x + y$$

and

$$(x + y) r_L (x + z) = \bigcup_{s \in x+y, t \in x+z} (s r_L t) \supset \bigcup_{s \in x+y} s = x + y$$

So,

$$[x + (y r_L z)] \cap [(x + y) r_L (x + z)] \neq \varnothing, \forall x, y, z \in \mathbf{\Omega}$$

Similarly, the weak distributivity of $(+)$ with respect to (r_L) from the right side. $\square$

4 Special elements

Since $x \in x r_L y, \forall x, y \in \mathbf{\Omega}$, the next proposition is obvious:

Proposition 4.1 *(a) All the elements of* $\mathbf{\Omega}$ *are right unit elements with respect to* (r_L)*.*
(b) All the elements of $\mathbf{\Omega}$ *are left absorbing-like elements with respect to* (r_L)*.*

Proposition 4.2 *The left scalar elements of the* H_v*-semigroup* $(\mathbf{\Omega}, r_L)$*, are left absorbing elements.*

Proof. Let $u \in \mathbf{\Omega}$ be a left scalar unit element, then $u r_L x = x, \forall x \in \mathbf{\Omega}$. But since $u \in u r_L x, \forall x \in \mathbf{\Omega}$, we get that $u r_L x = u, \forall x \in \mathbf{\Omega}$.$\square$

Proposition 4.3 *The right scalar unit elements of the* H_v*-semigroup* $(\mathbf{\Omega}, r_L)$*, are idempotent elements.*

Proof. Let $\alpha \in \mathbf{\Omega}$ be a right scalar unit element, then $xr_L\alpha = x, \forall x \in \mathbf{\Omega}$. So, $\alpha r_L \alpha = \alpha \Rightarrow \alpha^2 = \alpha$.□

Proposition 4.4 *If there exists $x \in \mathbf{\Omega}$ such that f(x)=x or [x]=x, then x is left absorbing element and every element of $(\mathbf{\Omega}, r_L)$ is right scalar unit of x.*

Proof. Suppose there exist $x \in \mathbf{\Omega}$ such that [x]=x, then $\forall y \in \mathbf{\Omega}$:

$$xr_Ly = [x] = x.$$

That means that x is left absorbing element and every element of $(\mathbf{\Omega}, r_L)$ is right scalar unit of x.□

Since all the elements of the H_v-semigroup $(\mathbf{\Omega}, r_L)$ are right unit elements, let us denote [7] by $I^l_{r_L}(x, y)$ the set of the left inverses of the element $x \in \mathbf{\Omega}$, associated with the right unit $y \in \mathbf{\Omega}$, with respect to the hyperoperation (r_L). The set of the right inverses of the element $x \in \mathbf{\Omega}$, associated with the right unit $y \in \mathbf{\Omega}$, with respect to the hyperoperation (r_L), is denoted by $I^r_{r_L}(x, y)$.

Proposition 4.5 $y \in I^l_{r_L}(x, y)$

Proof. Let $x' \in \mathbf{\Omega}$ such that $x' \in I^l_{r_L}(x, y) \Rightarrow y \in x'r_Lx$. But $\forall x \in \mathbf{\Omega}$ the relation $y \in yr_Lx$ is valid. That means that $y \in I^l_{r_L}(x, y)$.□

Proposition 4.6 $I^r_{r_L}(x, y) = \mathbf{\Omega}$ *if and only if* $y \in [x]$.

Proof. Let $y \in \mathbf{\Omega}$ be right unit element and $x \in \mathbf{\Omega}$,then

$$y \in [x] \Leftrightarrow y \in xr_Lx', \forall x' \in \mathbf{\Omega} \Leftrightarrow x' \in I^r_{r_L}(x, y), \forall x'\mathbf{\Omega} \Leftrightarrow I^r_{r_L}(x, y) = \mathbf{\Omega}.\square$$

Since $x \in [x], \forall x \in \mathbf{\Omega}$,the following is obvious.

Corollary 4.1 $I^r_{r_L}(x, x) = \mathbf{\Omega}$

Remark 4.1 *Notice that, according to the example 2.1, the elements x_4 and x_9 are idempotent elements, since $x_4^2 = x_4$ and $x_9^2 = x_9$. They are, also, left absorbing elements, since $x_4r_Lx = x_4$ and $x_9r_Lx = x_9, \forall x \in \mathbf{\Omega}$. Also, taking for example, the element x_2 of $\mathbf{\Omega}$, notice that $I^l_{r_L}(x, x_2) = \{x_1, x_2, x_3\}$, $\forall x \in \mathbf{\Omega}$. Even more, since $x_2 \in [x_1]$ we get that $I^r_{r_L}(x_1, x_2) = \mathbf{\Omega}$ and $I^r_{r_L}(x_1, x_1) = \mathbf{\Omega}$.*

5 Applications

As we mentioned above, the noise pollution in urban areas coming from a spot, annoys a certain area in which the noisy spot belongs to. That was the motivation which led to the mathematical expression $xr_Ly = [x], \ \forall x, y \in \mathbf{\Omega}$. That means that if a city is considered as a set $\mathbf{\Omega}$ with elements its buildings (or spots which could produce noise pollution), then every building (or a spot) x, which is a source of noise pollution, together with any other building (or a spot) y of the city, will affect anyhow the noise pollution area [x], where $x \in [x]$ and maybe y. It is clear, that the source of the noise pollution x, could not be seen as the center of a cyclic disk, but as any spot of a certain area which is affected by x.

We shall try to explain some of the properties of the noise hyperoperation (r_L) developed above, in terms of noise pollution problems in urban areas. The property $x \in xr_Ly, \forall y \in \mathbf{\Omega}$ means that the building x, as a source of noise pollution, first of all, annoys the residents of the building x.
The property $r_L[y] = [x]$ means that the source of noise pollution x together with any region [y] is not only independent on the spots of the region [y] but the noise pollution region remains [x], as well.
The property $[x]r_Ly = [x]r_L[y] \supset [x]$ means that the noise pollution region that results when either the noise pollution region [x] operates with the spot y or with the region [y], is the same and anyhow this noise pollution region is bigger than [x].
The property $xr_Ly = xr_Lz$ means that [x] remains the noise pollution region when x as a source of noise pollution affects any other spot of the city $\mathbf{\Omega}$. Continuously, the relation $xr_L\mathbf{\Omega} \neq \mathbf{\Omega}$ means that, the noise pollution region coming from spot x, can't affect the whole city $\mathbf{\Omega}$. The weak associativity which is expressed by the inclusion on the left parenthesis, i.e. $(xr_Ly)r_Lz \supset xr_L(yr_Lz)$ actually means that, the noise pollution region coming from the noise pollution region [x] together with any spot, is not only bigger than that one which comes from the noise pollution spot x together with any other region but includes it, as well.
An absorbing element, as in the relation $\alpha r_Lx = \alpha$, could be considered as a spot surrounded by a wall or a forest, which doesnt annoy any other spot of the city $\mathbf{\Omega}$.

Since the weak associativity is valid, the concept of transitive closure can be applied here, in order to obtain the fundamental β^* classes. The actual meaning of this situation is that the city $\mathbf{\Omega}$ can be divided, using the noise hyperoperation, in a partition, where every fundamental class does not annoy any other blocks of flats from other fundamental classes. The next example gives an idea:

Example 5.1 *According to the example 2.1, consider now that* $\mathbf{\Omega}$ *is a city where* $\mathbf{\Omega} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$. *From "multiplication" table of* (r_L),*we obtain that* $\beta^*(x_1) = \{x_1, x_2, x_3\}$,

$$\beta^*(x_4) = \{x_4\}, \beta^*(x_5) = \{x_5, x_6, x_7\}, \beta^*(x_8) = \{x_8, x_9\}$$

. *So,the fundamental semi-group* $\mathbf{\Omega}/\beta^*$ *is:*

$$\mathbf{\Omega}/\beta^* = \big\{\{x_1, x_2, x_3\}, \{x_4\}, \{x_5, x_6, x_7\}, \{x_8, x_9\}\big\}$$

and the "multiplication" table is:

$\circ$	$\underline{\mathbf{x_1}}$	$\underline{\mathbf{x_4}}$	$\underline{\mathbf{x_5}}$	$\underline{\mathbf{x_8}}$
$\underline{\mathbf{x_1}}$	$\underline{x_1}$	$\underline{x_1}$	$\underline{x_1}$	$\underline{x_1}$
$\underline{\mathbf{x_4}}$	$\underline{x_4}$	$\underline{x_4}$	$\underline{x_4}$	$\underline{x_4}$
$\underline{\mathbf{x_5}}$	$\underline{x_5}$	$\underline{x_5}$	$\underline{x_5}$	$\underline{x_5}$
$\underline{\mathbf{x_8}}$	$\underline{x_8}$	$\underline{x_8}$	$\underline{x_8}$	$\underline{x_8}$

In other words and beyond the mathematical content of the present example, the city $\mathbf{\Omega}$ was divided into four regions, where every region (fundamental class) does not annoy any other spot belonging to the rest regions. So, one could consider that among the four regions there exists a green park, full of trees, which absorbs the possible noise pollution caused by any of the four regions. Since $\beta^*(x_4) = \{x_4\}$, the element $x_4 \in \mathbf{\Omega}$ (spot or building of the city) is a single element and that means that it doesn't annoy any other spot of the city $\mathbf{\Omega}$, so it can be considered as the remotest spot of the city.

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