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Class of Semihyperrings from Partitions of a Set

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Abstract

In this paper we show that a partition $\{P_\alpha : \alpha \in \Lambda\}$ of a non-empty set S, where Λ is an ordered set with the least element α_0 and P_{α_0} is a singleton set, induces a hyperaddition $+$ such that $(S, +)$ is a commutative hypermonoid. Also by using a collection of subsets of S, induced by the partition of the set S, we define hypermultiplication on S so that $(S, +, \cdot)$ is a semihyperring.

Key words: hypermonoid, semihyperring, $*$-collection.

MSC 2010: 20N20.

1 Introduction

The theory of hyperstructures has been introduced by the French Mathematician Marty [11] in 1934 at the age of 23 during the 8^{th}congress of Scandinavian Mathematicians held in Stockholm. Since then many researchers have worked on this new area and developed it.

The theory of hyperstructure has been subsequently developed by Corsini [4, 5, 6], Mittas [13], Stratigopoulos [16] and various authors. Basic definitions and results about the hyperstructures are found in [5, 6]. Some researchers, namely, Davvaz [7], Massouros [12], Vougiouklis [18] and others developed the theory of algebraic hyperstructures.

There are different notions of hyperrings $(R, +, \cdot)$. If the addition $+$ is a hyperoperation and the multiplication $\cdot$ is a binary operation then we say the hyperring is an Krasner (additive) hyperring [10]. Rota [15] introduced a multiplicative hyperring, where $+$ is a binary operation and $\cdot$ is a hyperoperation. De Salvo [8] introduced a hyperring in which addition and multiplication are hyperoperations. These hyperrings are studied by Rahnamani Barghi [14] and by Asokkumar and Velrajan [1, 2, 17]. Chvalina [3] and Hoskova [3, 9], studied $h\nu$-groups, $H\nu$-rings.

In this paper, by using different partitions of a set, we construct different semihyperrings $(S, +, \cdot)$ where both $+$ and $\cdot$ are hyperoperations.

2 Preliminaries

This section explains some basic definitions that have been used in the sequel.

A *hyperoperation* $\circ$ on a non-empty set H is a mapping of $H \times H$ into the family of non-empty subsets of H (i.e., $x \circ y \subseteq H$, for every $x, y \in H$). A *hypergroupoid* is a non-empty set H equipped with a hyperoperation $\circ$. For any two subsets A, B of a hypergroupoid H, the set $A \circ B$ means $\bigcup_{\substack{a \in A \\ b \in B}} (a \circ b)$.

A hypergroupoid $(H, \circ)$ is called a *semihypergroup* if $x \circ (y \circ z) = (x \circ y) \circ z$ for all $x, y, z \in H$(the associative axiom). A semihypergroup H is said to be regular (in the sense of Von Neumann) if $a \in a \circ H \circ a$ for every $a \in H$. A hypergroupoid $(H, \circ)$ is called a *quasihypergroup* if $x \circ H = H \circ x = H$ for every $x \in H$(the reproductive axiom). A reproductive semihypergroup is called a *hypergroup*(in the sense of Marty).A comprehensive review of the theory of hypergroups appears in [5].

Definition 2.1. A *semihyperring* is a non-empty set R with two hyperoperations $+$ and $\cdot$ satisfying the following axioms:

(1) $(R, +)$ *is a commutative hypermonoid*, that is,

(a) $(x + y) + z = x + (y + z)$ for all $x, y, z \in R$,

(b) there exists $0 \in R$, such that $x + 0 = 0 + x = \{x\}$ for all $x \in R$,

(c) $x + y = y + x$ for all $x, y \in R$.

(2) $(R, \cdot)$ is a semihypergroup, that is, $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ for all $x, y, z \in R$.

(3) The hyperoperation $\cdot$ is distributive with respect to hyperoperation '+', that is, $x \cdot (y + z) = x \cdot y + x \cdot z$ and $(x + y) \cdot z = x \cdot z + y \cdot z$ for all $x, y, z \in R$.

(4) There exists element $0 \in R$, such that $x \cdot 0 = 0 \cdot x = 0$ for all $x \in R$.

Definition 2.2. Let S be a semihyperring, An element $a \in S$ is said to be regular if there exists an element $y \in S$ such that $x \in xyx$. A semihyperring S is said to be regular if each element of S is regular.

3 Semihyperring constructed from a $*$-collection.

In this section, for a given commutative hypermonoid $(S,+)$, we define hyperoperation $\cdot$ on S suitably so that $(S,+,\cdot)$ is a regular semihyperring.

Definition 3.1. Let S be a commutative hypermonoid. A collection of non-empty subsets $\{S_a : a \in S\}$ of S satisfying the following conditions is called a $*$-collection if (i) $S_a = \{0\}$ if and only if $a = 0$, (ii) if $a \neq 0$ then $\{0,a\} \subseteq S_a$, (iii) $\bigcup_{x\in S_a} S_x = S_a$ for every $a \in S$, (iv) $S_a + S_a = S_a$ for every $a \in S$ and (v) $\bigcup_{x\in a+b} S_x = S_a + S_b$ for every $a,b \in S$.

Example 3.2. Consider the set $S = \{0,a,b\}$. If we define a hyperoperation $+$ on S as in the following table, then (S,+) is a commutative hypermonoid.

+	0	a	b
0	0	a	b
a	a	{a,b}	{a,b}
b	b	{a,b}	{a,b}

Now it is easy to see that $S_0 = \{0\}; S_a = S; S_b = S$ is a $*$-collection.

Example 3.3. Consider the set S={0,a,b}. If we define a hyperoperation + on S as in the following table, then (S,+) is a commutative hypermonoid.

+	0	a	b
0	0	a	b
a	a	{a}	{a,b}
b	b	{a,b}	{b}

Now it is easy to see that $S_0 = \{0\}; S_a = S; S_b = S$ is a $*$-collection. Now we show that $S_0 = \{0\}; S_a = \{a,0\}; S_b = \{b,0\}$ is another $*$-collection.
For each $a \in S$, $\bigcup_{x\in S_a} S_x = \bigcup_{x\in\{a,0\}} S_x = S_a \bigcup S_0 = \{a,0\} \bigcup \{0\} = \{a,0\} = S_a$. Also $S_0 + S_0 = \{0\} + \{0\} = \{0\} = S_0; S_a + S_a = \{0,a\} + \{0,a\} = \{0,a\} = S_a; S_b + S_b = \{0,b\} + \{0,b\} = \{0,b\} = S_b$. Further, for $a,b \in S$, we get $\bigcup_{x\in a+b} S_x = \bigcup_{x\in\{a,b\}} S_x = S_a \bigcup S_b = \{0,a,b\} = S_a + S_b$.

Example 3.4. Consider the set S={0,a,b}. If we define a hyperoperation + on S as in the following table, then (S,+) is a commutative hypermonoid.

+	0	a	b
0	0	a	b
a	a	{0,a}	S
b	b	S	{0,b}

It is easy to show that $S_0 = \{0\}$; $S_a = S$ for every $a \neq 0 \in S$, is a $*$-collection and $S_0 = \{0\}$; $S_a = \{a, 0\}$ for every $a \neq 0 \in S$ is another $*$-collection

Example 3.5. Consider the set S={0,a,b,c}. If we define a hyperoperation + on S as in the following table, then (S,+) is a commutative hypermonoid.

+	0	a	b	c
0	{0}	{a}	{b}	{c}
a	{a}	{a}	{a, b}	{a, c}
b	{b}	{a, b}	{b}	{b, c}
c	{c}	{a, c}	{b, c}	{c}

In this commutative hypermonoid, each one of the following is a $*$-collection.

$S_0 = \{0\}$; $S_a = \{a, 0\}$ for every $a \neq 0 \in S$,
$S_0 = \{0\}$; $S_a = S$ for every $a \neq 0 \in S$,
$S_0 = \{0\}; S_a = \{0, a\}; S_b = \{0, b, a\}; S_c = \{0, c, a\}$,
$S_0 = \{0\}; S_a = \{0, a, b\}; S_b = \{0, b\}; S_c = \{0, c, b\}$,
$S_0 = \{0\}; S_a = \{0, a, c\}; S_b = \{0, b, c\}; S_c = \{0, c\}$.

Theorem 3.6. *Let S be a commutative hypermonoid with the additive identity 0 with the condition that $x + y = \{0\}$ for $x, y \in S$ implies either $x = 0$ or $y = 0$. Let $\{S_a : a \in S\}$ be a $*$-collection on S. For $a, b \in S$, if we define a hypermultiplication on S as*

$$a \cdot b = \begin{cases} S_a & \text{if } a \neq 0, b \neq 0, \\ 0 & \text{otherwise} \end{cases}$$

then $(S, +, .)$ is a regular semihyperring.

Proof. From the definition of the hypermultiplication, $a \cdot 0 = 0 \cdot a = 0$ for all $a \in S$. Let $a, b, c \in S$. If any one of a, b, c is 0, then $a \cdot (b \cdot c) = \{0\} = (a \cdot b) \cdot c$. If $a \neq 0, b \neq 0$ and $c \neq 0$, then $a \cdot (b \cdot c) = a \cdot S_b = S_a$. Also, $(a \cdot b) \cdot c = S_a \cdot c = \bigcup_{x \in S_a} (x \cdot c) = \bigcup_{x \in S_a} S_x = S_a$. Thus $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. Therefore, $(S, \cdot)$ is a semihypergroup.

Let $a, b, c \in S$. If $a = 0$ or $b = 0$ or $c = 0$, then it is obvious that $a \cdot (b + c) = a \cdot b + a \cdot c$. Suppose $a \neq 0, b \neq 0$ and $c \neq 0$. If $0 \in b + c$, then $a \cdot (b + c) = S_0 \cup S_a = S_a = S_a + S_a = a \cdot b + a \cdot c$. If $0 \notin b + c$, then $a \cdot (b + c) = S_a = S_a + S_a = a \cdot b + a \cdot c$. Thus $a \cdot (b + c) = a \cdot b + a \cdot c$.

Now we prove $(a + b) \cdot c = a \cdot c + b \cdot c$. For, $(a + b) \cdot c = \bigcup_{x \in a+b} x.c = \bigcup_{x \in a+b} S_x = S_a + S_b = a \cdot c + b \cdot c$. Therefore, $(a + b) \cdot c = a \cdot c + b \cdot c$. Thus $(S, +, \cdot)$ is a semihyperring.

Let $x \neq 0 \in S$. Now, for any $y \neq 0 \in S$, we have $x \in S_x = x \cdot y \subseteq x \cdot S_y = x \cdot (y \cdot x)$. Hence the semihyperring is regular. □

Example 3.7. The semihyperring obtained by using the Theorem 3.1 in the Example 3.1 is as follows.

+	0	a	b
0	0	a	b
a	a	{a,b}	{a,b}
b	b	{a,b}	{a,b}

.	0	a	b
0	0	0	0
a	0	S	S
b	0	S	S

Example 3.8. The semihyperrings obtained by using the Theorem 3.1 in the Example 3.2 are as follows.

+	0	a	b
0	0	a	b
a	a	{a}	{a,b}
b	b	{a,b}	{b}

.	0	a	b
0	0	0	0
a	0	S	S
b	0	S	S

.	0	a	b
0	0	0	0
a	0	{0, a}	{0,a}
b	0	{0,b}	{0,b}

Example 3.9. The semihyperrings obtained by using the Theorem 3.1 in the Example 3.3 are as follows.

+	0	a	b
0	0	a	b
a	a	{0,a}	S
b	b	S	{0,b}

.	0	a	b
0	0	0	0
a	0	S	S
b	0	S	S

.	0	a	b
0	0	0	0
a	0	{0, a}	{0,a}
b	0	{0,b}	{0,b}

Theorem 3.10. *Let S be a commutative hypermonoid with the additive identity 0 with the condition that $x + y = 0$ for $x, y \in S$ implies either $x = 0$ or $y = 0$. Let $\{S_a : a \in S\}$ be a $*$-collection on S. For $a, b \in S$, if we define a hypermultiplication on S as*

$$a \cdot b = \begin{cases} S_b & \text{if } a \neq 0, b \neq 0, \\ 0 & \text{otherwise} \end{cases}$$

then $(S, +, .)$ is a regular semihyperring.

Proof. The proof follows by the same lines as in the Theorem 3.1. Let $x \neq 0 \in S$. Now, for any $y \neq 0 \in S$, we have $x \in S_x = y \cdot x \subseteq S_y \cdot x = (x \cdot y) \cdot x$. Hence the semihyperring is regular. □

Theorem 3.11. *Let S be a commutative hypermonoid with the additive identity 0 such that $x + y = 0$ for $x, y \in S$ implies either $x = 0$ or $y = 0$. Let $\{S_a : a \in S\}$ be a $*$-collection on S such that $S_a \cap S_b = X$ for all $a \neq 0, b \neq 0 \in S$ where X is a subset of S such that $X + X = X$. For $a, b \in S$, if we define a hypermultiplication on S as*

$$a \cdot b = \begin{cases} S_a \cap S_b = X & \text{if } a \neq 0, b \neq 0, \\ 0 & \text{otherwise} \end{cases}$$

then $(S, +, .)$ is a regular semihyperring.

Proof. Since $0 \in S_a$ and $0 \in S_b$, we get $0 \in S_a \cap S_b$. This implies that the set X is non-empty. From the definition of hypermultiplication, $a \cdot 0 = 0 \cdot a = 0$ for all $a \in S$. Let $a, b, c \in S$. If any one of a, b, c is 0, then $a \cdot (b \cdot c) = \{0\} = (a \cdot b) \cdot c$. If $a \neq 0, b \neq 0$ and $c \neq 0$, then $a \cdot (b \cdot c) = X = (a \cdot b) \cdot c$. Thus $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. Therefore, $(S, \cdot)$ is a semihypergroup.

If $a = 0$ or $b = 0$ or $c = 0$, then it is obvious that $a \cdot (b + c) = a \cdot b + a \cdot c$. Suppose $a \neq 0, b \neq 0$ and $c \neq 0$ then, $a \cdot (b + c) = X = X + X = a \cdot b + a \cdot c$. Similarly we have $(a + b) \cdot c = X = a \cdot c + b \cdot c$. Thus $(S, +, \cdot)$ is a semihyperring. Let $x \neq 0 \in S$. Since $x \in S_x$, we have $x \in S_x = x \cdot x \subseteq x \cdot S_x = x \cdot (x \cdot x)$. Hence the semihyperring is regular. □

Example 3.12. Using the Theorem 3.3 in the commutative hypermonoid given in the Example 3.4 and by using the following each $*$-collection

$S_0 = \{0\}; S_a = \{0, a\}; S_b = \{0, b, a\}; S_c = \{0, c, a\}$ with $X = \{0, a\}$,

$S_0 = \{0\}; S_a = \{0, a, b\}; S_b = \{0, b\}; S_c = \{0, c, b\}$ with $X = \{0, b\}$,

$S_0 = \{0\}; S_a = \{0, a, c\}; S_b = \{0, b, c\}; S_c = \{0, c\}$ with $X = \{0, c\}$, we get three hypermultiplications so that we get three semihyperrings.

4 Semihyperrings induced by a Partition.

In this section we show that a partition of a non-empty set S induces a hyperaddition $+$ such that, $(S, +)$ is a commutative hypermonoid and also the partition induces a $*$-collection. Using this $*$-collection,we define hypermultiplication $\cdot$ on the set S, so that $(S, +, .)$ a regular semihyperring.

Theorem 4.1. *Let S be any non-empty set and $\{P_\alpha\}_{\alpha \in \Lambda}$ be a partition of S, where Λ is an ordered set with the least element $\alpha_0 \in \Lambda$ and P_{α_0} be a singleton set, say, $\{0\}$. Define a hyperaddition "+" on S as follows: For all $a \in S$, $0 + a = a + 0 = \{a\}$. For $a \neq 0, b \neq 0 \in S$, suppose $a \in P_\alpha$ and $b \in P_\beta$ and $\gamma = max\ \{\alpha, \beta\}$,*

$$a + b = \begin{cases} P_\gamma & \text{if } \alpha \neq \beta, \\ P_\alpha = P_\beta & \text{if } \alpha = \beta \end{cases}$$

Then (i) $(S,+)$ is a commutative monoid and (ii) the partition $\{P_\alpha\}_{\alpha\in\Lambda}$ induces a $*$-collection.

Proof. It is clear that $a+b=b+a$ for all $a,b\in S$. Let $a,b,c\in S$. Suppose that $a\in P_\alpha,\ b\in P_\beta$ and $c\in P_\gamma$, where $\alpha,\beta,\gamma\in\Lambda$.

Case 1 : Suppose $\alpha<\beta<\gamma$.
Then $a+(b+c)=a+P_\gamma=P_\gamma$. Also, $(a+b)+c=P_\beta+c=P_\gamma$. Therefore, $a+(b+c)=(a+b)+c$.

Case 2 : Suppose $\beta<\alpha<\gamma$.
Then $a+(b+c)=a+P_\gamma=P_\gamma$. Also, $(a+b)+c=P_\alpha+c=P_\gamma$. Therefore, $a+(b+c)=(a+b)+c$.

Case 3 : Suppose $\alpha<\gamma<\beta$.
Then $a+(b+c)=a+P_\beta=P_\beta$. Also, $(a+b)+c=P_\beta+c=P_\beta$. Therefore, $a+(b+c)=(a+b)+c$.

Case 4 : Suppose $\gamma<\alpha<\beta$.
Then $a+(b+c)=a+P_\beta=P_\beta$. Also, $(a+b)+c=P_\beta+c=P_\beta$. Therefore, $a+(b+c)=(a+b)+c$.

Case 5 : Suppose $\gamma<\beta<\alpha$.
Then $a+(b+c)=a+P_\beta=P_\alpha$. Also, $(a+b)+c=P_\alpha+c=P_\alpha$. Therefore, $a+(b+c)=(a+b)+c$.

Case 6 : Suppose $\beta<\gamma<\alpha$.
Then $a+(b+c)=a+P_\gamma=P_\alpha$. Also, $(a+b)+c=P_\alpha+c=P_\alpha$. Therefore, $a+(b+c)=(a+b)+c$.

Case 7 : Suppose $\alpha=\beta<\gamma$.
Then $a+(b+c)=a+P_\gamma=P_\gamma$. Also, $(a+b)+c=P_\beta+c=P_\gamma$. Therefore, $a+(b+c)=(a+b)+c$.

Case 8 : Suppose $\gamma<\alpha=\beta$.
Then $a+(b+c)=a+P_\alpha=P_\alpha$. Also, $(a+b)+c=P_\alpha+c=P_\alpha$. Therefore, $a+(b+c)=(a+b)+c$.

Case 9 : Suppose $\alpha=\gamma<\beta$.
Then $a+(b+c)=a+P_\beta=P_\beta$. Also, $(a+b)+c=P_\beta+c=P_\beta$. Therefore, $a+(b+c)=(a+b)+c$.

Case 10 : Suppose $\beta<\alpha=\gamma$.
Then $a+(b+c)=a+P_\alpha=P_\alpha$ Also, $(a+b)+c=P_\alpha+c=P_\alpha$. Therefore, $a+(b+c)=(a+b)+c$.

Case 11 : Suppose $\beta=\gamma<\alpha$.
Then $a+(b+c)=a+P_\gamma=P_\alpha$. Also, $(a+b)+c=P_\alpha+c=P_\alpha$. Therefore, $a+(b+c)=(a+b)+c$.

Case 12 : Suppose $\alpha<\beta=\gamma$.
Then $a+(b+c)=a+P_\gamma=P_\gamma$. Also, $(a+b)+c=P_\beta+c=P_\gamma$. Therefore, $a+(b+c)=(a+b)+c$.

Case 13 : Suppose $\alpha = \beta = \gamma$.
Then $a+(b+c) = P_\alpha = P_\beta = P_\gamma = (a+b)+c$. Therefore, $a+(b+c) = (a+b)+c$.

Thus the hyperoperation $+$ is associative. So, $(S,+)$ is a commutative hypermonoid. Let $S_0 = P_{\alpha_0} = \{0\}$. For $a \neq 0 \in S$, then $S_a = \bigcup_{\alpha_0 \leq t \leq \alpha} P_t$ where $a \in P_\alpha$. It is clear that $S_a = \bigcup_{x \in S_a} S_x$. For $a \neq 0 \in S$, and $a \in P_\alpha$, then $S_a + S_a = \bigcup_{\alpha_0 \leq t \leq \alpha} P_t + \bigcup_{\alpha_0 \leq t \leq \alpha} P_t = \bigcup_{\alpha_0 \leq t \leq \alpha} P_t = S_a$. Also $S_0 + S_0 = \{0\} + \{0\} = \{0\} = S_0$. If either $a = 0$ or $b = 0$, then $\bigcup_{x \in a+b} S_x = S_a + S_b$. Let $a \neq 0, b \neq 0 \in S$. Then $a \in P_\alpha$ and $b \in P_\beta$ for some $\alpha, \beta \in \Lambda$.

Case 1 : Suppose $\alpha \neq \beta$, say $\alpha < \beta$, then $a + b = P_\beta$. Now $x \in a + b$ implies $x \in P_\beta$. Therefore, $S_x = \bigcup_{\alpha_0 \leq t \leq \beta} P_t$. Hence

$$\bigcup_{x \in a+b} S_x = \bigcup_{x \in a+b} \Big(\bigcup_{\alpha_0 \leq t \leq \beta} P_t \Big) = \bigcup_{\alpha_0 \leq t \leq \alpha} P_t = \bigcup_{\alpha_0 \leq t \leq \alpha} P_t + \bigcup_{\alpha_0 \leq t \leq \beta} P_t = S_a + S_b.$$

Case 2 : Suppose $\alpha = \beta$ then $a + b = P_\alpha$. Therefore, $\bigcup_{x \in a+b} S_x = \bigcup_{x \in P_\alpha} S_x = S_a + S_b$. Therefore, $\bigcup_{x \in a+b} S_x = S_a + S_b$. Thus $\{S_a : a \in S\}$ is a $*$-collection.

□

Remark 4.2. Let S be any non-empty set and $x_0 \in S$. Let $P_0 = \{x_0\}$ and $\{P_1, P_2, P_3, \cdots, P_n, \cdots\}$ be a partition of $S \setminus \{x_0\}$. Then the partition $\{P_0, P_1, P_2, \cdots, P_n, \cdots\}$ of S induces a hyperoperation $+$ on S so that $(S,+)$ is a commutative hypermonoid and $\{P_0, P_1, P_2, \cdots, P_n, \cdots\}$ induces a $*$-collection.

Theorem 4.3. *Let S be any non-empty set and $\{P_\alpha\}_{\alpha \in \Lambda}$ be a partition of S, where Λ is an ordered set with the least element α_0 and P_{α_0} is a singleton set. Then the partition induces a semihyperring.*

Proof. By the Theorem 4.1, the partition induces a hyperaddition $+$ such that $(S,+)$ is a commutative hypermonoid and it also induces a $*$-collection. Hence by the Theorem 3.1, we get a regular semihyperring. □

Example 4.4. We illustrate the construction of semihyperrings from the following examples. Let $S = \{0, a, b\}$. Consider a partition $P_1 = \{0\}, P_2 = \{a\}, P_3 = \{b\}$ of S. Here, the indexing set is $\Lambda = \{1, 2, 3\}$ which is an ordered set. The commutative hypermonoid induced by this partition is given by the following Caley table.

+	0	a	b
0	0	a	b
a	a	{a}	{b}
b	b	{b}	{b}

The $*$-collection induced by this partition is $S_0 = \{0\}, S_a = \{0, a\}, S_b = \{0, a, b\}$ and the hypermultiplication induced by the $*$-collection is given in the Caley table.

.	0	a	b
0	0	0	0
a	0	{0,a}	{0,a}
b	0	{0,a,b}	{0,a,b}

Example 4.5. Let $S = \{0, a, b\}$. Consider a partition $P_1 = \{0\}, P_2 = \{b\}, P_3 = \{a\}$ of S. Here, the indexing set is $\Lambda = \{1, 2, 3\}$ which is an ordered set. The commutative hypermonoid induced by this partition is given by the following Caley table.

+	0	a	b
0	0	a	b
a	a	{a}	{a}
b	b	{a}	{b}

The $*$-collection induced by this partition is $S_0 = \{0\}, S_a = \{0, a, b\}, S_b = \{0, b\}$ and the hypermultiplication induced by the $*$-collection is given in the Caley table.

.	0	a	b
0	0	0	0
a	0	{0,a,b}	{0,a,b}
b	0	{0,b}	{0,b}

Example 4.6. Let $S = \{0, a, b\}$. Consider a partition $P_1 = \{0\}, P_2 = \{a, b\}$ of S. Here, the indexing set is $\Lambda = \{1, 2\}$ which is an ordered set. The commutative hypermonoid induced by this partition is given by the following Caley table.

+	0	a	b
0	0	a	b
a	a	{a,b}	{a,b}
b	b	{a,b}	{a,b}

The $*$-collection induced by this partition is $S_0 = \{0\}, S_a = \{0, a, b\}, S_b = \{0, a, b\}$ and the hypermultiplication induced by the $*$-collection is given in the Caley table.

.	0	a	b
0	0	0	0
a	0	{0,a,b}	{0,a,b}
b	0	{0,a,b}	{0,a,b}

Thus we have a regular semihyperring.

Conclusion : In the section 3 of this paper, for the given commutative hypermonoid, given $*$-collection, we construct three semihyperrings. In the section 4, by the Theorem 4.1, a partition of a set S induces a hyperaddition $+$ such that $(S, +)$ is a commutative hypermonoid and it also induces a $*$-collection. Hence by the Theorem 3.1, we get a semihyperring. Thus we get semihyperrings depending on the partitions of the set satisfies the conditions of the Theorem 4.2. All the semihyperrings so constructed are regular.

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