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Fuzzy bi-objective optimization model for multi-echelon distribution network

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Abstract

It is important for modern businesses to search the ways for continuous improvement in performance of their supply chains. The effective coordination and integrated decision making across the supply chain enhances the performance among its various partners in a multi stage network. The partners considered in this paper are product suppliers, processing points (PP), distribution centres (DC) and retail outlets (RO). The network addresses an uncertain environment threatened by different sources in order to captivate the real world conditions. The uncertain demand of deteriorating products and its dependent costs develop uncertainties in the environment. On the other hand, suppliers and processing points have restricted capacities for the retail outlets' order amount happened in each period. A bi-objective non-linear fuzzy mathematical model is developed in which the uncertainties are represented by the fuzzy set theory. The proposed model shows cost minimization and best supplier selection coordination under the conditions of capacity constraints, uncertain parameters and product's deteriorating nature. The fish and fish products give good examples for the proposed model. To solve, the model is converted into crisp form and solved with the help of fuzzy goal programming.

Key words: multi stage, supplier selection, processing point, fuzzy goal programming, supply chain, Bi-objective.

1 Introduction

With the growing importance of supply chain management (SCM) in enterprise development and in the operation of socio-economic systems, cost management has become a strategic business issue in recent years. It involves not only the financial flows but also the associated material flows and information flows among supply chain partners. Moreover, it plays an indispensable role in bringing profits and competitive advantage to firms, and consequently receives increasing attention from both supply chain managers and academics. Activities in supply chain system consist of transforming natural resources, raw materials and components into finished product and their final delivery to the end customers. Most of these economic activities form an integral part of the value chain. From this view point, cost management in supply chains is not limited to individual enterprises, but extends to all the purchasing, warehousing, production and distribution activities along the chain. Its goal is to provide a management tool and method to design the integrated chain, to promote its development and to reduce the total cost of supply chain system. However, a lot more complexity is involved in effectively integrating all the supply chain activities in a cost efficient manner owing to shorter life cycle of products and increased competition among suppliers who are offering different opportunities to the retailer. The uncertain demand of deteriorating products and their dependent costs creates uncertainty in the environment and consequently results in an indecisive and unsure environment for the decision makers. Choosing high level of procured quantity and inventory to avoid shortages will definitely lead to an immense increase in the cost of purchase and inventory holding. In this regard, operations management practices and mathematical models provide a sound framework for effective and integrative decision making across supply chain. For minimizing the cost and improving the overall performance, major functions considered are economic ordered quantity decisions, supplier selection decisions, inventory & capacity decisions and transportation policies in multi periods and for multi products. While economic ordered quantity decisions aim to minimize the cost of procurement, inventory and transportation, the intent of supplier selection and transportation policy selection decisions is to maximize inbound logistics performance by attaining a high degree of quality and delivery performance. Due to the inherent interdependency among these decisions, a firm cannot optimize them separately. Hence the main purpose of this paper is to develop a model addressing above issues i.e. to characterize the optimal decisions that each partner in supply chain should adopt to motivate the chain partners to coordinate so that everyone benefits from the improved performance of the system.

Though procurement functions need to consider cost minimization objective, yet in doing so one cannot compromise on quality and delivery related criteria. Nowadays, quality and delivery related objectives are being given higher priority than cost criterion during procurement decisions. Suppliers' performance on quality and delivery criteria has a significant influence on the ordered quantity and the total transportation costs. Taking into account the above observations, in this study we develop a fuzzy bi-objective non-linear programming model for an integrated economic ordered quantity, supplier selection and transportation policy problem. We investigate a problem in which multi products are procured from multiple suppliers in multiple periods considering limitations on capacity at supplier point and processing point for deteriorating products. We also incorporate cost of inventory at distribution centres & retail outlets and transportation cost and policy concepts in one stage to another. Imprecise demand and other uncertain known parameters make the environment of model uncertain and fuzzy. To summarize the above discussions, the present work shows (1) a fuzzy bi-objective multi stage non-linear optimization model that includes computation of cost of procurement, processing, holding and transportation as first objective and the other objective shows the process to choose best supplier on the basis of delivery and quality; (2) the coordination among multi stages, i.e. (i) procurement stage; (ii) processing stage constituted of (a) Receiving & Scanning, (b) Sorting & Packaging & (c) Scanning & Dispatching; (iii) distribution centres and (iv) retail outlets; (3) transportation policies and minimum cost per weight from processing stage to distribution centres and transportation cost per unit from distribution centre to retail outlet; (4) fuzzy set theory to coordinate uncertain parameters; (5) coordination in procurement, demand and inventory so the zero shortage is ensured.

2 Literature Review

There are vast researches working on supplier selection problems with different approaches. One of the most important decisions related to procurement operations is supplier evaluation and selection. There are several factors involved such as price offered by the supplier, lead time, the quality of items, the capacity of supplier and the geographical location of supplier while making supplier evaluation and selection decisions (Ho et al., 2010). Ho et al. (2010), the three most important criteria considered while selecting suppliers are product quality, delivery lead time and price. Hassini (2008) studies a lot sizing and supplier selection problem when supplier capacity reservation dependent on lead time. Ravindran, Bilsel, Wadhwa, and Yang (2010) study supplier selection and order allocation considering incremental price breaks. Liao and Rittscher (2007) propose a multi objective programming model for supplier selection, procurement lot sizing and carrier selection decisions. Razmi and Maghool (2010) propose a fuzzy bi-objective model for multiple items, multiple period, supplier selection and purchasing problem under capacity constraint and budget limitation. Zhang and Zhang (2011) formulate a mixed integer programming model for selecting suppliers and allocating the ordering quantity properly among the selected suppliers to minimize the selection, purchase and inventory costs. Jolai, Yazdian, Shahanaghi, and Khojasteh (2011) proposed a two-phase approach for supplier selection and order allocation problem under fuzzy environment for multiple products from multiple suppliers in multiple periods. Pal, Sana, and Chaudhuri (2012) addressed a multi-echelon suppler chain with two suppliers in which the main supplier may face supply disruption and the secondary supplier is reliable but more expensive, and the manufacturer may produce defective items. Kilic (2013) discussed an integrated approach including fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and a mixed integer linear programming model is developed to select the best supplier in a multi-item/multi-supplier environment.

Few of the studies have addressed problems having multi objectives and with fuzziness. Madronero, Peidro, and Vassant (2010) used S-curve membership functions for Fuzzy aspiration levels for objective functions, maximum capacity of the vendors as RHS, budget amount allocated to vendors as RHS with Fuzzy programming by using modified Werner's fuzzy or operator. Wu, Zhang, Wu and Olson (2010) used Trapezoidal membership functions for Fuzzy model parameters as objective function coefficients and right hand side (RHS) constants with Sequential quadratic programming. Arikan (2011) used Triangular and Right triangular membership functions for Fuzzy aspiration levels for objective functions and demand level as RHS with Lai and Hwang's augmented max–min model. Concerning with multi-choice goals, decision-making behaviour and limit of resources, Lee, Kang, and Chang (2009) develop a fuzzy multiple goal programming model to help downstream companies to select thin film transistor liquid display suppliers for cooperation. They used triangular membership functions for fuzzy aspiration levels for objective functions. Further, a multi-objective model for supplier selection in multi-service outsourcing is developed by Feng, Fan, and Li (2011). A multi objective mathematical model has been discussed by Seifbarghy and Esfandiari (2013), which includes minimizing the transaction costs of purchasing from suppliers as well as other objectives as minimizing the purchasing cost, rejected units, and late delivered units, and maximizing the evaluation scores of the selected suppliers. The problem is converted into single objective using weighting method and solved using meta-heuristics. Aghai, Mollaverdi and Saddagh (2014), outlined a fuzzy multi-objective programming model to propose supplier selection taking quantitative, qualitative, and risk factors into consideration. Also quantity discount has been considered to determine the best suppliers and to place the optimal order quantities among them.

From the literature, it is evident that most studies have not paid much attention to uncertainty in supplier's information and many problematic criteria in the conditions of multi product, transportation modes and multiple sourcing. The main purpose of this paper has been outlined as (1) to propose a fuzzy bi-objective mathematical model to choose the supplier with best performance on the basis of quality & delivery percentages and to keep the cost optimum while procurement, processing of products and transportation, the ideal number of inventory items so that shortages does not take place, and optimum quantity from suppliers subject to the constraints pertaining to demand, suppliers capacity, processing capacity and inspection, (2) the objectives are conflicting in nature as minimization of cost and performance maximization of the supplier. Because of uncertain parameters the environment of the problem becomes fuzzy, for which, fuzzy goal programming method has been used to solve the mathematical model of cost minimization and suppliers selection with maximum performance.

3 Problem Definition

To manage different entities to minimize their cost and simultaneously measuring the suppliers' performances in the environment of uncertainty, the current paper presents a fuzzy bi-objective mixed integer non-liner model. The first objective of the proposed model minimizes the cost of integration of procurement and distribution. This comprises of multi source (suppliers), two processing points, multi distribution centres & multi retail outlets and incorporating transportation costs and policies. The second objective focuses on performance and selection of suppliers on the bases of on-time delivery percentage and acceptance percentage of the ordered quantity.

The first stage of first objective explains procurement cost as per optimum procured quantity from the active suppliers, processing cost per unit in three levels at processing point. At this point receiving, scanning, sorting and packing of goods takes time, hence holding cost is included in the processing cost. The second stage shows the fuzzy cost of holding at distribution centres and cost of transportation of goods from processing points to distribution centres which is completed through two modes of transportation as full truck load (TL) mode and truck load (TL) & less than truck load (LTL) mode. In truck load transportation mode, the cost is fixed of one truck up to a given capacity. In this mode, the company may use less than the capacity available but cost per truck will not be reduced. However, sometimes the weighted quantity may not be large enough to corroborate the cost associated with a TL mode. In such situation, a LTL mode may be used. LTL is defined as a shipment of weighted quantity which does not fill a truck. In such a case, transportation cost is taken on the bases of per unit weight. The third stage includes inspection, fuzzy holding cost at retail outlet and transportation cost per unit in the account from distribution centres to retail outlet. The second objective is to find best suppliers with the combination of fuzzy on-time delivery percentage and fuzzy acceptance percentage of the ordered quantity.

The model integrates inventory, procurement and transportation mechanism to minimize all costs discussed above and also chooses the best supplier. In the model, all the co-ordinations among supply chain partners are being managed under one buyer who is taking care of processing points, distribution centres and retail outlets but not sources (suppliers) directly. The total cost of the model becomes fuzzy due to fuzzy holding cost and demand. On the other hand, performance level is also fuzzy as percentage of on-time delivery and acceptances are fuzzy. Hence, the model discussed above is fuzzy bi-objective mixed integer non-linear model. In the solution process, the fuzzy model is converted into crisp and further fuzzy goal programming approach is employed where each objective could be assigned a different weight.

4 Proposed Model Formulation

The model is based on following assumptions:

- Finite planning horizon
- Demand at retail outlet is uncertain and no shortages are allowed
- Initial inventory at the beginning of planning horizon is zero
- Inventory at retail outlet deteriorates at constant rate
- Inspection cost of received goods at retail out is fixed
- No transportation cost is discussed as it is considered as part of purchasing cost
- Holding cost is part of processing cost at processing point

4.1 Sets

Set	Cardinality	Index
Product	P	i
Supplier	J	j
Processing Point	Z	z
Distribution Centre	M	m
Retail outlet	O	o
Time period	T	t

4.2 Parameters

$\widetilde{C}$: Fuzzy total cost

C_0 & C_0^* : Aspiration & Tolerance level of fuzzy total cost

$\widetilde{PR}$: Fuzzy performance of supplier

PR_0 & PR_0^* : Aspiration & Tolerance level of fuzzy performance of supplier

$\underset{imt}{\widetilde{HD}}$ & $\overline{HD}_{imt}$: Fuzzy & Defuzzified holding cost per unit of product i for t^{th} period at m^{th} distribution centre

φ_{ijzt} : Unit purchase cost for i^{th} product in t^{th} period from supplier j for z^{th} processing point

A: Cost per weight of transportation in LTL policy

K_{zmt} : Fixed freight cost for each truck load in period $t$from processing point z to distribution centre m

TC_{imot} : Transportation cost for unit in period $t$from distribution centre m to retail outlet o

$\underset{iot}{\widetilde{HR}}$ & $\overline{HR}_{iot}$: Fuzzy & defuzzified holding cost per unit of product i for t^{th} period at retail outlet o

λ_{iot} : Inspection cost per unit of product i in period t at retail outlet o

$\underset{iot}{\widetilde{D}}$ & $\overline{D}_{iot}$: Fuzzy & defuzzified demand at retail outlet o for product i in period t

IN_{izt}: Initial Inventory processing point z in beginning of planning horizon for product i

η : Deterioration percentage of i^{th} product at retail outlet

w_i: Per unit weight of product i

ω : Weight transported in each full truck

$\underset{ijzt}{\widetilde{DT}}$ & $\overline{DT}_{ijzt}$: Fuzzy & defuzzified percentage of on-time delivery time for product i in period t for supplier j for processing point z

$\underset{ijzt}{\widetilde{AC}}$ & $\overline{AC}_{ijzt}$: Fuzzy & defuzzified percentage of acceptance for product i in period t for supplier j for processing point z

δ_{ijz}: Capacity at supplier j for product $i$for z^{th} processing point

α_{izrt} : Capacity of Receiving & Scanning level (r) at z^{th} processing point for product i in period t

C_{izrt} : Cost of Receiving & Scanning level (r) at z^{th} processing point for product i in period t

β_{izst} : Capacity of Sorting & Packing level (s) at z^{th} processing point for product i in period t

C_{izst} : Cost of Sorting & Packing (s) at z^{th} processing point for product i in period t

γ_{izdt} : Capacity of Scanning & Dispatching level (d) at z^{th} processing point for product i in period t

C_{izdt} : Cost of Scanning & Dispatching (d) at z^{th} processing point for product i in period t

4.3 Decision Variable

X_{ijzt} : Optimum ordered quantity of product i ordered in period $t$from supplier j transported to processing point z

V_{ijt}: If ordered quantity is procured by active supplier j for product i in period $t$then the variable takes value 1 otherwise zero

u_{zmt}: Usage of modes, either TL & LTL mode (value is 1) or only TL mode (value is 0)

4.4 Operating Variables

Y_{izt} : Procured quantity reached at Receiving & Scanning level of zth processing point from all the active suppliers

A_{izt} : Goods moved to Sorting & Packaging from Receiving & Scanning level at z^{th} processing point

E_{imt} : Goods reaching at m^{th} distribution centre from all processing points

j_{zmt} : Total number of truck loads in period t from processing point z to distribution centre m

Q_{zmt} : Weighted quantity in excess of truckload capacity

G_{iot} : Total quantity reached at retail outlet o from all distribution centres

I_{izt} : Inventory at processing point in period t for product i

I_{imt} : Inventory at distribution centre in period t for product i

I_{iot} : Inventory at retail outlet in period t for product i

B_{izmt} : Quantity of product i shipped from z^{th} processing point to m^{th} distribution centre in period t

F_{imot} : Quantity of product i shipped from m^{th} distribution centre to o^{th} retail outlet in period t

L_{zmt} : Weighted quantity transported from z^{th} processing point to m^{th} distribution centre in period t

4.5 Fuzzy Optimization Model Formulation

Fuzzy dependent environment with respect to uncertain independent variables cannot be quantified by Crisp mathematical programming approaches. Fuzzy optimization approach permits adequate solutions of real problems in the presence of vague information by defining the mechanisms to quantify uncertainties directly. Therefore, we formulate fuzzy optimization model for vague aspiration levels on cost, demand, on-time delivery percentage and acceptance percentage the decision maker may decide the aspiration and tolerance levels on the basis of past experience and knowledge.

4.5.1 Formulation of objectives

Initially a bi-objective fuzzy model is formulated which discusses about fuzzy total cost and performance of the suppliers. The first objective of the model minimizes the total cost, consisting of procurement cost of goods from supplier, processing cost, holding cost at distribution centres, transportation cost from processing point to distribution centres and further to retail outlets, holding cost at retail outlets and finally inspection cost of the reached quantity at retail outlets.

$$
\begin{aligned}
Minimize,\ \widetilde{C} = & \sum_{t=1}^{T}\sum_{z=1}^{Z}\sum_{j=1}^{J}\sum_{i=1}^{P} \varphi_{ijzt} X_{ijzt} V_{ijzt} \\
& + \sum_{t=1}^{T}\sum_{z=1}^{Z}\sum_{i=1}^{P} \left[\sum_{r=1}^{R} C_{izrt} Y_{izt} + \left(\sum_{s=1}^{S} C_{izst} + \sum_{d=1}^{D} C_{izdt} \right) A_{izt} \right] \\
& + \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{i=1}^{P} \widetilde{HD}_{imt}\, E_{imt}
\end{aligned}
$$

$$+\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{z=1}^{Z}\left[\left(AQ_{zmt}+j_{zmt}K_{zmt}\right)u_{zmt}+\left(j_{zmt}+1\right)K_{zmt}\left(1-u_{zmt}\right)\right]$$

$$+\sum_{t=1}^{T}\sum_{o=1}^{O}\sum_{m=1}^{M}\sum_{i=1}^{P}TC_{imot}F_{imot}+\sum_{t=1}^{T}\sum_{o=1}^{O}\sum_{i=1}^{P}\underset{iot}{\tilde{HR}}\,I_{iot}+\sum_{t=1}^{T}\sum_{o=1}^{O}\sum_{i=1}^{P}\lambda_{iot}G_{iot}$$

The second objective discusses the performance of suppliers and maximizes the performance percentage of supplier as per on-delivery time percentage and acceptance percentage of ordered quantity.

$$Maximize\ \tilde{PR}=\sum_{t=1}^{T}\sum_{z=1}^{Z}\sum_{j=1}^{J}\sum_{i=1}^{P}\left(\underset{ijzt}{\tilde{DT}}+\underset{ijzt}{\tilde{AC}}\right)V_{ijzt}$$

4.5.2 Constraint Formulation

All the suppliers must have enough capacity to fulfil the orders. The following equation ensures that the active supplier shall have enough capacity to complete the orders from processing point.

$$X_{ijzt}\leq\delta_{ijz}V_{ijzt}\quad\forall\,i,j,\,z,t$$

Next equation ensures that only one supplier can be active for a particular product in a period. However, same supplier can be active again in next period.

$$\sum_{j=1}^{J}V_{ijzt}=1\quad\forall\ i,\,t,z$$

Goods are reaching at z$^{\text{th}}$ processing point from all the suppliers.

$$Y_{izt}=\sum_{j=1}^{J}X_{ijzt}\quad\forall\,i,\,t,z$$

At Receiving & Scanning level in processing point, 2% from each lot is rejected and removed.

$$A_{izt}=0.98Y_{izt}\quad\forall\,i,\,t,z$$

Quantity dispatched from z$^{\text{th}}$ processing point is being transported to all distribution centres.

$$A_{izt}=\sum_{m=1}^{M}B_{izmt}\quad\forall i,\,z,\,t$$

Goods reaching at mth distribution centre are transported from all the processing points.

$$E_{imt} = \sum_{z=1}^{Z} B_{izmt} \quad \forall i,\, m,\, t$$

Goods are transported from mth distribution centre to all the retail outlets.

$$E_{imt} = \sum_{o=1}^{O} F_{imot} \quad \forall i,\, m,\, t$$

Goods reaching at oth retail outlets Eiot are transported from all the distribution centres

$$G_{iot} = \sum_{m=1}^{M} F_{imot} \quad \forall i,\, o,\, t$$

Following three equations explain the capacities in processing point at all the levels respectively i.e. Receiving and Scanning level, Sorting & Packaging level and Scanning and Dispatching level.

$$Y_{izt} \leq \alpha_{izrt} \quad \forall\, i,\, z, t, r$$

$$A_{izt} \leq \beta_{izst} \quad \forall\, i,\, t, z, s$$

$$A_{izt} \leq \gamma_{izdt} \quad \forall\, i,\, t, z, d$$

Next three equations show balancing equations at Processing Point, which also takes care of no shortages assumption. First two equations of the set calculate inventory at end of the period with respect to quantity reached at receiving and scanning level from the supplier and quantity sent to sorting & packaging level. The third equitation takes care of the shortages by balancing the quantity between the two levels discussed above.

$$I_{izt} = I_{izt-1} + Y_{izt} - A_{izt} \quad \forall\, i,\, t > 1, z$$

$$I_{izt} = IN_{izt} + Y_{izt} - A_{izt} \quad \forall\, i,\, t = 1, z$$

$$\sum_{t=1}^{T} I_{izt} + \sum_{t=1}^{T} Y_{izt} \geq \sum_{t=1}^{T} A_{izt} \quad \forall\, i, z$$

Balancing at distribution centres have been discussed in next three equation, where assumption of no shortages has also been taken care of.

$$I_{imt} = I_{imt-1} + E_{imt} - \sum_{o=1}^{O} F_{imot} \quad \forall\, i,\, t > 1, m$$

$$I_{im1} = 0 \quad \forall\, i, m$$

$$\sum_{t=1}^{T} I_{imt} + \sum_{t=1}^{T} E_{imt} \geq \sum_{t=1}^{T} \sum_{o=1}^{O} F_{imot} \quad \forall\, i, m$$

At retail outlets also, inventory has been balanced with respect to the received quantity and demand.

$$I_{iot} = I_{iot-1} + G_{iot} - \underset{iot}{\tilde{D}} - \eta I_{iot} \quad \forall\, i,\, t > 1, o$$

$$I_{io1} = 0 \quad \forall\, i, o$$

$$(1 - \eta) \sum_{t=1}^{T} I_{iot} + \sum_{t=1}^{T} G_{iot} \underset{\sim}{\geq} \sum_{t=1}^{T} \underset{iot}{\tilde{D}} \quad \forall\, i, o$$

Following equation is an integrator and calculates the weighted quantity which is to be transported from processing point to distribution centres.

$$L_{zmt} = \sum_{i=1}^{P} \omega_i B_{izmt} \quad \forall\, z, t, m$$

The next equation finds out transportation policy as per the weighted quantity. Here, the costs of TL policy and TL&LTL policy are compared as per the weight.

$$L_{zmt} \leq \left(Q_{zmt} + j_{zmt} w\right) u_{zmt} + \left(j_{zmt} + 1\right) w \left(1 - u_{zmt}\right) \quad \forall\, z, m, t$$

The calculation of overhead quantity in TL&LTL policy is calculated by comparing total weighted quantity with total number of full truck loads as per weight is discussed in following equation.

$$L_{zmt} = Q_{zmt} + j_{zmt} w \quad \forall z, m,\ t$$

Lastly, describing the nature of decision variables and enforcing the binary and non-negative restrictions to them.

$X_{ijzt},\ Y_{izt},\ A_{izt},\ E_{imt},\ F_{imot},\ G_{iot},\ L_{zmt} \geq 0;\ V_{ijzt},\ u_{zmt} \in [0,\ 1]\,;$

$I_{imt},\ I_{iot},\ I_{izt},\ Q_{zmt},\ j_{zmt}$ are integer.

4.5.3 Formulated Model

$$Minimize\ \tilde{C} = \sum_{t=1}^{T}\sum_{z=1}^{Z}\sum_{j=1}^{J}\sum_{i=1}^{P}\varphi_{ijzt}X_{ijzt}V_{ijzt}$$
$$+\sum_{t=1}^{T}\sum_{z=1}^{Z}\sum_{i=1}^{P}\left[\sum_{r=1}^{R}C_{izrt}Y_{izt}+\left(\sum_{s=1}^{S}C_{izst}+\sum_{d=1}^{D}C_{izdt}\right)A_{izt}\right]$$
$$+\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{i=1}^{P}\underset{imt}{\widetilde{HD}}\,E_{imt}$$
$$+\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{z=1}^{Z}\left[\left(AQ_{zmt}+j_{zmt}K_{zmt}\right)u_{zmt}+\left(j_{zmt}+1\right)K_{zmt}\left(1-u_{zmt}\right)\right]$$
$$+\sum_{t=1}^{T}\sum_{o=1}^{O}\sum_{m=1}^{M}\sum_{i=1}^{P}TC_{imot}F_{imot}+\sum_{t=1}^{T}\sum_{o=1}^{O}\sum_{i=1}^{P}\underset{iot}{\widetilde{HR}}\,I_{iot}+\sum_{t=1}^{T}\sum_{o=1}^{O}\sum_{i=1}^{P}\lambda_{iot}G_{iot}$$

$$Maximize\ \widetilde{PR} = \sum_{t=1}^{T}\sum_{z=1}^{Z}\sum_{j=1}^{J}\sum_{i=1}^{P}\left(\underset{ijzt}{\widetilde{DT}}+\underset{ijzt}{\widetilde{AC}}\right)V_{ijzt}.$$

Subject to $X_{ijzt} \le \delta_{ijz}V_{ijzt} \quad \forall\, i, j,\ z, t$ $\sum_{j=1}^{J} V_{ijzt} = 1 \quad \forall\ i,\ t, z$

$$Y_{izt} = \sum_{j=1}^{J} X_{ijzt} \quad \forall\, i,\ t, z$$
$$A_{izt} = 0.98Y_{izt} \quad \forall\, i,\ t, z$$
$$A_{izt} = \sum_{m=1}^{M} B_{izmt} \quad \forall i,\ z,\ t$$
$$E_{imt} = \sum_{z=1}^{Z} B_{izmt} \quad \forall i,\ m,\ t$$
$$E_{imt} = \sum_{o=1}^{O} F_{imot} \quad \forall i,\ m,\ t$$
$$G_{iot} = \sum_{m=1}^{M} F_{imot} \quad \forall i,\ o,\ t$$
$$Y_{izt} \le \alpha_{izrt} \quad \forall\, i,\ z, t, r$$
$$A_{izt} \le \beta_{izst} \quad \forall\, i,\ t, z, s$$
$$A_{izt} \le \gamma_{izdt} \quad \forall\, i,\ t, z, d$$
$$I_{izt} = I_{izt-1} + Y_{izt} - A_{izt} \quad \forall\, i,\ t > 1, z$$
$$I_{izt} = IN_{izt} + Y_{izt} - A_{izt} \quad \forall\, i,\ t = 1, z$$
$$\sum_{t=1}^{T} I_{izt} + \sum_{t=1}^{T} Y_{izt} \ge \sum_{t=1}^{T} A_{izt} \quad \forall\, i, z$$

$I_{imt} = I_{imt-1} + E_{imt} - \sum_{o=1}^{O} F_{imot} \quad \forall\, i,\, t > 1, m$
$I_{im1} = 0 \quad \forall\, i, m$
$\sum_{t=1}^{T} I_{imt} + \sum_{t=1}^{T} E_{imt} \geq \sum_{t=1}^{T} \sum_{o=1}^{O} F_{imot} \quad \forall\, i, m$
$I_{iot} = I_{iot-1} + G_{iot} - \underset{iot}{\tilde{D}} - \eta I_{iot} \quad \forall\, i,\, t > 1, o$
$I_{io1} = 0 \quad \forall\, i, o$
$(1-\eta) \sum_{t=1}^{T} I_{iot} + \sum_{t=1}^{T} G_{iot} \underset{\sim}{\geq} \sum_{t=1}^{T} \underset{iot}{\tilde{D}} \quad \forall\, i, o$
$L_{zmt} = \sum_{i=1}^{P} \omega_i B_{izmt} \quad \forall\, z, t, m$
$L_{zmt} \leq (Q_{zmt} + j_{zmt} w)\, u_{zmt} + (j_{zmt} + 1)\, w\, (1 - u_{zmt}) \quad \forall\, z, m, t$
$L_{zmt} = Q_{zmt} + j_{zmt} w \quad \forall z, m,\ t$
$X_{ijzt},\ Y_{izt},\ A_{izt},\ E_{imt},\ F_{imot},\ G_{iot},\ L_{zmt} \geq 0;\ V_{ijzt},\ u_{zmt} \in [0,\ 1]\,;$
$I_{imt},\ I_{iot},\ I_{izt},\ Q_{zmt},\ j_{zmt}$ are integer.

5 Solution Algorithm

5.1 Fuzzy Solution Algorithm

In following algorithm by Zimmermann (1976) specifies the sequential steps to solve the fuzzy mathematical programming problems.

Step 1. Compute the crisp equivalent of the fuzzy parameters using a defuzzification function. Here, ranking technique is employed to defuzzify the parameters as

$$F_2(A) = (a_l + 2a_m + a_u)/4,$$

where a_l, a_m, a_u are the Triangular Fuzzy Numbers (TFN).

Let $\underset{iot}{\bar{D}}$ be the defuzzified value of $\underset{iot}{\tilde{D}}$ and $(D^1_{iot}, D^2_{iot}, D^3_{iot})$ for each i, o & t be triangular fuzzy numbers then,$\overline{D}_{iot} = \left(D^1_{iot} + 2D^2_{iot} + D^3_{iot}\right)/4$. Similarly, $\underset{imt}{\bar{HD}}$ and $\underset{iot}{\bar{HR}}$ are defuzzified aspired holding cost at warehouse and destination.

Step 2. Since industry is highly volatile and customer demand changes in every short span, a precise estimation of cost and performance aspirations is a major area of discussion. Hence, a better way to come out of such situation is to incorporate tolerance and aspiration level with the main objectives. The model discussed in section 4.5.3 can thus be re-written as follows:

Find X,

$X \in\ S$

$(1-\eta)\sum_{t=1}^{T} I_{iot} + \sum_{t=1}^{T} G_{iot} \underset{\sim}{\geq} \sum_{t=1}^{T} \bar{D}_{iot} \quad \forall\, i, o$
$C(X) \underset{\sim}{\leq} C_0$
$PR \underset{\sim}{\geq} PR_0$
$X_{ijzt},\, Y_{izt},\, A_{izt},\, E_{imt},\, F_{imot},\, G_{iot},\, L_{zmt} \geq 0;\; V_{ijzt},\, u_{zmt} \in [0,\, 1]\,;$
$I_{imt},\, I_{iot},\, I_{izt},\, Q_{zmt},\, j_{zmt}$ are integer.

Step3. Define appropriate membership functions for each fuzzy inequalities as well as constraint corresponding to the objective functions.

$$\mu_C(X) = \begin{cases} 1\ ;\ \mathrm{C}(X) \leq C_0 \\ \frac{C_0^* - C(X)}{C_0^* - C_0}\ ;\ \mathrm{C}_0 \leq C(X) < C_0^* \\ 0\ ;\ \mathrm{C}(X) > C_0^* \end{cases},$$

$$\mu_{PR}(X) = \begin{cases} 1\ ;\ PR \geq PR_0 \\ \frac{PR - PR_0^*}{PR_0 - PR_0^*}\ ;\ PR_0^* \leq PR < PR_0 \\ 0\ ;\ PR < PR_0^* \end{cases}$$

$$\mu_{I_{iot}}(X) = \begin{cases} 1\ ;\ I_{iot}(X) \geq \overline{D}_0 \\ \frac{I_{iot}(X) - \overline{D}_0^*}{\overline{D}_0 - \overline{D}_0^*}\ ;\ \overline{D}_0^* \leq I_{iot}(X) < \overline{D}_0 \\ 0\ ;\ I_{iot}(X) > \overline{D}_0^* \end{cases}$$

Where $\overline{D}_0 = \sum_{t=1}^{T}\sum_{o=1}^{O} \overline{D}_{iot}$is the aspiration and $\overline{D}_0^*$is the tolerance level to inventory constraints.

Step4. Employ extension principle to identify the fuzzy decision, which results in a crisp mathematical programming problem given by

Maximize α
Subject to $\mu_c(X) \geq \alpha$,
$\mu_{PR}(X) \geq \alpha$,
$\mu_{I_{iot}}(X) \geq \alpha$,
$X \in S$

Where α represents the degree up to which the aspiration of the decision-maker is met. The above problem can be solved by the standard crisp mathematical programming algorithms.

Step5. Following Bellman and Zadeh (1970), while solving the problem following steps 1-4, the objective of the problem is also treated as a constraint. Each constraint is considered to be an objective for the decision-maker and the problem can be looked as a fuzzy bi-objective mathematical programming problem. Further, each objective can have a different level of importance and can be assigned weight to measure the relative importance. The resulting problem can be solved by the weighted min max approach. On substituting the values for $\mu_{PR}(x)$ and $\mu_C(x)$the problem becomes

Maximize α

subject to

$PR(x) \geq PR_0 - (1 - w_1\alpha)(PR_0 - PR_0^*)$

$C(x) \leq C_0 + (1 - w_2\alpha)(C_0^* - C_0)$ (P1)

$\mu_{I_{iot}}(X) \geq \alpha$

$X \in S$

$w_1 \geq 0,\ w_2 \geq 0,\ \ w_1 + w_2 = 1, \alpha \in [0, 1]$

Step6. If a feasible solution is not obtained for the problem in Step 5, then we can use the fuzzy goal programming approach to obtain a compromised solution given by Mohamed (1997). The method is discussed in detail in the next section.

5.2 Fuzzy Goal Programming Approach

On solving the problem, we found that the problem (P1) is not feasible; hence the management goal cannot be achieved for a feasible value of α[0,1]. Then, we use the fuzzy goal programming technique to obtain a compromised solution. The approach is based on the goal programming technique for solving the crisp goal programming problem given by Mohamed (1997). The maximum value of any membership function can be 1; maximization of α[0,1] is equivalent to making it as close to 1 as best as possible. This can be achieved by minimizing the negative deviational variables of goal programming (i.e., η) from 1. The fuzzy goal programming formulation for the given problem (P1) introducing the negative and positive deviational variables η_j & ρ_j is given as

Minimize u

subject to $\mu_{PR}(X) + \eta_1 - \rho_1 = 1$

$\mu_C(X) + \eta_2 - \rho_2 = 1$

$u \geq w_j * \eta_j \quad j = 1, 2$

$\eta_j * \rho_j = 0 \quad j = 1, 2$

$w_1 + w_2 = 1$

$\alpha = 1 - u$

$\eta_j, \rho_j \geq 0; X \in S; u \in [0, 1]; w_1, w_2 \geq 0$

6 Case Study

Fish is a highly perishable food which needs proper handling and preservation if it is to have a long shelf life and also retain a desirable quality and its nutritional value. The central concern of fish processing is to prevent fish from deterioration. When fish are captured or harvested for commercial purposes, they need some pre-processing so they can be delivered to the next part of the supply chain in a fresh and undamaged condition. This means, for example, that fish caught by a fishing vessel need handling so they can be stored safely until the boat lands the fish on shore. Some of the methods to preserve and process fish and fish products include control of temperature using ice, refrigeration or freezing, sorting and grading, chilling, storing the chilled fish. The model is validated for the case on fish and fish products. Case is taken for two suppliers, two processing points, three distribution centres and three retail outlets for three time periods. Each processing point has its own internal three stages i.e. Receiving & Scanning, Sorting & Packing and Scanning & Dispatching. At processing point, fish products are received and scanned, which have been pre-processed to reduce the deterioration percentage. Afterwards, they are sorted as per quality checks and packed and further sent to the next stage for final scanning before dispatching to the distribution centres. The objectives include minimizing the cost of procurement, processing, transportation and inventory by obtaining the optimal ordered quantity, transportation weights & minimum inventory and maximizing the performance of procurement by choosing the best supplier on the basis of delivery and quality. The data on cost of procurement from suppliers, processing cost, transportation cost from one stage to another, cost of inspection and inventory carrying cost has been discussed.

Three types of fish have been discussed in the case are Rohu, Katle and Pomfret which are ranging from Rs.80 to Rs.190 per kg. In the case, uncertain parameters are performance parameters, holding cost and demand. Further, defuzzified holding costs at all distribution centres and retail outlets are Rs.14, Rs.8 and Rs.8 for three fish types respectively in all the periods. The capacity at both the suppliers is 300 and 380 packets for fish type 'Rohu', 370 and 390 packets for fish type 'Katle' and 360 and 380 packets for fish type 'Pomfret'. In processing stage, the costs of receiving & scanning, sorting & packing and scanning & dispatching are Rs.1, Rs.2 and Rs.2.5 respectively per packet. Inspection cost per packet is Rs.2 and deterioration percentage is constant with 3% deterioration cost.

	Product Type		
Supplier	Rohu	Katle	Pomfret
Supplier 1	134	90	190
Supplier 2	185	85	185

Table 1: Purchase Cost in all periods and at all processing points

	Product Type		
Processing Point	Rohu	Katle	Pomf
PP 1	320	310	300
PP 2	355	275	245

Table 2: Capacity at all stages in processing point for all periods

	Supplier 1 to PP1 & PP2					
Product Type	Period 1		Period 2		Period 3	
	AC	DT	AC	DT	AC	DT
Rohu	0.93	0.98	0.93	0.98	0.93	0.98
Katle	0.99	0.98	0.99	0.98	0.99	0.98
Pomfret	0.95	0.98	0.95	0.98	0.95	0.98
	Supplier 2 to PP1 & PP2					
Product Type	Period 1		Period 2		Period 3	
	AC	DT	AC	DT	AC	DT
Rohu	0.95	0.99	0.95	0.99	0.95	0.99
Katle	0.93	0.97	0.93	0.97	0.93	0.97
Pomfret	0.95	0.97	0.95	0.97	0.95	0.97

Table 3: De-fuzzified Delivery time (DT) and Acceptance (AC) Probabilities

	Distribution Centre		
Processing Point	DC 1	DC 2	DC 3
PP 1	2000	2500	2500
PP 2	2200	2900	2400

Table 4: Transportation cost per truck

	Retail Outlet		
Distribution Centre	RO 1	RO 2	RO 3
DC 1	2	2.2	1.9
DC 2	2.2	2.5	2.1
DC 3	1.9	1.8	2

Table 5: Transportation cost per packet from DC to RO

	Product Type		
Retail Outlet	Rohu	Katle	Pomfret
RO 1	100	160	140
RO 2	110	150	135
RO 3	105	170	150

Table 6: De-fuzzified demand in all time periods

Truckload per truck is 250kg. Overhead quantity transportation cost is Rs.9 per packet.

6.1 Results and Managerial Implications

The model helps company to provide minimum total cost incurred co-ordinating all the entities. Rs. 1085767 is the total cost which consists of holding cost at distribution centres as Rs.65758, procurement cost of Rs.856600, processing cost of Rs.33001, cost of transportation from processing point to distribution centres of Rs.76588, holding cost at retail outlets of Rs.28015.63, cost of transportation from distribution centres to retail outlets of Rs.13848.80 and finally inspection cost of Rs.11956. It is observed from the results that highest proportion is of the cost of procurement, which clearly validates the requirement of supplier selection. Further, keeping a valid track of transportation polices is equally important as the second highest portion in the cost is due to the transportation cost only. Next observation is towards the impact of the product's nature as holding cost at distribution centre contributes towards the third highest portion in the cost. To prevent the over valuation of cost, the aspiration and tolerance level have been considered as Rs.950000 and Rs.1220000. As validated with the help of cost, the suppliers' performance is second objective of the model which is a combination of on-time delivery and acceptance percentage of the suppliers. The higher the performance of the supplier, better the performance of the company. Keeping the aspiration level of suppliers' performance as 39 and tolerance as 30, the performance level of suppliers obtained is 35.04. The model tries to activate the high performers to procure ordered quantity so that uncertainty in the environment can be managed. Nearby 78% of the aspiration level of cost and performance has been attained which makes the environment more certain and crisp for future decisions.

	Processing Point 1					
	Per. 1		Per. 2		Per. 3	
Pr.T.	S1	S2	S1	S2	S1	S2
Rohu	0	350	0	350	0	350
Katle	350	0	350	0	350	0
Pomfret	350	0	350	0	350	0
	Processing Point 2					
	Per. 1		Per. 2		Per. 3	
Pr.T.	S1	S2	S1	S2	S1	S2
Rohu	0	350	0	350	0	150
Katle	350	0	350	0	350	0
Pomfret	350	0	350	0	350	0

Table 7: Optimum ordered quantity from supplier (S1-S2)

In Table 7, the positive ordered quantity indicates the active supplier to supply goods as he has the highest performance percentage between the two suppliers on the bases of on-time delivery, acceptance percentage and capacity. It can help in reducing the procurement cost and making the process smooth in further echelon.

Tables 8 and 9 shows ending inventory at processing points and retail outlets, which ensures no shortages in the case of unexpected demand. It is observed that at second retail outlet, storage capacity and infrastructure is better as well as the cost of holding is also low, hence inventory is higher at this outlet in comparison to others. Inventory at distribution is not discussed as no inventory was leftover at any of the distribution centres.

While transporting weighted quantity to distribution centres, the policy type, number of trucks and overhead weights are to be checked as each of them incurs cost. In the Table 10 it is observed that while transporting

	Processing Point					
	Period 1		Period 2		Period 3	
Product Type	PP1	PP2	PP1	PP2	PP1	PP2
Rohu	7	7	14	14	21	21
Katle	7	7	14	14	21	21
Pomfret	3	7	10	14	17	21

Table 8: Inventory at processing points (in packets)

	Retail Outlet								
	Period 1			Period 2			Period 3		
ProductType	RO1	RO2	RO3	RO1	RO2	RO3	RO1	RO2	RO3
Rohu	0	0	0	112	171	78	11	698	1
Katle	0	0	0	131	69	0	2	317	75
Pomfret	0	0	0	144	58	51	5	487	8

Table 9: Inventory at retail outlets (in packets)

from processing point 1 to distribution centre 1 in period 2, only Truckload (T*) policy is used as 250kg can be transported by 1 truck. In this case, LTL policy will become expensive. On the other side, transporting from processing point 1 to distribution centre 1 in period 1, TL & LTL$^?$ policy is used as 49kg should be transported as per unit weight. In the case of TL&LTL policy, if overhead weighted quantity is transported through full truckload, the cost of transportation will become much higher than using LTL policy.

Where TL & LTL is indicated as TLT and only TL is indicated as T.

Some more operational variables who helped in smooth process of goods from one level to other are as follows:

7 Conclusion

In the emerging business scenario, the concepts of time, volume and capacity become even more essential to the managerial decision-making. Customers are more sensitive to delivery times and service quality. The coordination among the members of the chain helps them to make a cost-effective procurement and distribution network as well as better response to the cus-

	Distribution Centre 1					
	Period 1		Period 2		Period 3	
	PP1	PP2	PP1	PP2	PP1	PP2
Tpt Quantity	49	7	250	0	329	250
No. of Trucks	0	0	1	0	1	0
Tpt Mode	TLT?	TLT	T*	T	TLT	T
Qty Overhead	49	7	0	0	79	0
	Distribution Centre 2					
	Period 1		Period 2		Period 3	
	PP1	PP2	PP1	PP2	PP1	PP2
Tpt Quantity	749	761	752	1000	686	500
No. of Trucks	2	3	3	4	2	2
Tpt Mode	TLT	TLT	TLT	T	T	T
Qty Overhead	249	11	2	0	186	0
	Distribution Centre 3					
	Period 1		Period 2		Period 3	
	PP1	PP2	PP1	PP2	PP1	PP2
Tpt Quantity	35	261	27	29	14	279
No. of Trucks	0	1	0	0	0	1
Tpt Mode	TLT	TLT	TLT	TLT	TLT	TLT
Qty Overhead	35	11	27	29	14	29

Table 10: Transported quantity, no. of trucks, transportation mode, overhead quantity

$\boldsymbol{E}_{imt}$	Period 1			Period 2			Period 3		
Dis.C.	Rohu	Katle	Pomf	Rohu	Katle	Pomf	Rohu	Katle	Pomf
DC 1	18	16	22	94	144	12	1	0	578
DC 2	451	634	425	544	534	674	657	421	108
DC 3	217	36	43	48	8	0	28	265	0

Table 11:

G_{iot}	Period 1			Period 2			Period 3		
R.O.	Rohu	Katle	Pomf	Rohu	Katle	Pomf	Rohu	Katle	Pomf
RO 1	0	35	0	215	295	288	0	31	1
RO 2	641	647	426	286	221	195	658	408	578
RO 3	45	4	64	185	170	203	28	247	107

Table 12:

tomers' demand. The authors explain the coordination among many entities of supply chain. As mentioned in the objectives of this study, the main plan of this research is to find optimum quantity from the best suppliers under fuzzy environment to develop an optimum coordination among multi supplier, multi processing points, multi distribution centres and multiple number of retail outlets. To attain the objective, a fuzzy bi-objective mathematical model is formulated with objective functions of cost and combination of timely delivery & acceptance of lot, keeping the constraints as supplier capacity, processing capacity, deteriorating nature of the product and truck capacity. The parameters in study as holding cost, consumption, delivery time and acceptance percentage are fuzzy in nature. To handle the issues of uncertainty and fuzziness, the model is converted into crisp form with the help of membership functions of fuzzy modeling. The parameters are also converted into crisp form by using triangular fuzzy numbers. To obtain the solutions, a fuzzy goal programming is employed. Hence, the current study is able to find a balance between minimum cost and best performed supplier. The proposed model was validated by applying to the real case study data.

References

[1] Ho, W., Xu, X., and Dey, P.K. (2010). Multi-criteria decision making approaches for supplier evaluation and selection: a literature review. European Journal of Operational Research 202 (1), 16-24.

[2] Hassini, E. (2008). Order lot sizing with multiple capacitated suppliers offering lead time-dependent capacity reservation and unit price discounts. Production Planning & Control, 19(2), 142–149.

[3] Ravindran, A. R., Bilsel, R. U., Wadhwa, V., and Yang, T. (2010). Risk adjusted multi criteria supplier selection models with applications. International Journal of Production Research, 48(2), 405–424.

[4] Liao, Z., and Rittscher, J. (2007). Integration of supplier selection, procurement lot sizing and carrier selection under dynamic demand conditions. International Journal of Production Economics, 107, 502–510.

[5] Razmi, J., and Maghool, E. (2010). Multi-item supplier selection and lot-sizing planning under multiple price discounts using augmented e-constraint and Tchebycheff method. International Journal of Advances in Manufacturing Technology, 49, 379–392.

[6] Zhang, J.-L., and Zhang, M.-Y. (2011). Supplier selection and purchase problem with fixed cost and constrained order quantities under stochastic demander. International Journal of Production Economics, 129, 1–7.

[7] Jolai, F., Yazdian, S. A., Shahanaghi, K., and Khojasteh, M. A. (2011). Integrating fuzzy TOPSIS and multi-period goal programming for purchasing multiple products from multiple suppliers. Journal of Purchasing & Supply Management, 17, 42–53.

[8] Pal, B., Sana, S. S., and Chaudhuri, K. (2012). A multi-echelon supply chain model for reworkable items in multiple-markets with supply disruption. Economic Modelling, 29(5), 1891–1898.

[9] Kilic, H. S. (2013). An integrated approach for supplier selection in multi-item/ multi-supplier environment. Applied Mathematical Modelling, 37, 7752–7763.

[10] Lee, A. H. I., Kang, H. Y., and Chang, C. T. (2009). Fuzzy multiple goal programming applied to TFT-LCD supplier selection by downstream manufacturers. Expert System with Applications, 36, 6318–6325.

[11] Madronero, M. D., Peidro, D., and Vassant, P. (2010). Vendor selection problem by using an interactive fuzzy multi-objective approach with modified S-curve membership functions. Computers and Mathematics with Applications, 60,1038–1048.

[12] Wu, D. D., Zhang, Y., Wu, D., and Olson, D. L. (2010). Fuzzy multi-objective programming for supplier selection and risk modeling: A possibility approach. European Journal of Operations Research, 200, 774–787.

[13] Arikan, F. (2011). An Augmented Max–min Model For Multiple Objective Supplier Selection, 9th International Congress on Logistics and

Supply Chain Management, 27–29 October, Yas_ar University, Ces_me-_Izmir-Turkey. The congress proceeding including full-texts is in the publication process.

[14] Feng, B., Fan, Z., and Li, Y. (2011). A decision method for supplier selection in multi-service outsourcing, International Journal of Production Economics, 132, pp 240-250.

[15] Seifbarghy, M. and Esfandiari, N. (2013). Modeling and solving a multi-objective supplier quota allocation problem considering transaction costs, Journal of Intelligent Manufacturing, 24(1), 201-209.

[16] Aghai, S., Mollaverdi, N., and Sabbagh, M. S. (2014), A fuzzy multi-objective programming model for supplier selection with volume discount and risk criteria, The International Journal of Advanced Manufacturing Technology, 71(5-8), 1483-1492.

[17] Zimmermann, H. J. 1976. "Description and optimization of fuzzy systems." International Journal of General Systems 2 (4): 209–215.

[18] Bellman, R. E., and Zadeh, L. A. (1970). "Decision-making in a fuzzy environment." Management Science 17 (4): 141-164.

[19] Mohamed, R. H. 1997. "The relationship between goal programming and fuzzy programming." Fuzzy Sets and Systems 89 (2): 215-222.