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# On Characterization of $\delta$-Topological Vector Space

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**Abstract**

The main objective of this paper is to present the study of $\delta$-topological vector space. $\delta$-topological vector space is defined by using $\delta$-open sets and $\delta$-continuous mapping which was introduced by J.H.H. Bayati[3] in 2019. In this paper, along with basic inherent properties of the space, $\delta$-closure and $\delta$-interior operators are discussed in detail. We characterize some important properties like translation, dilation of the $\delta$-topological vector space and an example of $\delta$-topological vector space is also established.
**Keywords**: Regular open set, $\delta$-open set, $\delta$-closed set, $\delta$-continuous mapping and $\delta$-topological vector space.
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# 1 Introduction

In functional analysis, topological vector space is one of the fundamental space being investigated by mathematicians due to the significant role played by it in other branches of mathematics such as operator theory, fixed point theory, variational inequality etc. The formalism of topological vector space belongs to Kolmogroff [4] who was the first to introduce a well structured notion of topological vector space in his pioneering work done in 1934. Since then, it has evolved further and many mathematicians have developed different generalizations of topological vector space. In 2015, the notion of s-topological vector space is being developed by M. Khan et.al.[5], which is one of the generalization of topological vector space. Later, many other significant generalizations of topological vector space are being introduced such as irresolute topological vector space [2], $\beta$-topological vector space [11], strongly preirresolute topological vector space [9], almost s-topological vector spaces [10], etc.

# 2 Preliminaries

In this paper, $(X,\tau)$(or simply X) always means topological space on which no separation axioms are assumed unless stated explicitly. For a subset D of a space X , we denote closure and interior by $Cl(D)$ and $Int(D)$ respectively and neighborhood and $\delta$-neighborhood of an element x in any topological space X is denoted by $N(x)$ and $N_\delta(x)$.

**Definition 2.1.** *Let $B$ be a subset of a topological space $(X,\tau)$. Then $B$ is said to be*
*(a) Regular open [12] if $B = Int(Cl(B))$*
*(b) Pre-open [6] if $B \subseteq Int(Cl(B))$*
*(c) $\beta$-open [1] if $B \subseteq Cl(Int(Cl(B)))$.*

**Definition 2.2.** *A subset $C$ of a topological space $X$ is called*
*(a) Regular closed if $X \setminus C$ is open i.e. $C = Cl(Int(C))$*
*(b) Pre-closed if $Cl(Int(C)) \subseteq C$*
*(e) $\beta$-closed if $Int(Cl(Int(C))) \subseteq C$.*

**Definition 2.3.** *A subset $D$ of a topological space $X$ is said to be $\delta$-open [13] if for each $x \in D$, there exist a regular open set $R$ such that $x \in R \subseteq D$.*

**Remark 2.1.** *Every regular open set is open and every open set is pre-open, while the converse need not be true.*

**Example 2.1.** *Let $\mathbb{R}$ be a set of real numbers with usual topology. Then $Int(Cl(\mathbb{Z})) = \emptyset$, which implies $\mathbb{Z}$ is not regular in topological space $(\mathbb{R}, \tau_u)$. Also, set of rational number denoted by $\mathbb{Q}$ is pre-open but neither regular open nor open set in topological space $(\mathbb{R}, \tau_u)$.*

The complement of $\delta$-open set is $\delta$-closed. The concept of $\delta$-closure and $\delta$-interior are introduced by Velicko [13] in 1968. The intersection of all $\delta$-closed sets in X containing a subset $D \subseteq X$ is called $\delta$-closure of D and is denoted by $Cl_\delta(D)$. A point $x \in Cl_\delta(D)$ if and only if $D \cap R \neq \emptyset$, for a regular open set R in X containing x. A subset C of X is $\delta$-closed if and only if $C = Cl_\delta(C)$. The union of all $\delta$-open sets in X that are contained in $D \subseteq X$ is called $\delta$-interior of D and is denoted by $Int_\delta(D)$. A point $x \in X$ is called $\delta$-interior of $D \subseteq X$ if there exist a $\delta$-open set U in X such that $x \in U \subseteq D$.

**Definition 2.4.** *[4] Let X be a vector space over the field $\mathbb{F}(\mathbb{R}$ or $\mathbb{C})$. Let $\tau$ be a topology on X such that*
*1) for each $x, y \in X$, and for each open neighborhood $W$ of $x+y$ in $X$, there exist open neighborhoods $U$ and $V$ of x and y respectively in X such that $U + V \subseteq W$*
*2) for each $\lambda \in \mathbb{F}$, $x \in X$ and for each open neighborhood $W$ of $\lambda \cdot x$ in X, there exist open neighborhoods $U$ of $\lambda$ in $\mathbb{F}$ and $V$ of $x$ in $X$ such that $U \cdot V \subseteq W$.*
*Then, the $(X_{(\mathbb{F})}, \tau)$ is called topological vector space.*

# 3 $\delta$-Topological Vector Space

In this section, we give an examples of $\delta$-topological vector space and further illustrate the properties of this space.

**Definition 3.1.** *[3] Let $X$ be a vector space over field $\mathbb{F}(\mathbb{R}$ or $\mathbb{C})$ with standard topology. Let $\tau$ be a topology over $X$ such that the following conditions hold:*
*(a) For each $x, y \in X$ and each open set R containing $x + y$, there exist $\delta$-open set P and Q containing x and y respectively, such that $P + Q \subseteq R$;*
*(b) For each $\lambda \in \mathbb{F}$, $x \in X$ and each open set R containing $\lambda x \in X$, there exist $\delta$-open set P and Q containing $\lambda$ and x respectively such that $P.Q \subseteq R$.*
*Then the pair $(X_{(\mathbb{F})}, \tau)$ is said to be $\delta$-topological vector space.*

Following are some examples of $\delta$-topological vector.

**Example 3.1.** *Let $\mathbb{K} = \mathbb{R}$ with usual topology. Let $X = \mathbb{R}$ with base $B = \{(a, b) : a, b \in \mathbb{R}\}$. We shall show that $(X_{(\mathbb{K})}, \tau)$ is $\delta$-topological vector space. For, we will check the following:*
*(i) Let $x, y \in X$. Consider open set $R = (x + y - \epsilon, x + y + \epsilon)$ in X containing*

*$x+y$. Then we can choose $\delta$-open sets $P=(x-\eta, x+\eta)$ and $Q=(y-\eta, y+\eta)$ in X containing x and y respectively such that $P+Q \subseteq R$, for each $\eta < \frac{\epsilon}{2}$. This establish the first condition of the definition of $\delta$-topological vector space.*
*(ii) Let $\lambda \in \mathbb{R}$ and $x \in X$. Consider an open set $R=(\lambda x-\epsilon, \lambda x+\epsilon)$ in X containing $\lambda x$. Then, we have the following cases:*
*Case I: If $\lambda > 0$ and $x > 0$, then we can choose $\delta$-open set $P=(\lambda-\eta, \lambda+\eta)$ in $\mathbb{R}$ containing $\lambda$ and $Q=(x-\eta, x+\eta)$ in X containing x such that $P.Q \subseteq R$, for each $\eta < \frac{\epsilon}{\lambda+x+1}$.*
*Case II: If $\lambda < 0$ and $x < 0$, then $\lambda x > 0$. We choose $\delta$-open set $P=(\lambda-\eta, \lambda+\eta)$ of $\lambda$ in $\mathbb{R}$ and $Q=(x-\eta, x+\eta)$ of x in $\mathbb{R}$ such that $P.Q \subseteq R$, for $\eta \leq \frac{-\epsilon}{\lambda+x-1}$.*
*Case III: If $\lambda > 0$ and $x=0$, ($\lambda=0$ and $x > O$). We can choose $\delta$-open sets as $P=(\lambda-\eta, \lambda+\eta)$ (resp.$(-\eta, \eta)$) containing $\lambda$ in $\mathbb{K}$ and $Q=(\eta, \eta)$ (resp.$(x-\eta, x+\eta)$) containing x in $\mathbb{R}$ such that $P.Q \subseteq R$, for each $\eta < \frac{\epsilon}{\lambda+1}(resp.(\eta < \frac{\epsilon}{x+1}$)).*
*Case IV: If $\lambda=0$ and $x < 0, (\lambda=0$ and $x > 0)$. We can choose $\delta$-open sets as $P=(\eta, \eta)(resp.(\lambda-\eta, \lambda+\eta))$ containing $\lambda$ in $\mathbb{K}$ and $Q=(x-\eta, x+\eta)(resp.(-\eta, \eta))$ containing x in $\mathbb{R}$, we have $P.Q \subseteq R$, for every $\eta < \frac{\epsilon}{1-x}(resp.(\eta < \frac{\epsilon}{1-\lambda}))$.*
*Case V: If $\lambda=0$ and $x=0$. Then, for $\delta$-open set $P=(\eta, \eta)$ and $Q=(\eta, \eta)$ of $\lambda$ and x respectively such that $P.Q \subseteq R$, for each $\eta < \sqrt{\epsilon}$.*
*This proves that the pair $(X_{(\mathbb{K})}, \tau)$ is $\delta$-TVS.*

**Example 3.2.** *Consider a vector space $X=\mathbb{R}$ of real number over the field $\mathbb{K}$ with the topology $\tau=\{\phi, Q^c, \mathbb{R}\}$, where $Q^c$ denotes the set of irrational numbers and the field $\mathbb{K}$ is endowed with standard topology. Then $(X_{(\mathbb{K})}, \tau)$ is not $\delta$-topological vector space. For $x, y \in Q^c$ and open neighborhood $Q^c$ of $x+y$ in X, there doesn't exist any $\delta$-open sets P and Q containing x and y respectively such that $P+Q \subseteq Q^c$.*

**Theorem 3.1.** *[3] Let D be any open subset of $\delta$-topological vector space X. Then*
*(a) $x+D \in \delta O(X)$, for each $x \in X$;*
*(b) $\lambda D \in \delta O(X)$, for each non-zero scalar $\lambda$.*

**Theorem 3.2.** *Let C be any closed subset of a $\delta$-topological vector space X, then*
*(a) $x+C \in \delta C(X)$, for each $x \in X$;*
*(b) $\lambda C \in \delta C(X)$, for each non-zero scalar $\lambda$.*

**Proof:** (a) Let $y \in Cl_\delta(x+C)$, $z=-x+y$ and R be an open set in X containing z. Then there exist $\delta$-open set P and Q containing $-x$ and y respectively, such that $P+Q \subseteq R$. Also, $y \in Cl_\delta(x+C)$, $y \in Q$ and Q is $\delta$-open implies there exist regular open set $Q'$ such that $y \in Q' \subseteq Q$. So $(x+C) \cap Q' \neq \emptyset$. Let $a \in (x+C) \cap Q' \Rightarrow -x+a \in C \cap (P'+Q') \subseteq C \cap (P+Q) \subseteq C \cap R \neq \emptyset$.

Hence $z \in Cl(C) = C \Rightarrow -x + y \in C \Rightarrow y \in x + C$. Thus, $x + C \in \delta C(X)$, for each $x \in X$.

(b) Assume that $x \in Cl_\delta(\lambda C)$ and R be open neighborhood of $y = \frac{1}{\lambda}x \in X$. Since X is $\delta - TVS$, there exist $\delta$-open neighborhood P of $\frac{1}{\lambda}$ in $\mathbb{F}$ and Q of x in X such that $P.Q \subseteq R$. By hypothesis, $(\lambda C) \cap Q' \neq \emptyset$, for regular open set $Q'$ subset of Q containing x. Let $a \in (\lambda C) \cap Q'$. Now $\frac{1}{\lambda}a \in C \cap P'.Q' \subseteq C \cap P.Q \subseteq C \cap R$ $\Rightarrow C \cap R \neq \emptyset$ i.e. y is a limit point of C and so $y = \frac{1}{\lambda}x \in Cl(C) = C$, since C is closed subset of X. Hence $y \in \lambda C$. Since the inclusion $\lambda C \subseteq Cl_\delta(\lambda C)$ holds generally, so $Cl_\delta(\lambda C) = \lambda C$. Therefore, $\lambda C$ is $\delta$-closed set in X. This completes the proof.

**Theorem 3.3.** *For any subset D of $\delta$-topological vector space X,*
*(a) $Cl_\delta(x + D) \subseteq x + Cl(D)$, for each $x \in X$.*
*(b) $x + Cl_\delta(D) \subseteq Cl(x + D)$, for each $x \in X$.*

**Proof:** (a) Let $y \in Cl_\delta(x + D)$ and consider $z = -x + y$ in X. Let R be open neighborhood of z. By hypothesis, there exist $\delta$-open set P and Q containing -x and y respectively such that $P + Q \subseteq R$. Existence of $\delta$-open set confirms the existence of regular open set $P'$ and $Q'$ such that $-x \in P' \subseteq P$ and $y \in Q' \subseteq Q$. Since $y \in Cl_\delta(x + D)$, $(x + D) \cap Q' \neq \emptyset$. Let $a \in (x + D) \cap Q'$. Now, $-x+a \in D \cap (P'+Q') \subseteq D \cap (P+Q) \subseteq D \cap R \neq \emptyset$, which implies $z \in Cl(D)$. Hence $y \in x + Cl(D)$. This completes the proof.

(b) Let $z \in x + Cl_\delta(D)$. Then $z = x + y$, for some $y \in Cl_\delta(D)$. Let R be any open neighborhood of z in X, then there exist $\delta$-open neighborhood P and Q of x and y respectively such that $P + Q \subseteq R$. Also, $D \cap Q' \neq \emptyset$, for regular open set $Q' \subseteq Q$ containing y which implies $D \cap Q \neq \emptyset$. Let $a \in D \cap Q$. Then $x + a \in (x + D) \cap (P + Q) \subseteq (x + D) \cap R \neq \emptyset$, which implies z is a limit point of $x+D$ i.e $z \in Cl(x+D)$. Hence the inclusion holds for each $x \in X$.

**Theorem 3.4.** *For a subset D of $\delta$-topological vector space X, the following are valid:*
*(a) $x + Int(D) \subseteq Int_\delta(x + D)$, for each $x \in X$.*
*(b) $Int(x + D) \subseteq x + Int_\delta(D)$, for each $x \in X$.*

**Proof:** (a) Assume $y \in x + Int(D)$. Then,$-x + y \in Int(D)$. Since X is $\delta$-topological vector space, there exist $\delta$-open sets P containing -x and Q containing y in X such that $Q \subseteq Int(D)$. Also, $\delta$-openness of P and Q implies the existence of regular open set $P'$ and $Q'$ such that $-x \in P' \subseteq P$ and $y \in Q' \subseteq Q$ satisfying $P' + Q' \subseteq P + Q \subseteq Int(D)$. In particular,

$-x + Q' \subseteq Int(D) \subseteq D \Rightarrow Q' \subseteq x + D$. Thus there exist regular open set $Q'$ containing y such that $y \in Q' \subseteq x + D$, which implies y is $\delta$-interior point of $x + D$ i.e. $y \in Int_\delta(x + D)$. Hence the proof.

(b) Let $y \in Int(x + D)$, then $y = x + a$, for some $a \in D$. By definition of $\delta$-topological vector space, there exist $\delta$-open set P and Q such that $x \in P, a \in Q$ satisfying $P + Q \subseteq Int(x + D)$. Hence, $x + a \in P' + Q' \subseteq P + Q \subseteq Int(x + D)$, for each regular open set $P'$ and $Q'$ such that $x \in P' \subseteq P$ and $a \in Q' \subseteq Q$. Now $x + Q' \subseteq x + Q \subseteq Int(x + D) \subseteq x + D$, which implies $y \in x + Int_\delta(D)$. Hence the inclusion $Int(x + D) \subseteq x + Int_\delta(D)$ holds.

**Theorem 3.5.** *Let D be any subset of $\delta$-topological vector space X. Then the following holds:*
*(a) $\lambda Cl_\delta(D) \subseteq Cl(\lambda D)$, for every non-zero scalar $\lambda$.*
*(b) $Cl_\delta(\lambda D) \subseteq \lambda Cl(D)$, for every non-zero scalar $\lambda$.*

Proof The proof is trivial, omitted.

**Theorem 3.6.** *Let X be a $\delta$-topological vector space and D be any subset of X. Then the following holds:*
*(a) $Int(\lambda D) \subseteq \lambda Int_\delta(D)$, for every non-zero scalar $\lambda$.*
*(b) $\lambda Int(D) \subseteq Int_\delta(\lambda D)$, for every non-zero scalar $\lambda$.*

Proof The proof is trivial, omitted.

**Theorem 3.7.** *Let C and D be any subset of a $\delta$-topological vector space X. Then $Cl_\delta(C) + Cl_\delta(D) \subseteq Cl(C + D)$.*

**Proof:** Let $z \in Cl_\delta(C) + Cl_\delta(D)$. Then $z = x + y$, where $x \in Cl_\delta(C)$ and $y \in Cl_\delta(D)$. Let R be an open neighborhood of z in X. By definition of $\delta$-topological vector space, there exist $\delta$-open neighborhood P and Q of x and y respectively such that $P + Q \subseteq R$. Since $x \in Cl_\delta(C)$, $C \cap P' \neq \emptyset$ for regular open set $P'$ such that $x \in P' \subseteq P$ and also $y \in Cl_\delta(D)$, $D \cap Q' \neq \emptyset$ for regular open set $Q'$ satisfying $y \in Q' \subseteq Q$.
Let $a \in C \cap P'$ and $b \in D \cap Q' \Rightarrow (a + b) \in (C + D) \cap (P' + Q') \subseteq (C + D) \cap (P + Q) \subseteq (C + D) \cap R \Rightarrow (C + D) \cap R \neq \emptyset$. Thus z is a closure point of $(C + D)$ i.e. $z \in Cl(C + D)$. Hence the inclusion holds.

**Theorem 3.8.** *For any subsets C and D of $\delta$-topological vector space X. Then $C + Int(D) \subseteq Int_\delta(C + D)$.*

**Proof:** Let $z \in C + Int(D)$ be arbitrary. Then $z = x + y$, for some $x \in C, y \in Int(D)$, which results in $-x + z \in Int(D)$. By definition of $\delta$-TVS, there exist $\delta$-open neighborhood P and Q containing -x and z respectively such that $P + Q \subseteq Int(D)$. Hence, there exist regular open sets $P'$ and $Q'$ containing -x and z respectively satisfying $P' \subseteq P$, $Q' \subseteq Q$ and $P' + Q' \subseteq P + Q \subseteq Int(D)$. In particular, $-x + Q' \subseteq Int(D) \Rightarrow Q' \subseteq x + Int(D) \subseteq C + D$. Hence, there exist regular open set $Q'$ containing z such that $z \in Q' \subseteq C + D$. Therefore, z is $\delta$-interior point of $A + B$. Hence the proof.

**Definition 3.2.** *[8] A function $f : X \to Y$ is called $\delta$-continuous if for each $x \in X$ and each open neighborhood Q of f(x), there exist open neighborhood P of x such that $f(Int(Cl(P)) \subseteq Int(Cl(Q))$.*

**Lemma 3.1.** *[8] For a function $f : X \to X$, the following are equivalent:*
*(a) f is $\delta$-continuous.*
*(b) For each $x \in X$ and each regular open set V containing f(x), there exist a regular open set U containing x such that $f(U) \subseteq V$.*
*(c) $f([A]_\delta) \subset [f(A)]_\delta$, for every $A \subset X$.*
*(d) $[f^{-1}(B)]_\delta \subset f^{-1}([B]_\delta)$, for every $B \subset X$.*
*(e) For every regular closed set F of Y, $f^{-1}(F)$ is $\delta$-closed in X.*
*(f) For every $\delta$-closed set V of Y, $f^{-1}(V)$ is $\delta$-closed in X.*
*(g) For every $\delta$-open set V of Y, $f^{-1}(V)$ is $\delta$-open in X.*
*(h) For every regular-open set V of Y, $f^{-1}(V)$ is $\delta$-open in X.*

**Theorem 3.9.** *[3] Let X be $\delta$-topological vector space, then the following are true:*
*(a) the translation mapping $g_a : X \to X$ defined by $g_a(b) = a + b$, $\forall b \in X$ is $\delta$-continuous.*
*(b) the mapping $g_\lambda : X \to X$ defined by $g_\lambda(a) = \lambda a$, $\forall a \in X$ is $\delta$-continuous, where $\lambda$ is a fixed scalar.*

**Theorem 3.10.** *For a $\delta$-topological vector space X, the mapping $\Phi : X \times X \to X$ defined by $\Phi(x, y) = x + y$, $\forall x \in X \times X$ is $\delta$-continuous.*

**Proof:** Take arbitrary elements x, y in X and let R be regular open neighborhood of $x + y$ which implies R is open neighborhood of x+y. Then by hypothesis, there exist $\delta$-open neighborhood P and Q of x and y respectively such that $P + Q \subseteq R$.
Also, by definition of $\delta$-open set, there exist regular open neighborhood $P'$ and $Q'$ such that $x \in P' \subseteq P$ and $y \in Q' \subseteq Q$. This implies that $\Phi(P' \times Q') = P' + Q' \subseteq P + Q \subseteq R$. Since $P \times Q$ is regular open in $X \times X$(with respect to product topology), it follows that $\Phi$ is $\delta$-continuous.

**Theorem 3.11.** *For $\delta$-topological vector space X, the mapping $\Psi : \mathbb{K} \times X \to X$ defined by $\Psi(\lambda, x) = \lambda x$, $\forall(\lambda, x) \in \mathbb{K} \times X$ is $\delta$-continuous.*

**Proof:** Let $\lambda \in \mathbb{K}$ and $x \in X$ and R be a regular open neighborhood of $\lambda x$ in X. Then there exist $\delta$-open neighborhood P of $\lambda$ in $\mathbb{K}$ and $\delta$-open neighborhood Q of x in X such that $P.Q \subseteq R$. Also, $P'.Q' \subseteq P.Q \subseteq R$, for regular open set $P'$ and $Q'$ contained in P and Q containing $\lambda$ and x respectively. Since $P \times Q$ is regular in $\mathbb{K} \times X$, $\Psi(P'.Q') \subseteq R$. Hence, it follows that $\Psi$ is $\delta$-continuous for arbitrary element $\lambda \in \mathbb{K}$ and $x \in X$.

**Theorem 3.12.** *Let X be $\delta$-topological vector space and Y be topological vector space over the same field $\mathbb{K}$. Let $f : D_1 \to D_2$ be a linear map such that f is continuous at 0. Then f is $\delta$-continuous.*

**Proof:** Let $0 \neq x \in X$ and V be regular open set and hence open in Y containing f(x). Since translation of an open set is open in topological vector space, which implies $V - f(x)$ is open in Y containing 0. Since f is continuous at 0, there exist open set U in X containing 0 such that $f(U) \subseteq V - f(x)$. Also, by linearity of f implies that $f(x + U) \subseteq V$. By theorem 3.1, $x + U$ is $\delta$-open and hence there exist regular open set Q such that $Q \subseteq x + U$. Hence, $f(Q) \subseteq V$.

# 4 Conclusions

$\delta$-Topological vector space is an extension of topological vector space and this paper give an insight into this space. We presented the space with new examples and inherent properties. Moreover, important characterization of the space is studied in this paper.

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