Ratio Mathematica Volume 40, 2021, pp. 179-190

Intuitionistic FWI-ideals of residuated lattice Wajsberg algebras

R. Shanmugapriya*
A. Ibrahim†

Abstract

The notions of intuitionistic Fuzzy Wajsberg Implicative ideal (FWI –ideal) and intuitionistic fuzzy lattice ideal of residuated Wajsberg algebras are introduced. Also, we show that every intuitionistic FWI- ideal of residuated lattice Wajsberg algebra is an intuitionistic fuzzy lattice ideal of residuated lattice Wajsberg algebra. Further, we discussed its converse part.

Keywords: Wajsberg algebra; Lattice Wajsberg algebra; Residuated lattice Wajsberg algebra; WI–ideal; FWI –ideal; Intuitionistic FWI –ideal; Intuitionistic fuzzy lattice ideal.
2010 AMS subject classification: 06B10, 03E72, 03G10.

*Research Scholar, P. G. and Research Department of Mathematics, H. H. The Rajah's College, Pudukkotai, Affiliated to Bharathidasan University, Trichirappalli, Tamilnadu, India; priyasanmu@gmail.com
†Assistant Professor, P.G. and Research Department of Mathematics, H. H. The Rajah's College, Pudukkotai, Affiliated to Bharathidasan University, Trichirappalli, Tamilnadu, India; ibrahimaadhil@yahoo.com; dribra@hhrc.ac.in
†Received on January 12th, 2021. Accepted on May 12th, 2021. Published on June 30th, 2021. doi: 10.23755/rm.v40i1.587. ISSN: 1592-7415. eISSN: 2282-8214.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [13] in 1935. The concept of intuitionistic fuzzy set was introduced by Atanassov [1, 2]. The idea of Wajsberg algebra was introduced by Mordchaj Wajsberg [10]. The author [8] introduced the notions of FWI-ideals and investigated their properties with illustrations.

In the present paper, we introduce the notions of intuitionistic FWI –ideal and intuitionistic fuzzy lattice ideal of residuated lattice Wajsberg algebras. Also, we show that every intuitionistic FWI–ideal of residuated lattice Wajsberg algebra is an intuitionistic fuzzy lattice ideal of residuated lattice Wajsberg algebra. Further, we verify its converse part.

2. Preliminaries

In this section, we recall some basic definitions and properties which are helpful to develop our main results.

Definition 2.1[3]. Let $(A,\rightarrow,*,1)$ bean algebra with a binary operation " $\rightarrow$ " and a quasi-complement " $*$ ". Then it is called a Wajsberg algebra, if the following axioms are satisfied for all $x,y,z\in A$,

(i) $1\rightarrow x=x$
(ii) $(x\rightarrow y)\rightarrow y=((y\rightarrow z)\rightarrow(x\rightarrow z))=1$
(iii) $(x\rightarrow y)\rightarrow y=(y\rightarrow x)\rightarrow x$
(iv) $(x^*\rightarrow y^*)\rightarrow(y\rightarrow x)=1$.

Definition 2.2[3]. Let$(A,\rightarrow,*,1)$ be a Wajsberg algebra. Then the following axioms are satisfied for all $x,y,z\in A$,

(i) $x\rightarrow x=1$
(ii) If $(x\rightarrow y)=(y\rightarrow x)=1$ then $x=y$
(iii) $x\rightarrow 1=1$
(iv) $(x\rightarrow(y\rightarrow x))=1$
(v) If $(x\rightarrow y)=(y\rightarrow z)=1$ then $x\rightarrow z=1$
(vi) $(x\rightarrow y)\rightarrow\big((z\rightarrow x)\rightarrow(z\rightarrow y)\big)=1$
(vii) $x\rightarrow(y\rightarrow z)=y\rightarrow(x\rightarrow z)$
(viii) $x\rightarrow 0=x\rightarrow 1^*=x^*$
(ix) $(x^*)^*=x$
(x) $(x^*\rightarrow y^*)=y\rightarrow x$.

Definition 2.3[3]. Let $(A,\rightarrow,*,1)$ be a Wajsberg algebra. Then it is called a lattice Wajsberg algebra, if the following axioms are satisfied for all $x,y\in A$,

(i) The partial ordering " $\leq$ " on a Wajsberg algebra such that $x \leq y$ if and only if $x \rightarrow y = 1$

(ii) $x \vee y = (x \rightarrow y) \rightarrow y$

(iii) $x \wedge y = ((x^* \rightarrow y^*) \rightarrow y^*)^*$.

Thus, $(A, \vee, \wedge, *, 0, 1)$ is a lattice Wajsberg algebra with lower bound 0 and upper bound 1.

Proposition 2.4[3]. Let$(A, \rightarrow, *, 1)$ be a lattice Wajsberg algebra. Then the following axioms are satisfied for all $x, y, z \in A$,

(i) If $x \leq y$ then $x \rightarrow z \geq y \rightarrow z$ and $z \rightarrow x \leq z \rightarrow y$

(ii) $x \leq y \rightarrow z$ if and only $if\ y \leq x \rightarrow z$

(iii) $(x \vee y)^* = (x^* \wedge y^*)$

(iv) $(x \wedge y)^* = (x^* \vee y^*)$

(v) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$

(vi) $x \rightarrow (y \wedge z) = (x \rightarrow y) \wedge (x \rightarrow z)$

(vii) $(x \rightarrow y) \vee (y \rightarrow x) = 1$

(viii) $x \rightarrow (y \vee z) = (x \rightarrow y) \vee (x \rightarrow z)$

(ix) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$

(x) $(x \wedge y) \vee z = (x \vee z) \wedge (y \vee z)$

(xi) $(x \wedge y) \rightarrow z = (x \rightarrow y) \rightarrow (x \rightarrow z)$.

Definition 2.5[11]. Let$(A, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ be an algebra of type (2, 2, 2, 2, 0, 0). Then it is called a residuated lattice, if the following axioms are satisfied for all $x, y, z \in A$,

(i) $(A, \vee, \wedge, 0, 1)$ is a bounded lattice,

(ii) $(A, \otimes, 1)$ is commutative monoid,

(iii) $x \otimes y \leq z$ if and only if $x \leq y \rightarrow z$.

Definition 2.6[3]. Let $(A, \vee, \wedge, *, \rightarrow, 1)$ be a lattice Wajsberg algebra. If a binary operation " $\otimes$ " on A satisfies $x \otimes y = (x \rightarrow y^*)^*$ for all $x, y \in A$. Then $(A, \vee, \wedge, \otimes, \rightarrow, *, 0, 1)$ is called a residuated lattice Wajsberg algebra.

Definition 2.7[4]. Let A be a lattice Wajsberg algebra. Let I be a non-empty subset of A, then I is called a*WI*-ideal of lattice Wajsberg algebra A, if the following axioms are satisfied for all$x, y \in A$,

(i) $0 \in I$

(ii) $(x \rightarrow y)^* \in I$and $y \in I$ imply $x \in I$.

Definition 2.8[4]. Let L be a lattice. An ideal I of L is a non-empty subset of L is called a lattice ideal, if the following axioms are satisfied for all $x, y \in A$,

(i) $x \in I, y \in L$ and $y \leq x$ imply $y \in I$

(ii) $x, y \in I$ implies $x \vee y \in I$.

Definition 2.9[7]. Let A be a residuated lattice Wajsberg algebra and I be a non-empty subset of A.Then I is called a WI-ideal of residuated lattice Wajsberg algebra A, if the following axioms are satisfiedfor all $x, y \in A$,
(i) $0 \in I$
(ii) $x \otimes y \in I$ and $y \in I$ imply $x \in I$
(iii) $(x \to y)^* \in I$ and $y \in I$ imply $x \in I$.

Definition 2.10[13]. Let A be a set. A function $\mu: A \to [0, 1]$ is called a fuzzy subset on A for each $x \in A$, the value of $\mu(x)$ describes a degree of membership of x in μ.

Definition 2.11[5]. Let A be a lattice Wajsberg algebra. Then the fuzzy subset μ of A is called a fuzzy WI-ideal of A, if the following axioms are satisfied for all $x, y \in A$,

(i) $\mu(0) \geq \mu(x)$
(ii) $\mu(x) \geq \min\{\mu((x \to y)^*), \mu(y)\}$.

Definition 2.12[5].A fuzzy subsetμ of a lattice Wajsberg algebra A is called a fuzzy lattice ideal if for all $x, y \in A$,
(i) If $y \leq x$ then $\mu(y) \geq \mu(x)$
(ii) $\mu(x \vee y) \geq \min\{\mu(x), \mu(y)\}$.

Definition 2.13[8]. Let A be a residuated lattice Wajsberg algebra. Then the fuzzy subset μ of A is called a FWI-ideal of residuated lattice Wajsberg algebra A, if the following axioms are satisfied for all $x, y \in A$,
(i) $\mu(0) \geq \mu(x)$
(ii) $\mu(x) \geq \min\{\mu(x \otimes y), \mu(y)\}$
(iii) $\mu(x) \geq \min\{\mu((x \to y)^*), \mu(y)\}$.

Definition 2.14[2]. An intuitionistic fuzzy subset S is a non-empty set X is an object having the form $S = \{(x, \mu_s(x), \gamma_s(x)) | x \in X\} = (\mu_s, \gamma_s)$where the functions $\mu_s(x): X \to [0, 1]$denote the degree of membership and the degree of non-membership respectively and $0 \leq \mu_s(x) + \gamma_s(x) \leq 1$ for any $x \in X$.

Definition 2.15[13]. If μ and v are fuzzy sets in A, define $\mu \leq$ v if and only if $\mu(x) \leq \mathrm{v}(x)$ for all $x \in A$.

Definition 2.16[13]. The level set μ_t defined by $\mu_t = \{x \in A / \mu(x) \geq t\}$, where$t \in [0, 1]$, then μ_t is also denoted by $U(\mu; t)$.

3. Properties of Intuitionistic *FWI*-ideal of a residuated lattice Wajsberg algebra

In this section, we introduce the concept of an intuitionistic *FWI*-ideal and intuitionistic fuzzy lattice ideals. Also, we obtain some properties of an intuitionistic FWI-ideal.

Definition 3.1. Let A be a residuated lattice Wajsberg algebra. An intuitionistic fuzzy set $S = (\mu_s, \gamma_s)$ of A is called an intuitionistic *FWI*-ideal of residuated lattice Wajsberg algebra A if it satisfies the following inequalities for all $x, y \in A$,

(i) $\mu_s(0) \geq \mu_s(x)$ and $\gamma_s(0) \leq \gamma_s(x)$
(ii) $\mu_s(x) \geq \min\{\mu_s(x \otimes y),\ \mu_s(y)\}$
(iii) $\gamma_s(x) \leq \max\{\gamma_s(x \otimes y),\ \gamma_s(y)\}$
(iv) $\mu_s(x) \geq \min\{\mu_s((x \rightarrow y)^*, \mu_s(y)$
(v) $\gamma_s(x) \leq \max\{\gamma_s((x \rightarrow y)^*, \gamma_s(y)\}$.

Example 3.2. Consider a set A=$\{0, a, b, c, d, r, s, t, 1\}$. Define a partial ordering "$\leq$" on A, such that $0 \leq a \leq b \leq c \leq d \leq r \leq s \leq t \leq 1$ with a binary operations"$\otimes$"and"$\rightarrow$"and a quasi-complement "$*$"on A as in following tables 3.1 and 3.2.

x	x^*
0	1
a	t
b	b
c	r
d	d
r	c
s	b
t	a
1	0

Table 3.1: Complement

$\rightarrow$	0	a	b	c	d	r	s	t	1
0	1	1	1	1	1	1	1	1	1
a	t	1	1	t	1	1	t	1	1
b	b	t	1	s	t	1	s	t	1
c	r	r	r	1	1	1	1	1	1
d	d	r	r	t	1	1	t	1	1
r	c	d	r	s	t	1	s	t	1
s	b	b	b	r	r	r	1	1	1
t	a	b	b	d	r	r	t	1	1
1	0	a	b	c	d	r	s	t	1

Table 3.2: Implication

Define $\vee$ and $\wedge$ operations on A as follows:
$(x \vee y) = (x \rightarrow y) \rightarrow y$,
$(x \wedge y) = (x^* \rightarrow y^*) \rightarrow y^*)^*$,

$x \otimes y = (x \rightarrow y^*)^*$ for all $x, y \in A$.
Then, A is a residuated lattice Wajsberg algebra.
Consider an intuitionistic fuzzy set $S = (\mu_s, \gamma_s)$ on A as,

$$\mu_s(x) = \begin{cases} 1 & \text{if } x \in (0, q) \quad \text{for all } x \in A \\ 0.54 & \text{otherwise} \quad \text{for all } x \in A \end{cases};$$

$$\gamma_s(x) = \begin{cases} 0 & \text{if } x \in (0, q) \quad \text{for all } x \in A \\ 0.36 & \text{otherwise} \quad \text{for all } x \in A \end{cases}$$

Then, S is an intuitionistic *FWI*-ideal of A.
In the same Example 3.2, we consider an intuitionistic fuzzy set $S = (\mu_s, \gamma_s)$ on A as,

$$\mu_s(x) = \begin{cases} 1 & \text{if } x \in \{0, a, b\} \quad \text{for all } x \in A \\ 0.55 & \text{otherwise} \quad \text{for all } x \in A \end{cases};$$

$$\gamma_s(x) = \begin{cases} 0 & \text{if } x \in \{0, a, b\} \quad \text{for all } x \in A \\ 0.42 & \text{otherwise} \quad \text{for all } x \in A \end{cases}$$

Then, S is not an intuitionistic FWI-ideal of A.
Since $\mu_s(x) \ngeq \min\{\mu_s(s \otimes b), \mu_s(b)\}$ and $\gamma_s(x) \nleq \max\{\gamma_s(s \otimes b), \ \gamma_s(b)\}$.

Proposition 3.3. Every intuitionistic FWI-ideal $S = (\mu_s, \gamma_s)$ of residuated lattice Wajsberg algebra A is an intuitionistic monotonic. That is, if $x \leq y$, then $\mu_s(x) \geq \mu_s(y)$ and $\gamma_s(x) \leq \gamma_s(y)$.

Proof. Let $S = (\mu_s, \gamma_s)$ be an intuitionistic FWI-ideal of A.
Let $x, y \in A, x \leq y$.
Then $x \otimes y = (x \rightarrow y^*)^*$ [From the definition 2.6]
$= (x \rightarrow x)^* = 1^* = 0$ [From (i) of definition 2.2]
$\mu_s(x) \geq \min\{\mu_s(x \otimes y), \mu_s(y)\}$ [From (ii) of definition 3.1]
We have $\mu_s(x) \geq \mu_s(y)$
Now, $\gamma_s(x) \leq \max\{\gamma_s(x \otimes y), \gamma_s(y)\}$ [From (iii) of definition 3.1]
$= \max\{\gamma_s(0), \gamma_s(y)\} = \gamma_s(y)$ [From the definition 2.6]
Hence $\gamma_s(x) \leq \gamma_s(y)$
And $\mu_s(x) \geq \min\{\mu_s(x \rightarrow y)^*, \mu_s(y)\}$ [From (iv) of definition 3.1]
$= \min\{\mu_s(0), \mu_s(y)\} = \mu_s(y)$ [From (ii) of definition 2.7]
We have $\mu_s(x) \geq \mu_s(y)$
Now, $\gamma_s(x) \leq \max\{\gamma_s(x \rightarrow y)^*, \gamma_s(y)\}$ [From (v) of definition 3.1]
$= \max\{\gamma_s(0), \gamma_s(y)\} = \gamma_s(y)$ [From (ii) of definition 2.7]
Therefore, $\gamma_s(x) \leq \gamma_s(y)$. ■

Example 3.4. Let A be a residuated lattice Wajsberg algebra defined in example 3.2, define an intuitionistic fuzzy set $S = (\mu_s, \gamma_s)$ of A as follows,

(i) $\mu_s(0) = \mu_s(c) = 1$
(ii) $\mu_s(x) = m$ for any $x \in \{a,\ b,\ c,\ d,\ r,\ s,\ t,\ 1\}$
(iii) $\gamma_s(0) = \gamma_s(c) = 0$
(iv) $\gamma_s(x) = n$ for any $x \in \{a,\ b,\ c,\ d,\ r,\ s,\ t,\ 1\}$.

Where $m, n \in [0, 1]$ and $m + n \leq 1$. Then $S = (\mu_s, \gamma_s)$ is an intuitionistic FWI-ideal of A.

Example 3.5. Consider a set$A = \{a,\ b,\ p,\ q,\ c,\ d, 1\}$. Define a partial ordering "$\leq$" on A, such that $0 \leq a \leq b \leq p \leq q \leq c \leq d \leq 1$ with a binary operations"$\otimes$"and "$\rightarrow$"and a quasi-complement "$*$"on A as in following tables 3.3 and 3.4.

x	x^*
0	1
a	b
b	a
p	0
q	0
c	0
d	0
1	0

Table 3.3: Complement

$\rightarrow$	0	a	b	p	q	c	d	1
0	1	1	1	1	1	1	1	1
a	b	1	b	1	1	1	1	1
b	a	a	1	1	1	1	1	1
p	0	a	b	1	1	1	1	1
q	0	a	b	p	1	1	1	1
c	0	a	b	p	d	1	d	1
d	0	a	b	p	c	c	1	1
1	0	a	b	p	q	c	d	1

Table 3.4: Implication

Define $\vee$ and $\wedge$ operations on A as follows:
$(x \vee y) = (x \rightarrow y) \rightarrow y$,
$(x \wedge y) = (x^* \rightarrow y^*) \rightarrow y^*)^*$,
$x \otimes y = (x \rightarrow y^*)^*$ for all $x, y \in A$.
Then, A is a residuated lattice Wajsberg algebra.
Consider an intuitionistic fuzzy set $S = (\mu_s, \gamma_s)$ on A as,

$$\mu_s(x) = \begin{cases} 1 \quad \text{if } x \in (0, q) & \text{for all } x \in A \\ 0.54 \quad \text{otherwise} & \text{for all } x \in A \end{cases};$$

$$\gamma_s(x) = \begin{cases} 0 \ \text{ if } x \in (0, q) & \text{for all } x \in A \\ 0.36 \quad \text{otherwise} & \text{for all } x \in A \end{cases}$$

Then, S is an intuitionistic FWI-ideal of A.
In the same Example 3.5, we consider an intuitionistic fuzzy set $S = (\mu_s, \gamma_s)$ on A as,

$$\mu_s(x) = \begin{cases} 1 & \text{if } x \in \{0, a, b\} \quad \text{for all } x \in A \\ 0.55 & \text{otherwise} \quad \text{for all } x \in A \end{cases};$$

$$\gamma_s(x) = \begin{cases} 0 & \text{if } x \in \{0, a, b\} \quad \text{for all } x \in A \\ 0.42 & \text{otherwise} \quad \text{for all } x \in A \end{cases}$$

Then, S is not an intuitionistic FWI-ideal of A.
Since $\mu_s(x) \ngeq \min\{\mu_s(c \otimes a), \mu_s(a)\}$ and $\gamma_s(x) \nleq \max\{\gamma_s(c \otimes a),\ \gamma_s(a)\}$.

Proposition 3.6. Let $S = (\mu_s, \gamma_s)$ be an intuitionistic FWI-ideal of residuated lattice Wajsberg algebra A. For any $x, y, z \in A$ which satisfies $x \leq y^* \to z$ then $\mu_s(x) \geq \min\{\mu_s(y), \mu_s(z)\}$ and $\gamma_s(x) \leq \max\{\gamma_s(y), \gamma_s(z)\}$.

Proof. Let$S = (\mu_s, \gamma_s)$ be an intuitionistic FWI-ideal of A. If $x \leq y^* \to z$

Then, we have $1 = x \to (y^* \to z) = z^* \to (x \to y)$
$= (x \to y)^* \to z$ for all $x, y, z \in A$ [From (x) of definition 2.2]
And $((x \to y)^* \to z)^*) = 0$.
It follows that,
$\mu_s(x) \geq \min\{\mu_s(x \otimes y), \mu_s(y)\}$ [From (ii) of definition 3.1]
$\geq \min\{\min\{\mu_s((x \otimes y) \to z), \mu_s(z)\}, \mu_s(y)\}$
$= \min\{\min\{\mu_s((0) \to z), \mu_s(z)\}, \mu_s(y)\}$ [From the definition 2.6]
$= \min\{\min\{\mu_s(0), \mu_s(z)\}, \mu_s(y)\} = \min\{\mu_s(y), \mu_s(z)\}$
[From (ii) of definition 3.1]
We have $\mu_s(x) \geq \min\{\mu_s(y), \mu_s(z)\}$ for all $x, y, z \in A$
Now,$\gamma_s(x) \leq \max\{\max\{\gamma_s((x \otimes y), \gamma_s(y))\}$
$\leq \max\{\max\{\gamma_s\left(((x \otimes y) \to z), \gamma_s(z)\right\}, \gamma_s(y)\}$
$= \max\{\max\{\gamma_s((0) \to z), \gamma_s(z)\}, \gamma_s(y)\}$ [From the definition 2.6]
$= \max\{\max\{\gamma_s(0), \gamma_s(z)\}, \gamma_s(y)\}$
$= \max\{\gamma_s(y), \gamma_s(z)\}$ [From (iii) of definition 3.1]
Hence $\gamma_s(x) \leq \max\{\gamma_s(y), \gamma_s(z)\}$ for all $x, y, z \in A$
Now, $\mu_s(x) \geq \min\{\mu_s((x \to y)^*), \mu_s(y)\}$ [From (iv) of definition 3.1]
$\geq \min\{\min\{\mu_s(x \to y)^* \to z)^*), \mu_s(z)\}, \mu_s(y)\}$
$= \min\{\min\{\mu_s(0),\ \mu_s(z)\}, \mu_s(y)\}$
$= \min\{\mu_s(y), \mu_s(z)\}$ [From (ii) of definition 3.1]
We have $\mu_s(x) \geq \min\{\mu_s(y), \mu_s(z)\}$ for all $x, y, z \in A$
And $\gamma_s(x) \leq \max\{\gamma_s((x \to y^*), \gamma_s(y))\}$ [From (v) of definition 3.1]
$\leq \max\{\max\{\gamma_s((x \to y^*) \to z)^*), \gamma_s(z)\}, \gamma_s(y)\}$
$= \max\{\max\{\gamma_s(0), \gamma_s(z)\}, \gamma_s(y)\}$
$= \max\{\gamma_s(y), \gamma_s(z)\}$ [From (iii) of definition 3.1]
Hence, $\gamma_s(x) \leq \max\{\gamma_s(y), \gamma_s(z)\}$ for all $x, y, z \in A$.■

Definition 3.7. An intuitionistic fuzzy set $S = (\mu_s, \gamma_s)$ of residuated lattice Wajsberg algebra A is called an intuitionistic fuzzy lattice ideal of A if it satisfies the following axioms for all$x, y \in A$,

(i) $S = (\mu_s, \gamma_s)$ is intuitionistic monotonic
(ii) $\mu_s(x \vee y) \geq \min\{\mu_s(x), \mu_s(y)\}$
(iii) $\gamma_s(x \vee y) \leq \max\{\gamma_s(x), \gamma_s(y)\}$.

Remark 3.8. In the Definition 3.7(ii) and (iii) can be equivalently replaced by $\mu_s(x \vee y) = \min\{\mu_s(x), \mu_s(y)\}$ and $\gamma_s(x \vee y) = \max\{\gamma_s(x), \gamma_s(y)\}$ respectively by γ.

Example 3.9. Let A be a residuated lattice Wajsberg algebra defined in the Example 3.2 and $S = (\mu_s, \gamma_s)$ be an intuitionistic fuzzy set of A defined by

$$\mu_s(x) = \begin{cases} 1 & \text{if } x \in (0, d) \quad \text{for all } x \in A \\ m & \text{otherwise} \quad \text{for all } x \in A \end{cases},$$
$$\gamma_s(x) = \begin{cases} 0 & \text{if } x \in (0, d) \quad \text{for all } x \in A \\ n & \text{otherwise} \quad \text{for all } x \in A \end{cases}$$

Where $m, n \in [0, 1]$ and $m + n \leq 1$. [From the definition 3.11]
Then, $S = (\mu_s, \gamma_s)$ is an intuitionistic fuzzy lattice ideal of residuated lattice Wajsberg algebra A.

Proposition 3.10. Let$A$be a residuated lattice Wajsberg algebra. Every intuitionistic FWI-ideal of A is an intuitionistic fuzzy lattice ideal of A.

Proof. Let $S = (\mu_s, \gamma_s)$ be an intuitionistic fuzzy lattice ideal of A.
Then we have $S = (\mu_s, \gamma_s)$ is intuitionistic monotonic. [From proposition 3.6]
Now $((x \vee y) \to y)^* = (((x \to y) \to y)) \to y)^*$ From (ii) of definition 2.3]
$= (x \to y)^* \leq (x^*)^*$ for all $x, y \in A$ [From (ix) of proposition 2.2]
It follows that

$$\mu_s(x \vee y) \geq \min\{\mu_s(x \vee y) \otimes y, \mu_s(y)\}$$
[From definition 3.1 and definition 3.7]
$$\geq \min\{\mu_s(x \to y) \to y) \otimes y, \mu_s(y)\}$$
[From (ii) of definition 2.3]
$$\geq \min\{\mu_s(0), \mu_s(y)\}$$
$$\geq \min\{\mu_s(x), \mu_s(y)\} \text{for all } x, y \in A$$
[From (i) of proposition 2.10]
$$\gamma_s(x) \leq \max\{\gamma_s((x \vee y) \otimes y), \gamma_s(y)\}$$
$$\leq \max\{\gamma_s(((x \to y) \to y) \otimes y), \gamma_s(y)\}$$
[From (ii) of definition 2.3]
$$\leq \max\{\gamma_s(0), \gamma_s(y)\}$$

$\leq \max\{\gamma_s(x), \gamma_s(y)\}$for all $x, y \in A$

[From (ii) of definition 2.10]

And we have

$\mu_s(x \vee y) \geq \min\{\mu_s(x \vee y) \to y)^*), \mu_s(y)\} \geq \min\{\mu_s(x), \mu_s(y)\}$

$\gamma_s(x) \leq \max\{\gamma_s\big((x \vee y) \to y)^*\big), \gamma_s(y)\} \leq \max\{\gamma_s(x), \gamma_s(y)\}$

for all $x, y \in A$.

Hence, we have $S = (\mu_s, \gamma_s)$is an intuitionistic fuzzy lattice ideal of residuated lattice Wajsberg algebra A. ■

Proposition 3.11. Let A be a residuated lattice Wajsberg algebra. An intuitionistic fuzzy set $S = (\mu_s, \gamma_s)$ is an intuitionistic *FWI*-ideal of A if and only if the fuzzy subsets μ_s and γ_s^c are FWI-ideal of A, where $\gamma_s^c(x) = 1 - \gamma_s(x)$for all $x \in A$.

Proof. Let $S = (\mu_s, \gamma_s)$ be an intuitionistic *FWI*-ideal of A.

Then μ_s is a *FWI*-ideal of A.

Now, we have $\gamma_s^c = 1 - \gamma_s(0)$

$\geq 1 - \gamma_s(x)$ [From (i) of proposition 2.10]

$\gamma_s^c(0) = \gamma_s^c(x)$ for all $x, y \in A$

And $\gamma_s^c(x) = 1 - \gamma_s(x)$

$\geq 1 - \max\{\gamma_s(x \otimes y), \gamma_s(y)\}$

$= \min\{1 - \gamma_s(x \otimes y), 1 - \gamma_s(y)\}$

$= \min\{\gamma_s^c(x \otimes y), \gamma_s(y)\}$

$\gamma_s^c(x) = 1 - \gamma_s(x)$

$\geq 1 - \max\{\gamma_s((x \to y)^*), \gamma_s(y)\}$

$= \min\{1 - \gamma_s((x \to y)^*), 1 - \gamma_s(y)\}$

$\gamma_s^c(x) = \min\{\gamma_s^c((x \to y)^*), \gamma_s(y)\}$for all $x, y \in A$

Hence, we have γ_s^c is a *FWI*-ideal of A.

Conversely, assume that μ_s and γ_s^c are *FWI*-ideal of A.

Then, we have $\mu_s(0) \geq \mu_s(x)$and $1 - \gamma_s(0) = \gamma_s^c(0) \geq \gamma_s^c(x) = 1 - \gamma_s(x)$

$\gamma_s(0) \leq \gamma_s(x)$ for all $x, y \in A$

Now, $\mu_s(x) \geq \min\{\mu_s^c(x \otimes y), \mu_s^c(y)\}$

$= \min\{1 - \mu_s(x \otimes y), 1 - \mu_s(y)\}$

$= 1 - \max\{\mu_s(x \otimes y), \mu_s(y)\}$

$\gamma_s(x) \leq \max\{\gamma_s(x \otimes y), \gamma_s(y)\}$ for all $x, y \in A$

$\mu_s(x) \geq \min\{\mu_s^c(x \to y)^*, \mu_s^c(y)\}$

$= \min\{1 - \mu_s((x \to y)^*), 1 - \mu_s(y)\}$

$= 1 - \max\{\mu_s((x \to y)^*), \mu_s(y)\}$

$\gamma_s(x) \leq \max\{\gamma((x \to y)^*), \gamma_s(y)\}$for all $x, y \in A$

Hence, we have $S = (\mu_s, \gamma_s)$ is an intuitionistic FWI-ideal of A.■

Proposition 3.12. Let A be a residuated lattice Wajsberg algebra and $S = (\mu_s, \gamma_s)$ is an intuitionistic FWI-ideal of$A$. Then $S = (\mu_s, \gamma_s)$ is an intuitionistic FWI-ideal of A if and only if (μ_s, μ_s^c) and (γ_s^c, γ_s) are intuitionistic FWI-ideal of A.

Proof. Let $S = (\mu_s, \gamma_s)$ be an intuitionistic FWI-ideal of A.
Then, μ_s and γ_s^c are FWI-ideal of A[From proposition 3.11]
Hence, we have (μ_s, μ_s^c) and (γ_s^c, γ_s) are intuitionistic FWI-ideal of A.
Conversely, if (μ_s, μ_s^c) and (γ_s^c, γ_s) are intuitionistic FWI-idealof A
[From proposition 3.11]
Then, the fuzzy sets μ_s and γ_s^c are FWI-ideal of A
Hence, $S = (\mu_s, \gamma_s)$ is an intuitonistic FWI-ideal of A. ■

Proposition 3.13. Let A be residuated lattice Wajsberg algebra, V a non-empty subset of $[0, 1]$ and $\{I_t / \ t \in V\}$ a collection of FWI -ideal of A such that

(i) $A = \underset{t \in v}{\cup} I_t$
(ii) $r > t$ if and only if $I_r \subseteq I_t$ for any $r, t \in V$ then the intuitionistic fuzzy set $S = (\mu_s, \gamma_s)$ of A defined by $\mu_s = \sup\{t \in V / x \in I_t\}$ and$\gamma_s = \inf\{t \in V / x \in I_t\}$ for any $x \in A$ is intuitionistic FWI -ideal of A.

Proof. According to proposition 3.10, it is sufficient to show that μ_s and γ_s^c are FWI–idealof A for all$x \in A$.
$\mu_s(0) = \sup\{t \in V / 0 \in I_t\} = \sup V \geq \mu_s(x)$ [From (i) of definition 3.1]
If there exists $x, y \in A$ such that $\mu_s(x) < \min\{\mu_s(x \otimes y), \mu_s(y)\}$ and
$\mu_s(x) < \min\{\mu_s((x \to y)^*), \mu_s(y)\}$.
There exists t_1 such that $\mu_s(x) < t_1 < \min\{\mu_s(x \otimes y), \mu_s(y)\}$ and
$\mu_s(x) < t_1 < \min\{\mu_s((x \to y)^*), \mu_s(y)\}$
It follows that t_1 such that$t_1 < \mu_s(x \otimes y)$,$t_1 < \mu_s((x \to y)^*)$,$t_1 < \mu_s(y)$ and
Hence, there exist $t_2, t_3 \in V$, $t_2 > t_1$, $t_3 > t_1$, $(x \otimes y) \in I_{t_2}, (x \to y)^*) \in I_{t_2}$ and $y \in I_{t_3}$
It follows that $(x \otimes y) \in I_{t_2 \wedge t_3}$, $(x \to y)^*) \in I_{t_2 \wedge t_3}$ and $y \in I_{t_2 \wedge t_3}$
Now, we have $x \in I_{t_2 \wedge t_3}$
That is, $\mu_s(x) = \sup\left\{t \in \frac{V}{x} \in I_t\right\} \geq t_2 \wedge t_3 > t_1$ [From definition 2.16]
Therefore, $\mu_s(x) > t_1$
This is a contradiction.
Hence, we have μ_s is a FWI -ideal of A. $\gamma_s^c$is aFWI -ideal, which can be proved by similar method. ■

4. Conclusions

In this paper, we have introduced the notions of intuitionistic FWI –ideal and intuitionistic fuzzy lattice ideal of residuated Wajsberg algebras. Also, we have shown that every intuitionistic FWI- ideal of residuated lattice Wajsberg algebra is an intuitionistic fuzzy lattice ideal of residuated lattice Wajsberg algebra. Further, we have discussed its converse part.

References

[1] K. T. Atanassav, Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 20(1):87-96,1986.

[2] K. T. Atanassov, New Operations Defined Over the Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems,61(2):137-142, 1994.

[3] J. M. Font, A. J. Rodriguez and A. Torrens, Wajsberg Algebras, STOCHASTICA, 8 (1): 5-31, 1984.

[4] A. Ibrahim and C. Shajitha Begum, On WI-Ideals of Residuated Lattice Wajsberg Algebras, Global Journal of Pure and Applied Mathematics, 13(10): 7237-7254, 2017.

[5] A.Ibrahim and C.Shajitha Begum, Fuzzy and Normal Fuzzy WI-ideals of Lattice Wajsberg Algebras, International Journal of Mathematical Archive, 8(11):122-130, 2017.

[6] A. Ibrahim and C. Shajitha Begum, Intuitionistic Fuzzy WI-ideals of Lattice Wajsberg algebras, International Journal of Mathematical Archive, 7-17, 2018.

[7] A. Ibrahim and R.Shanmugapriya, WI-Ideals of Residuated Lattice Wajsberg Algebras Journal of Applied Science and Computations, 792-798, 2018.

[8] A. Ibrahim and R. Shanmugapriya, FWI-Ideals of Residuated Lattice Wajsberg algebras, Journal of Computer and Mathematical Sciences, 491-501, 2019.

[9] A. Ibrahim and R. Shanmugapriya, Anti FWI-Ideals of Residuated Lattice Wajsberg Algebras, Advances in Mathematics: Scientific Journal 8:300-306, 2019.

[10] M. Wajsberg, Beitrage zum Metaaussagenkalkul, Monat. Mat. Phy. 42: 221-242, 1935.

[11] M. Word and R. P. Dilworth, Residuated Lattices, Transactions the American Mathematical Society 45: 335-354, 1939.

[12] Yi Liu, Ya Qin, Qin Xiaoyan and Xu Yang, Ideals and Fuzzy Ideals on Residuated Lattices, International Journal of Machine Learning and Cybernetics, Volume 8(1): 239-253, 2017.

[13] L. A. Zadeh, Fuzzy Sets, Information and Control, 8(3):338-353, 1965.