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A mathematical assessment on the ontology of time♣

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Abstract

In this work, we develop and propose an ontological formal definition of time, based on a topological analysis of the formal mathematical description of time, coming from approaches to both quantum theories and Relativity; thus, being compatible with all physical epistemological theories. We find out a mathematical topological invariability, thus establishing a rigorous definition of time, as fundamental generic magnitude. Very preliminary analysis of physical epistemology is provided; likely highlighting a path towards a final common vision between Quantum and Cosmology ontology and human feeling of time.

Keywords: Time essence, topology, parameterization, metaphysics, science.[1]

♣ The author would like to acknowledge this analysis to the first scientist in the Modern Age who studied "machines to measure time" systematically, through his scientific method, *Galileo.*

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1. Introduction

In Epistemology of Physics, the problem of time is a conceptual conflict between "general relativity" and "quantum mechanics" theories. In the latter, flow of time is regarded as universal and absolute, whereas general relativity regards the flow of time as malleable and relative [1]. This problem raises the question of what time really is in a physical sense and whether it is truly a real, distinct phenomenon. It also involves the related question of why time seems to flow in a single direction [2], [3], [4]. Though, it is recognized for macroscopic systems the directionality of time is directly linked to "first principles" such as the Second law of Thermodynamics, thus Universe concerned [5].

In classical mechanics, a special status is assigned to time in the sense that it is treated as a classical background parameter, external to the system itself. This special role is seen in the standard formulation of quantum mechanics. It is regarded as part of an a priori given classical background with a well defined value. In fact, the classical treatment of time is deeply intertwined with the "Copenhagen interpretation" (V Solvay Conference) of quantum mechanics, and, thus, with the conceptual foundations of quantum theory: all measurements of observables are made at certain instants of time and probabilities are only assigned to such measurements [6]. Special relativity has modified the notion of time. But from a fixed Lorentz observer's viewpoint, time still remains a distinguished, absolute, external, global parameter [7]. In consequence, the Newtonian notion of time essentially carries over to special relativistic systems, hidden in the space-time structure. The last is a consequence of the pure geometric nature of the essence of the Relativity theory. Nevertheless, there is a substantially different nuance: the imaginary magnitude of quantitative time description. This, as we will see later, is the initial key point to identify a common ontological definition of time, since the mathematical essential source of the magnitude has drastically changed. In Relativity, since Geometry is an intrinsic physical feature of the system, time cannot be observed as external to the system, as Quantum (and thus, classical) approximations consider. The last has been basically, the root of the "problem of time": the ontological discrepancies for time definition.

Attempts to define "time" as an observable parameter in a Quantum approach (external approximation to define "time") have fundamental limitations: Pauli's theorem [8]. This theorem imposes a serious limitation to define time as an observable due to Heisenberg's uncertainty relation between $[\hat{H}, \hat{T}]$, being $\hat{H}$ energy of the system and $\hat{T}$ operator related to observable "time".[9]. Basically, the limitation comes from the fact that values that observable energy could take are unlimited, which obviously has no physical meaning, [10] [11]. The initiatives to find novel approaches to overcome this major limitation haven't achieved to relevant successful results, to the best of our knowledge, from the Epistemological point of view, being one of the most relevant approaches the so called "Positive Operator-Valued Measure." (POVM) approach [12].

A third intermediate approach (neither internal nor external ones), the so called "covariant theories", does not have a notion of a distinguished physical time with respect to which everything evolves [13], [14]. However, it is not needed for the full formulation and interpretation of the theory. The dynamical laws are determined by correlations which are sufficient to make predictions. But then a mechanism is needed which explains how the familiar notion of time eventually emerges from the timeless structure to become such an important ingredient of the macroscopic world we live in as well as of our conscious experience. The "thermal time hypothesis" has been put forward as a possible solution. It postulates that physical time flow is not an a priori given fundamental property of the theory, but is a macroscopic feature of thermodynamical origin [15].

In the following section, a unique and mathematical consistent definition of time is shown. From a topological approach to the different conceptions of time, we will propose a generalized mathematical definition; thus including "quantum mechanics" (classical theory as a limit of this is, in consequence, concerned) and cosmological existing approaches. The success is achieved by distinguishing parameterization of the magnitude time, including the "time thermal hypotheses" as the natural flow of time, with definition of the nature of time as a physical magnitude mathematical modeling. Preliminary consequences following this definition for the mathematical configurational space for time are finally provided, including for Metaphysics of Science.

2. The difference between time as magnitude and its parameterization

The purpose here is to highlight the difference between a magnitude itself and its formal mathematical representation under a parameterization including a continuous function, consequence of its mathematical definition. Effectively, for the trivial case of time as external parameter (quantum vision and in the limit, classical approaches), this parameterization is naturally provided by the human feeling of time as a continuous succession of ordered events. The natural parameterization then could preliminary be conceived as [0, a], being "a" $\subset \mathbb{R}$, "a">0. This parameterization is induced naturally for whatever system is considered when time is external to the system identified. One then could identify the parametric function as the identity one, which is inaccurate. Reason comes from historic quantum consideration about the well-known "the problem of the measure", inducing a perturbation in all physical systems during the observational stage in the measurement fact [16], [17]. Effectively, under these recognized assumptions, we can consider that system itself comes back after the measurement to the unperturbed original point in its essence: the initial point in time is equivalent to the final one, thus defining in the set of magnitude set of time a quotient space [0, a)/ $\mathcal{R}$. Being "$\mathcal{R}$" the equivalence relation induced by the quantum-based "problem of the measure". This quotient space becomes to [0,a), mathematically modeled.

Now, we will see that Cosmology will also accept the same parameterization as the above quotient space [0, a)/ $\mathcal{R}$, both with the topology induced in the set by the continuous function, projection *"p"*:

$$\text{“}p\text{”}: [0,a] \subset \mathbb{R} \longrightarrow [0, a]/\mathcal{R} = [0, a).$$

In Cosmology, there is consensus that the beginning of time is localized behind us in a finite steps of time [4], and coincides with the beginning of space according to Einstein's Relativity [18]; in fact, we definitively date the origin of the universe into temporal units, according to established methodology [19]. So it makes sense to assign and include in our parameterization a beginning, zero. In principle, this is the well-known *"problem of initial conditions"* for Cosmology [4].

As far as the other edge of the interval is concerned, there are, in principle, two possibilities; either, there is an end of the Universe in a finite sequence or this ending is in a series of infinite number of steps: it will always be, *eternal universe*. For this last case, taking into account the previous paragraph, the parameterization is in consequence, [0, ∞).

This parameterization form has a natural homeomorphism "f" (biyective, "f" "f^{-1}" continuous) well identified:

$$f: [0,\infty) \rightarrow [0,a)$$

$$f(x) \rightarrow \begin{cases} \frac{x}{2}, & x \in [0,a) \\ \frac{a(2x-a)}{2x}, & x \in [a,\infty) \end{cases}$$

This homeomorphism preserves the topological properties between the two intervals, being able to analyze each other interchangeably. Thus, it is shown that the parameterization of time in this case can be described as [0, a). "a" finite.

On the other hand, if Universe ends in a future finite moment, this last step, where space-time disappears will be equivalent (and here is the key, again) at the beginning mathematically speaking; "nothing can be said of it". Thus, the beginning and end are again equivalent under this statement inducing once more an equivalence relation between the edges of the interval. Directly, then, the parameterization of time is again the quotient space [0,a).

Therefore, under the proper formal definition of the so called "problem of initial conditions" in Cosmology, time parameterization is mathematically expressed by $[0, a]/\mathcal{R} = [0, a)$.

Summarizing the above, we propose to reformulate the so-called "problem of initial conditions" to "problem of boundary conditions" by mathematical formal arguments characterizing the parameterization of the physical magnitude of time. As a consequence, a single equivalent parameterization of time is achieved given by the *quotient spaces* $[0, a]/\mathcal{R} = [0,a) \subset \mathbb{R}$. Being $\mathcal{R}$ *the equivalence relations* provided above by this reformulation in each approximation. Order inside the parameterization set is complete ($t_n < t_{n+1}$), provided by the human existence, which orders naturally cognitive events related to itself. This natural auto induced order is also supported and justified by Second Law of Thermodynamics principle also proposed valid in both Cosmology [5] and in the covariant theories (*thermal time hypothesis*). If Second Law is cosmologically valid everywhere, we provide evidences of mathematical coherence with human natural experience of time flow order induced by its existence; thus, this thermal hypothesis is consistent and provides a natural flow-time in its parameterization set $\subset\mathbb{R}$.

Finally, mathematically, under all circumstances and for the whole Physics Epistemology , we can define the parameterization of time as:

$$T = \{t \in \mathbb{R}: t \geq 0\} => [0, a)$$

Here, a concise comment is worth. The equivalent relation pointed out between “0” and “a” does not imply we argue an approach physically to the origin for the universe at the end. We rather assess that, mathematically, both points are equivalent to accurate define the mathematical configurational space: nothing can be said on them because space time configuration has no sense either before the beginning, at “0” or in the epilogue of Cosmos, “a”.

3. Comments on time as physical fundamental magnitude

Is the purpose of this part to demonstrate that the above definition of time fully ordered is perfectly consistent with time defined in Relativity.

Cosmological time under Einstein's Relativity is mathematically defined as:

$$\tau=\{x\in\mathbb{C}:Re(x)=0,Im(x)\geq 0\},\ \ x=Re(x)+iIm(x)\quad Re(x)\ \wedge Im(x)\in\mathbb{R}$$

That is, a pure imaginary number, a subset of the well known "complex numbers".

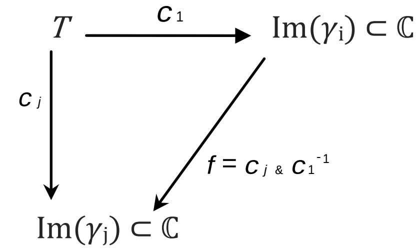
Mathematically, this definition is a particularization of a subset $Im(\gamma) \subset\mathbb{C}$, under the generic homeomorphism (γ biyective, γ and γ^{-1} continuous):

$$\gamma(s):I\rightarrow C,\text{with } I \subset\mathbb{R},\text{interval } [0,a)$$

Where the flow of the parameterization is given directly by the homeomorphism condition of the function, which in fact is a mathematical parameterization of 1-d manifold, subset of $\mathbb{C}$ (equivalent $\mathbb{R}^2$). More precisely, the above definition is always consistent with time in Quantum theories and classical ones IF we consider the interval [0,a) previously commented as another particular subset of $\mathbb{C}$ (equivalent $\mathbb{R}^2$). The particularity for the latest is that the homeomorphism γ here is the identity function, “I”.

As a direct consequence, it can be observed that there is a mathematical topological (in consecuence,ontological) equivalence between all parameterizations of magnitude time in Physics Epistemology: all are 1-dimension varieties, topological subspaces of the topological space (ℂ, *Tu*), where "*Tu*" is the Euclidean usual topology induced by Euclidean distances. Being ℂ isomorphic to $\mathbb{R}^2$ as euclidean metric spaces.

What is more, taking into account that given a specific parameterization "T", defines a specific relationship "*f*" between the different varieties 1-dimensional as follows ($i \neq j$):



Being "*f*", in consequence, homeomorphism. This homeomorphism identified induces a further equivalence relationship among the different Im(γ_i) ⊂ ℂ, thus inducing an additional quotient space defined as:

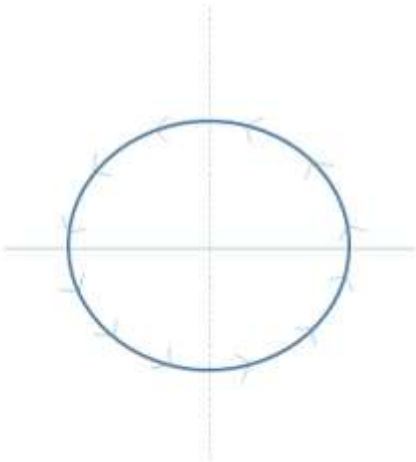
Be $\mathcal{P}$(Im(γ_i)) set incluiding all Im(γ_i) ⊂ ℂ, with γ_i homeomorphism as describe above (i=1,2,3….). The defined equivalent relationship, $\mathcal{R}^*$, as "defining a *f* homeomorphism under the composition of $\gamma_{i,j}$ above shown" induces the following quotient space,

$$\mathcal{P}(\mathrm{Im}(\gamma_i))/\,\mathcal{R}^*$$

In particular, at this point, we will point out that there is an specific Im(ϒ)⊂ℂ defined as the homeomorphism, representing the whole space (canonical representative):

$$S^1\,(s): I \rightarrow \exp(i\, 2\pi/a\; s) \subset \mathbb{C},\; I \subset \mathbb{R}\ \text{interval } [0,a)$$

Whose Im(S^1)⊂ℂ can be geometrical represented as, taking into account the preliminary arguments concerning human feeling naturally providing an standarised parameterization of time with its natural and unique flow, by "thermal time mathematical consistent hypothesis":



Im(S^1)⊂ℂ geometrical representation, as defined as 1-d oriented manifold with boundary and flow-time induced by natural human feeling of-time.

This provides, in consequence, a mathematical ontological-based definition of time: this unique and solely canonical element, valid and mathematically consistent with Physics Epistemology.

Linguistically speaking, this definition can be expressed as:

"Ontology of magnitude of time is represented as a 1-dimensional manifold (with boundary) in the $\mathbb{R}^2$ plane (or equivalent ℂ body set), oriented and embedded with respect to natural human parameterization".

The author points out that this definition is not equivalent to the cyclic S^1 circumference, as the circumference can be identified as a closed-loop in its parameterization. Thus, not injective as our identified 1-dimensional manifold. The mathematical difference between the circumference S^1 and 1-d manifold S^1 can be seen elsewhere in any undergraduate topology handbook [20]. Just we briefly point out here that is well recognized that only two types of 1 dimension manifolds exit for connected 1d spaces: the numerical line ℝ -or interval- and the circumference S^1. For interval sets in ℝ as the ones shown in this paper, 1d manifold with boundary (the S^1 manifold proposed) appears coming from deconstructionist topological arguments in the identified quotient spaces [0,a). S^1 manifold with boundary is a topological set completely different (non homeomorphic) to the circumference S^1. Being the main difference already pointed out: the circumference is a closed-loop (not injective) and

S^1 is homeomorphic to [0,a) as quotient space, topological deconstruction under equivalence relations expressed.

Following to this argument, Nielsen et al. [5] had already rejected the S^1 circumference as mathematical expression of time, but with a mathematical inconsistence in the arguments provided by its definition of time as the whole set $\mathbb{R} = (-\infty, \infty)$. Topologically, this set, as defined is not homeomorphic (thus, not topologically equivalent) to the following 1-d manifold: [0,a) we have defined. In fact, we remind here that $\mathbb{R}$ is a connected space, being homeomorphic to $(0,\infty)$, mathematically inconsistent to Nielsen et al. decomposition shown: $(-\infty,0)$ and $[0,\infty)$. Consequence of this, we disagree with the authors in their artificial mathematical description of thermodynamics as a sum of entropy magnitudes running across $(-\infty,0)$ and $[0,\infty)$. On the other hand, we can agree in their second conclusion about Second Law of Thermodynamics, but with a more simple (and realistic) argument: the natural feeling for humans of time while ordering chronological events justifying the "Thermal time hypothesis" giving a natural induced flow time through the flow induced in the parameterization while ordering the human events. Briefing our comments to Nielsen et al.: topologically, we demonstrate time configurational mathematical space and its flow is unique; in consequence, allusions and comments about the lack of anthropic principle in Cosmology are unfounded.

4. Epistemological implications of time definition

Let us brief at this point now the implications and consequences achieved and derived up to now:

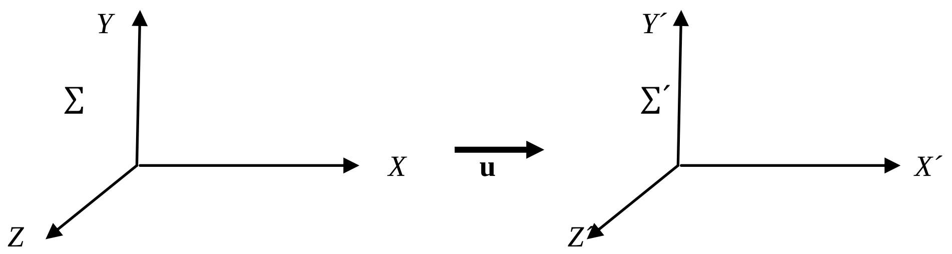
- The thermal time hypothesis is consistently sustained by the human induced feeling of time parameterization, thus giving sense a unique coherent flow of time.
- Consistent with all cosmological theories where there is a final for the Universe either in a finite or infinite steps. In any case, there is a mathematically equivalence of time as a physical fundamental magnitude under an identified quotient metric space, subset of the topological space ($\mathbb{C}$, *Tu*), where "*Tu*" is the Euclidean usual topology induced by Euclidean distances. Being $\mathbb{C}$ isomorphic to $\mathbb{R}^2$ as euclidean metric spaces. It is worth pointing out that even Conformal Cyclic Cosmology approach is compatible by this mathematical space configuration of the ontology of time [23], [24]. Cyclic periods of expansions and contractions of Cosmos are not excluded by this mathematical expression of time because equivalence relations identified only refers to circumstances when space-time has no

epistemological sense: before Cosmos existence and, if ever, beyond the end of Cosmos.

- The definition is unequivocally unique, taken into account the equivalency between the identified homeomorphisms, and the subsequence quotient space identified. The canonical element of this second quotient space identified provides us with the mathematical model of the magnitude "time" valid for all epistemological approaches in Science in general.

Now, let us consider the orthogonal group of the Lorentz Poincaré transformation of the Relativity. The so called semiorthogonal group, $O(3,1)$, subgroup of the general group $GL(4;\mathbb{R}^4)$ of all invertible 4x4 matrixes. As a reminder, be $A \in O(3,1) \leftrightarrow A^T \boldsymbol{\mathcal{E}} A = A \boldsymbol{\mathcal{E}} A^T = \boldsymbol{\mathcal{E}}$. Where $\boldsymbol{\mathcal{E}}$ in this case is diag(1,1,1,-1).

Be two reference systems ∑ y ∑′, with relative movement one to the other with uniform speed **u** in the x direction as shown in the schema below:



Schema of the movement proposed to facilitate comprehension of the analysis shown: a general u speed provides the same result complicating the mathematical formalism.

From generic algebraic undergraduate texts for Relativity [21], one can check that metrics satisfies:

$$ds^2=(dt)^2[c^2-(dx/dt)^2-(dy/dt)^2-(dz/dt)^2]=\text{const.}$$

The above expression for each reference system corresponds to:

$$(dt)^2[c^2-\sum_j\left(\frac{dx^j}{dt}\right)^2] = (dt')^2[c^2-\sum_j\left(\frac{dx'^j}{dt}\right)^2]$$

Suppose the system at rest is ∑ , as described in the schema above. Then,

$$(dt)^2c^2 = (dt')^2[c^2 - \boldsymbol{u}^2]$$

Thus, finally giving the famous expression of relativity of time, belonging always to S^1:

$$dt' = \frac{dt}{\sqrt{1-\beta^2}} \qquad \beta = \frac{|\boldsymbol{u}|}{c}$$

$$dt = \left|\left|dS^1(s)\right|\right|, being \left|\left|*\right|\right| \, modulus \, of \, dS^1(s) \, in \, \textstyle\sum \text{reference system.}$$

Thus, evidencing that our time definition is also consistent with relativistic semiorthogonal group, $O(3,1)$ for Lorentz Poincaré transformations.

In Epistemology, all the above immediately leads us to affirm:

- It does not matter what mathematical representation you are describing; Physics will be the same according to Lorentz-Poincaré transformation checked. This directly implies no sense for travels between different times. ALL ontologically are the same in the Universe: the one described by the identified quotient metric space, whose canonical element is proposed for simplicity the S^1 manifold.
- Direct implication is that Universe is unique. Physically, no travels between different Universes. According to thermal time hypothesis, consistent with our time definition, there is only a valid time flow. So, loops of time and travels between different times are non sense epistemology taking into account the consistency between the above hypothesis and our definition of "time".
- It is indeed consistent with the cosmological concept of "Time of Planck"[22], defined as "time from which the universe can exist as we know, with the current epistemology of Physics being applied. Mathematically, the parametric topological space is dense; in consequence, in the neighbourdhood of zero, $\varepsilon \underset{lim}{\longrightarrow} 0$, whatever the manifold 1-d is, we can identify the corresponding Planck Time. Also consistent with current theories about Cosmology description [3], [23], [24] based on Relativity background. For those, consistency is immediate due to geometrical arguments based on the homemorphism-based argumentation previously identified.
- Consistent with Heisenberg's uncertainty principle $\Delta t \Delta E \sim h$, being "t" parameter of time as 1-d manifold. This comes directly from the fact that γ_i^{-1}, for all i, provides directly "the operator time" $\hat{T} = \gamma_i^{-1}: Im(\gamma_i) \subset \mathbb{C} \rightarrow [0, \mathrm{a}) \subset \mathbb{R}$, where directly if we assume the set metrical space-time as defined in $\mathbb{R}^3 \oplus S^1$ (as

canonical element of the quotient space defined for time above), directly induced the definition of the eigenvector and the corresponding eigenvalue of "operator time", in Pauli's notation, as:

$$\hat{T}|Im(\gamma_i) >= t|Im(\gamma_i) >, t \; as \; parameter \; value \; in \; [a, b).$$

Where $\| \, Im(\gamma_i) > \|$ is defined as 1s.

This last point directly could open the door to the mathematical consistency of Relativity with Quantum Theory axioms from first and fundamental principles; thus, providing further support to research towards the "Single Field Theory" in Physics.

5. Conclusions

This work has shown a mathematical assessment, by topological analysis, of the ontological definition of the fundamental magnitude of time, being part of the mathematical configurational space of Cosmos in general. A formal definition unique and consistent with all epistemological fields in Science has been provided. The last eliminates the hypotheses of travels in time, thus eliminating definitively loops, by thermodynamics considerations supported by natural parameterization of time induced by human existence (ordering continuously facts related to existence). Additionally, a confirmation of compatibility with semiorthogonal group $O(3,1)$ of Lorentz-Poincaré transformations for Relativity has been checked. The universe, both finite and infinite based models for time description, can be mathematically conceived as unique with a single ontological definition of flow time. Finally, Cosmology could also assume the Axioms of Quantum Theories; thus, providing a further support towards "Single Field Theory" in Physics. Consequences in the theoretical limit of Physics Epistemology is in progress, though we have considered, keeping in mind the potential impact of this novel approximation to the ontology of time, to publish now our results up to now. Finally, by clearly splitting mathematical character of time, as a 1-d manifold (with boundary), and its natural parameterization provided by human feeling of time, we clearly indicate the way in Philosophy to fully understand the entity "space-time" in Science; thus, eliminating all cognitive misunderstandings among the various approaches to time entity human knowledge can do.

References

[1] Hilgevoord, J. (2005). Studies in History and Philosophy of Modern Physics, 36, 29-60.

[2] Hawking, S. Hertog, T. and Reall, H.(2000) Physical Review D 62 (4), 043501-043527.

[3] Jack, Ng Y. Christiansen, W A. and van Dam, H. (2003).The Astrophysical Journal Letters (The American Astronomical Society), 591 (2) 87-89.

[4] Borde, A. Guth, A H. and Vilenkin, A. (2003). Physical Review Letters 90, 151301-151304.

[5] Nielsen, H B. and Ninomiya, M. (2007). Prog.Theor.Phys.116, 851-863.

[6] Schwabl, F. (1992). Quantum Mechanics. Berlin :Springer Verlag, 37.

[7] Wangness, R.K. (1991). Electromagnetic Fields. N.York: John Wiley & Sons, 577-581.

[8] Pauli, W. (1933). Handbuch der Physik, 24, 83-86.

[9] Mandelstamm, I. T. L. (1945). Journal of Physics 9, 249-254.

[10] Kijowski, J. (1974). Reports on Mathematical Physics, 6 (3), 361-386.

[11] Galapon, E.A. (2002) Proceedings of the Royal Society of London, 458, 2671-2689.

[12] Holevo, A.S. (1985) Probabilistic and statistical aspects of quantum theory. Amsterdam: North-Holland Publishing Company.

[13] Rovelli, C. (1991). Phys. Rev. D 43 (2) 442-456.

[14] Rovelli,C. (1993) Class. and Quant. Grav. 10, 1549-1566.

[15] Connes, A. and Rovelli, C. (1994). Class.Quant.Grav. 11, 2899-2918.

[16] Schlosshauer, M. (2005). Rev.Mod.Phys.76, 1267-1305.

[17] Velavkin, V P. (2001) Progress in Quantum Elect. 25, 1-7.

[18] J.Azcárraga, J. (2005) "Revista de la Unión Iberoamericana de Sociedades de Física " 1, 35-53.

[19] Planck Collaboration (2018). Planck 2018 results. VI. Cosmological parameters. Arxiv:1807.06209v3 (2020).

[20] Crossley, M.D. (2010).Essential Topology. London: Springer Undergraduate Mathematics Series, cap.5.

[21] For instance, Golovina,L I. (1983). Linear algebra and some of its applications. Moscú: Mir, 224.

[22] Hawking, S. Hertog, T. and Reall, H. (2000). Physical Review D. 62, (4), 043501-043527; Hawking,S. and Hertog, T. Journal of High Energy Physics. (2018). Berlin: Springer Science & Bus. Med 4, 147.

[23] Penrose, R. (2007). Edimburgh: Procc. of EPAC 2006. Special contribution.

[24] Gurzadyan, V.G. and Penrose, R. (2013). , Eur.Phys.J. Plus 128, 22.