

# The measurement problem in quantum mechanics

Alessio Giuseppe Ferraioli<sup>1</sup>

Canio Noce<sup>2</sup>

## Abstract

In this paper, we discuss the importance of measurement in quantum mechanics and the so-called measurement problem. Any quantum system can be described as a linear combination of eigenstates of an operator representing a physical quantity; this means that the system can be in a superposition of states that corresponds to different eigenvalues, i.e., different physical outcomes, each one incompatible with the others. The measurement process converts a state of superposition (not macroscopically defined) in a well-defined state. We show that, if we describe the measurement by the standard laws of quantum mechanics, the system would preserve its state of superposition even on a macroscopic scale. Since this is not the case, we assume that a measurement does not obey to standard quantum mechanics, but to a new set of laws that form a “quantum measurement theory”.

**Keywords:** measurement theory; quantum mechanics; reduction postulate, quantum superposition.<sup>3</sup>

---

<sup>1</sup> Università degli Studi di Salerno, Salerno, Italy; a.ferraioli40@studenti.unisa.it.

<sup>2</sup> Dipartimento di Fisica “E. R. Caianiello”, Università degli Studi di Salerno, Salerno, Italy; cnoce@unisa.it.

<sup>3</sup> Received on April 29th, 2019. Accepted on June 25th, 2019. Published on June 30th, 2019.

doi: 10.23756/sp.v7i1.462. ISSN 2282-7757; eISSN 2282-7765. ©Ferraioli and Noce.

This paper is published under the CC-BY licence agreement.

## 1 Introduction

The act of measurement is a crucial point in the scientific method. It is the process by which we collect empirical information to formulate our hypothesis and build-up our models. Moreover, any physical quantity can be named as such only when a non-ambiguous measurement procedure is (at least in theory) defined. It is fair to say that measurement is one of the cornerstones of scientific progress *in toto*.

Nonetheless, until the last century, its importance has always been taken for granted, and its definition has been somewhat naïve, along the lines of “the process done by the experimenter in the laboratory with rulers, scales and such”. Then, in the XX century, quantum mechanics put the measurement process under the spotlight. The measurement serves as the bridge between the macroscopic world, that obeys the laws of classical physics, and the microscopic world, ruled by the counter-intuitive laws of quantum mechanics.

As stated by Bohr [10], “quantum mechanics occupies a very unusual place among physical theories: it contains classical mechanics as a limiting case, yet at the same time it requires this limiting case for its own formulation”. If we describe a macroscopic system with the laws of quantum mechanics, we quickly come to the paradoxical conclusion that an object could be in two or more different configurations at the same time. For example, a light bulb could be on and off at the same time; a football team could win and lose the same match; an unlucky cat could be dead and alive. This contradiction is called the “measurement problem”. To solve that, we assign a very peculiar role to measurement, defining a new set of rules known as the “quantum measurement theory”. A quantum object, such as an electron or an atom, evolves according to quantum mechanics (i.e. the Schrödinger equation) until a measurement is performed. At this point, the processes of quantum measurement theory come into play, translating the quantum state of the system in a macroscopic consequence of the measuring apparatus. This *ad hoc* set of rules can be easily unsettling: indeed, a macroscopic object is made up of smaller parts, molecules and atoms, behaving under the laws of quantum mechanics, so why would the system as a whole behave differently? Moreover, a measurement is not an elementary process, but it can be split into simpler interactions, eventually described by quantum mechanics: can we really center a theory around a process

so weakly defined? These and many more questions bother anyone who is faced with the study of quantum physics.

## 2 Physical quantities as operators and the wavefunction

Classical mechanics is both descriptive and predictive: if we know the initial conditions of an object and the forces acting upon it, we can define its trajectory. Mathematical difficulties aside, we can always say where it will be after any time. To do so, the instructions to follow are straightforward: let's suppose that we have a point object of mass  $m$ , and  $\mathbf{F}(\mathbf{r}, t)$  is the net force on it. Given the position and the momentum of the object at  $t = 0$ , we use Newton's 2<sup>nd</sup> law  $\mathbf{F} = m\mathbf{a}$  to study its motion. If we are dealing only with conservative forces, so the net force that can be described as the derivative of a potential energy function  $V(\mathbf{r}, t)$ , Newton's law of motion becomes  $-\nabla V = m\mathbf{a}$ . Therefore we can calculate the trajectory of the object, namely the position  $\mathbf{r}(t)$  and the momentum  $\mathbf{p}(t)$ .

In quantum mechanics, we cannot define these function for every  $t$ ; Newton's law is replaced by the **Schrödinger equation** [9]:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V\Psi, \quad (2.1)$$

where  $i$  is the imaginary unit, and  $\hbar$  is the Planck constant divided by  $2\pi$ . In this equation, we find neither the position of the particle nor the momentum; instead, we have  $\Psi(\mathbf{r}, t)$ , called the **wavefunction**. It is a continuous complex function of time and spatial variables, and it belongs to  $L^2(\mathbb{R}^3)$ , that is the set of all the square-integrable functions over the whole space. The wavefunction contains all the pieces of information quantum mechanics can offer. So how do we get them? How can we obtain from the wavefunction the physical quantities we are interested in, such as position, momentum, energy?

First of all, we assign to the wavefunction an abstract vector  $|\Psi\rangle$ , using Dirac notation. We then define a scalar product as follows:

$$\langle \Psi_1 | \Psi_2 \rangle = \int_{-\infty}^{+\infty} \Psi_1^* \Psi_2 dx \quad (2.2)$$

(it can be shown that this integral does not diverge as long as  $\Psi_1$  and  $\Psi_2$  are square integrable, so this product is well-defined). The set of all these abstract vectors constitutes a complete metric space, called **Hilbert space**. This is the space where the wavefunctions live.

For each dynamical quantity  $Q(\mathbf{r}, \mathbf{p})$  we define an operator  $\hat{Q}$  such that:

$$\langle Q \rangle = \int_{-\infty}^{+\infty} \Psi^* (\hat{Q}) \Psi d\mathbf{r}, \quad (2.3)$$

which we can write as a linear mapping onto the Hilbert space using Dirac notation:

$$\langle Q \rangle = \langle \Psi | \hat{Q} | \Psi \rangle. \quad (2.4)$$

This product gives the expectation value of  $Q(\mathbf{r}, \mathbf{p})$  over the state represented by  $\Psi$ : that means that if we had infinitely many copies of the same system, each described by the same wavefunction, and took a measurement of  $Q(\mathbf{r}, \mathbf{p})$  on each one, the average of the outcomes would be precisely  $\langle Q \rangle$ .

Since  $\langle Q \rangle$  represents the outcome of a measurement, it must be a real number; we can write:

$$\langle \Psi | \hat{Q} | \Psi \rangle = \langle \Psi | \hat{Q} | \Psi \rangle^* = \langle \hat{Q}^\dagger \Psi | \Psi \rangle = \langle \hat{Q} \Psi | \Psi \rangle. \quad (2.5)$$

It follows that any operator that represents a physical quantity, called an **observable**, must be self-adjoint. For example, we can assign to the physical quantity “position”  $\mathbf{r}$  the following observable:

$$\hat{\mathbf{r}} \equiv \mathbf{r},$$

therefore the expected value of position measured on the state  $\Psi$  is given by:

$$\langle \mathbf{r} \rangle = \int_{-\infty}^{+\infty} \Psi^*(\mathbf{r}) \Psi d\mathbf{r}. \quad (2.6)$$

### 3 The generalized statistical interpretation

In general, measurements made on identically prepared systems (i.e., with the same wavefunction) do not return the same result. However, a system can be in a defined state for an observable  $Q$ , in which a measurement always yields to the same outcome. In this case, the standard deviation is zero:

$$\begin{aligned} 0 = \sigma_Q^2 &= \langle (\hat{Q} - \langle Q \rangle)^2 \rangle = \langle \Psi | (\hat{Q} - \langle Q \rangle)^2 | \Psi \rangle = \langle (\hat{Q} - \langle Q \rangle) \Psi | (\hat{Q} - \langle Q \rangle) \Psi \rangle \\ &= |(\hat{Q} - \langle Q \rangle) \Psi|^2. \end{aligned}$$

The only vector which has norm zero is the null vector, so:

$$(\hat{Q} - \langle Q \rangle) | \Psi \rangle = 0 \quad \Leftrightarrow \quad \hat{Q} | \Psi \rangle = \langle Q \rangle | \Psi \rangle.$$

If  $| \Psi \rangle$  is an eigenstate of  $\hat{Q}$ , a measurement will produce the outcome  $\langle Q \rangle$  (the corresponding eigenvalue) with certainty.

We can extend this result, with the postulate known as the **generalized statistical interpretation**: if we measure the observable  $\hat{Q}$  on a system described by the state  $| \Psi \rangle$ , we are sure to get one of the eigenvalues of  $\hat{Q}$ . The probability of getting a specific eigenvalue  $\lambda$  is equal to the square norm of the  $\lambda$ -component of  $| \Psi \rangle$ , with respect to the orthonormal basis of eigenstates of  $\hat{Q}$ . To ensure this postulate is meaningful, the eigenstates must generate the whole space; this is undoubtedly the case for finite-dimensional spaces, but it is not a trivial question if we are dealing with infinite-dimensional ones. We will then consider an observable a valid one only if its eigenstates fulfil this condition.

The eigenvalues spectrum of an operator can be discrete or continue. If the spectrum is discrete, we can label the eigenvalues with a discrete index  $n$ :

$$\hat{Q} | e_n \rangle = \lambda_n | e_n \rangle, \quad \text{with } n = 1, 2, 3, \dots,$$

where vectors  $| e_n \rangle$  form an orthonormal basis of eigenvectors (the fact that  $\hat{Q}$  is self-adjoint guarantees the existence of such basis, as stated by the spectral theorem). Any state  $| \Psi \rangle$  can be written as:

$$| \Psi \rangle = \sum_{n=1}^{\infty} c_n | e_n \rangle. \tag{3.1}$$

The coefficients  $c_n$  are complex numbers that can be computed using the orthonormality of the basis:

$$c_n = \langle e_n | \Psi \rangle. \quad (3.2)$$

Therefore, the probability that a measurement will give a specific eigenvalue  $\lambda_n$  is:

$$|c_n|^2 = |\langle e_n | \Psi \rangle|^2. \quad (3.3)$$

Alternatively, if the spectrum is continuous, we label the eigenvalues with a real variable  $k$ :

$$\hat{Q}|e_k\rangle = \lambda_k|e_k\rangle, \quad \text{with } -\infty < k < +\infty.$$

The eigenfunctions  $|e_k\rangle$  are not normalizable, but satisfy the following, which is a sort of an orthonormality condition:

$$\langle e_k | e_l \rangle = \delta(k - l), \quad (3.4)$$

where  $\delta(k - l)$  is a Dirac delta function. In this case, we can write a generic state  $|\Psi\rangle$  not as a sum but as an integral:

$$|\Psi\rangle = \int_{-\infty}^{+\infty} c_k |e_k\rangle dk. \quad (3.5)$$

Similarly, we find the coefficients  $c_k$ :

$$c_k = \langle e_k | \Psi \rangle \quad (3.6)$$

and probabilities:

$$|c_k|^2 = |\langle e_k | \Psi \rangle|^2. \quad (3.7)$$

Therefore, we can write the wavefunction as a linear combination of eigenstates of a specific operator. Each one represents a possible state in which the system can be found by a measurement, with a coefficient linked to the probability that a particular state will occur. In general, a wavefunction can be written in many ways, with respect to the basis of eigenstates of several observables; those eigenstates are all and only the possible states in which the system can be found when we measure that quantity. The wavefunction mathematically expresses the concept of **quantum superposition** of states: a physical system can always be described by the sum of two or more different states, and vice-versa the sum of two or more different states is still a quantum state of the system. Quantum

superposition is a direct result of the linearity of the Schrödinger equation, which is a consequence of the principle of relativity. In Dirac's words [6]:

“The non-classical nature of the superposition process is brought out clearly if we consider the superposition of two states, A and B, such that there exists an observation which, when made on the system in state A, is certain to lead to one particular result, a say, and when made on the system in state B is certain to lead to some different result, b say. What will be the result of the observation when made on the system in the superposed state? The answer is that the result will be sometimes a and sometimes b, according to a probability law depending on the relative weights of A and B in the superposition process. It will never be different from both a and b [i.e., either a or b]. The intermediate character of the state formed by superposition thus expresses itself through the probability of a particular result for an observation being intermediate between the corresponding probabilities for the original states, not through the result itself being intermediate between the corresponding results for the original states.”

## 4 The measurement problem

The superposition principle states that a wavefunction can be written as a sum of states, each one representing a different physical situation. This peculiar aspect of quantum theory made possible understanding many phenomena, such as the double slit experiment: the wavefunction of the incident particle carries both the state in which the particle goes through the first slit and the state in which the particle goes through the second slit. The coexistence of two macroscopically incompatible states is what makes possible explaining this experiment, famous for being one of the first to undermine the foundations of classical physics. Even if it led to inestimable development of both theoretical and experimental physics, this approach hides an insidious complication concerning the act of measurement. If we admit the possibility of superposition of states, we occur in a series of contradictions known as the “measurement problem”. Let us see in detail what it is about.

Let us suppose that we have a microscopic object, initially described by wavefunction  $\phi_i$ . The object is monitored by a macroscopic measuring apparatus with initial wavefunction  $\psi_i$ , in order to measure a physical quantity represented by the operator  $\hat{Q}$ . Let  $\alpha_n$  be the eigenstates of  $\hat{Q}$ :

$$\hat{Q}\alpha_n = A_n\alpha_n, \tag{4.1}$$

These functions spawn the entire wavefunction space, and we suppose that the system is initially (before any measurement) in an eigenstate of  $\hat{Q}$ , for example:

$$\phi_{i_1} = \alpha_1$$

or maybe:

$$\phi_{i_2} = \alpha_2.$$

At the end of the measurement process, the measured system will be described by a new wavefunction,  $\phi_{f_1}$  or  $\phi_{f_2}$ , depending on the initial state, and the measuring apparatus will be described by, respectively,  $\psi_{f_1}$  or  $\psi_{f_2}$ . The product of the two functions gives the wavefunction of the whole system (microscopic object + macroscopic apparatus). We can represent a measurement schematically, as follows:

$$\alpha_1; \psi_i \rightarrow \phi_{f_1} \psi_{f_1} \tag{4.2a}$$

$$\alpha_2; \psi_i \rightarrow \phi_{f_2} \psi_{f_2}. \tag{4.2b}$$

We must take into account some properties of  $\phi_{f_i}$  and  $\psi_{f_i}$ . In order to perform a consistent and useful measure, the functions  $\psi_{f_i}$  must express that the apparatus registered an unambiguous result and produced an outcome accordingly. That means the apparatus must be in a well-defined macroscopic state, univocally linked to the value of the measured quantity, so the measurement has meaning. For instance, if the apparatus is a screen where a chemical emulsion produces a black dot when it detects a particle, the point where the screen turns black must reflect the initial state of the measured system. Therefore, we must be able to interpret the apparatus left in the state  $\psi_{f_i}$  as a result of the measured quantity, being  $A_i$  (the eigenvalue corresponding to  $\alpha_i$ ), without ambiguity. We cannot say much about  $\phi_{f_1}$  or  $\phi_{f_2}$ ; there is no particular reason they should be linked to the initial states because the measurement can alter the system in many ways. In the above example concerning the chemical emulsion, a particle after the measurement has an entirely different wavefunction because of the collision with the screen.

This schematic analysis of a measurement is simple and straightforward; however, if the object is not initially in an eigenstate  $\alpha_i$  of the operator  $\hat{Q}$  we

*The measurement problem in quantum mechanics*

have a much more complicated situation. Let us suppose that the initial wavefunction is a linear combination of eigenstates, for instance:

$$\phi_i = \frac{1}{\sqrt{2}}(\alpha_1 + \alpha_2). \quad (4.3)$$

Because of the linearity of the Schrödinger equation, the final state will also be represented by a linear combination. Schematically, the measurement is as follows:

$$\frac{1}{\sqrt{2}}(\alpha_1 + \alpha_2); \psi_i \rightarrow \frac{1}{\sqrt{2}}(\phi_{f_1}\psi_{f_1} + \phi_{f_2}\psi_{f_2}). \quad (4.4)$$

In the general case, in which:

$$\phi_i = \sum_n c_n \alpha_n, \quad (4.5)$$

the measurement is:

$$\phi_i; \psi_i \rightarrow \sum_n c'_n \phi_{f_n} \psi_{f_n}. \quad (4.6)$$

Unlike the previous case, where the system is in a well-defined state after the measurement (4.2), now the apparatus is left in a superposition of many states. Each state represents a different macroscopic situation, in which the apparatus produced different outcomes, like  $A_1$  or  $A_2$ . This contradicts the obvious fact that a measurement leads to a specific result – indeed this is what we expect from a good measure. If we apply Schrödinger equation to the measurement process, as we would with any physical process consisting of elementary interactions obeying quantum mechanics, we come to a paradoxical conclusion: if the measured object is in a superposition of states, the system object + apparatus will be in a superposition of states as well, because of the linearity of the equation. To imagine a superposition of macroscopic states is such a silly thing that the most famous thought experiment about it, Schrödinger's cat, is part of popular culture.

As we can see, there is a conflict between the mere linearity of the Schrödinger equation and the basic fact that a measurement should lead to a

clear outcome. This conflict is the notorious **measurement problem** of quantum physics. It seems like a physical object behaves in two different ways, mutually exclusive. On the one hand, we have the proper dynamic evolution expected from standard quantum mechanics (i.e., the Schrödinger equation), and on the other, we have a non-linear and non-reversible process any time we make a measurement. While the former preserves the quantum superposition, the latter converts a superposition of states in a “classical” well-defined single state. The measurement seemingly does not obey Schrödinger equation, but it is governed by a distinct theory, aptly named “**quantum measurement theory**”. Some authors, like Peres, doubt that such a theory is necessary since a measurement is not a primary process and is not strictly defined; moreover, a measurement is made up of simpler physical interactions that do obey quantum mechanics [11]:

“[...] there can be no quantum measurement theory—there is only quantum mechanics. Either you use quantum mechanics to describe experimental facts, or you use another theory. A measurement is not a supernatural event. It is a physical process, involving ordinary matter, and subject to the ordinary physical laws.”

Still, if we apply the quantum formalism to a measurement we do not get a defined outcome. As we will see in the next paragraphs, many possible answers have been given to what a measurement theory would look like; yet no interpretation has been globally accepted or has been present without significant flaws or drawbacks. Nonetheless, their contribution is priceless, since they made “working” with quantum mechanics possible, without lingering too much on unresolved questions and not being able to do anything.

## 5 Measurement of the first and the second kind

We did not pose any particular condition on the final wavefunction of the apparatus ( $\psi_f$ ) nor of the object ( $\phi_f$ ), except that  $\psi_f$  should be linked in some meaningful way to  $\phi_f$ , in order to perform a consistent measurement. To find what these functions look like is the job of the measurement theory. We would like to know how the final state of the object depends on its initial state or, equivalently, how  $\psi_f$  is linked to  $\phi_f$ . These questions are inherently connected to the question of repeated measurements: what happens if briefly after a measurement we perform a second identical one? That is, what can a measurement reveal about the function  $\phi_f$ ?

There is not a standard answer to these questions, but it strongly depends on the specific measurement procedure. For instance, in many cases the result of a hypothetical repeated measurement is trivial: if an electron collides with a screen, forming a black dot in order to measure its position, it is no more

## *The measurement problem in quantum mechanics*

available for a second measurement; even when a repeated measurement can be performed, it may give no useful information. We can think of the following example: measurement of momentum on a neutron can be achieved by completely stopping the neutron with a series of collisions and observing the recoil protons. The particle is available for a second measurement, but its momentum has become zero independently of  $\phi_i$ .

Finally, there are situations where a second measurement can be made, and the final state is linked in a relevant way to the initial state. Let us consider a Wilson cloud chamber [2]: an  $\alpha$ -particle passes through a supersaturated vapour of water, interacting with its particles with the formation of a black mark. Since the blackening is macroscopic, we can think of it as an approximate measurement of position – indeed, compared to the scale of the particle it carries an enormous uncertainty. Because of this lack of precision on the position, the uncertainty on the momentum remains small. Therefore, we can consider the measure causing essentially no perturbation to the momentum, which remains constant. For this reason, a black track of the passage of the particle forms in the chamber, made up of several consecutive position measurements. This is an example of measurement where  $\psi_f$  and  $\phi_f$  are clearly related: the blackening of a specific macroscopic point (described by  $\psi_f$ ) happens because at that moment the particle initial wavefunction  $\phi_i$  is localized in that point. Since the measurement does not alter the motion, we can conclude that the particle wavefunction after the first blackening  $\phi_f$  is located in the same place with the same momentum.

A measurement as above, where the final state of the system gives information to the initial state because they are directly connected, is called **measurement of the first kind**. Conversely, if the final state depends entirely on the measurement procedure and it is not linked in a significant way to the initial wavefunction we are dealing with a **measurement of the second kind**, as in the screen where an electron collides or the momentum measurement on a neutron that stops the particle. In a measurement of the second kind, the information about the initial state is lost, wiped out by the measurement. From now on we will speak mostly of measurements of the first kind, or “moral” measurements [2].

In addition to the Wilson chamber, we can think of some other practical situations that fit into the category of measurement of the first kind. Let us consider a spin- $\frac{1}{2}$  particle at high speed that goes through a Stern-Gerlach apparatus, a channel with one entry and two exits, in which there is a magnetic field along the  $z$ -axis. The magnetic field will deviate the particle towards one exit or the other, depending on its spin orientation. In both cases, it exits a detector is placed to indicate where the particle passes. In this apparatus, we directly measure the position, but doing so we obtain information on the spin of the particle: we perform an indirect but well-defined measurement of the third

component of the spin  $s_z$ . Since the measurement does not affect at all the measured quantity, we can reasonably assume that a second measurement on the same particle will lead to the same result – it is easy to connect a pair of Stern-Gerlach apparatus at the exits of the first and confirm this is the case. We can expand this idea by connecting  $N$  pairs of apparatus, thus building a device that performs the spin measurement  $N$  times and gets the same result every time.

Another example would be the measurement of radioactive decay. We surround a radioactive nucleus with detectors, in order to be 100% sure to detect an eventual decay. If at some point the detectors signal the nucleus is decayed, a successive measurement after the first would still show that the nucleus is indeed decayed. Vice-versa, if under inspection the nucleus is found to be non-decayed, then a second measurement made shortly afterwards will return the same outcome.

By these simple representations of real measurements, we understand by intuition and common sense that a repeated measurement should give the same result (provided that we take it after a time so small the system would not be altered by the physical interactions it is subject to). Dirac [6] held this concept in high regard, asserting it is a matter of physical continuity, a requirement for any measurement theory. While it is undoubtedly desirable, is there a formal justification to such a necessity? If we think about a measurement of position, special relativity implies that repeated measurements must yield to the same result. Otherwise, a particle would be able to travel a finite distance in zero time. In reality, if we observe a particle in a point  $P$ , a second measurement made after a time  $t$  could find the particle to be everywhere within a distance of  $ct$  from  $P$ . We should remark that an exact measurement of position causes a total uncertainty on the momentum; therefore we limit the precision of a position measurement to a range of points where we can observe the particle. Doing so, we reduce the momentum uncertainty and expect that a second measurement performed after a small enough time falls within the same range. For other observables, it is not so clear why repeated measurement should give the same result, and those arguments could be as well called “moral” [2].

## 6 Possible solutions to the measurement problem

The measurements of the first kind preserve information about the initial state of the system. If we want to measure the observable  $\hat{Q}$  and the measured system is in the eigenstate  $\alpha_1$ , the measurement is:

$$\alpha_1; \psi_i \rightarrow \alpha_1 \psi_f. \quad (6.1)$$

### *The measurement problem in quantum mechanics*

A measurement of the first kind does not alter the system, maintaining the eigenstate. As we mentioned, if the initial wavefunction is a superposition of states, for instance  $\phi_i = \frac{1}{\sqrt{2}}(\alpha_1 + \alpha_2)$ , we face the measurement problem:

$$\frac{1}{\sqrt{2}}(\alpha_1 + \alpha_2); \psi_i \rightarrow \frac{1}{\sqrt{2}}(\alpha_1 + \alpha_2)\psi_f. \quad (6.2)$$

The most common approach to solving this problem involves postulating the existence of a **reduction of the wavefunction**. It is a process that turns a state of superposition (not macroscopically defined) into an eigenstate, whenever we make a measurement. It is irreversible and non-linear; therefore, it does not obey the Schrödinger equation. The reduction postulate states that measurements always leave the system in a well-defined state, “reducing” the wavefunction to only one of the eigenstates that constitute the superposition. Let us suppose the measurement outcome is  $A_2$ , the eigenvalue relative to  $\alpha_2$ ; the reduction of the wavefunction will result in:

$$\frac{1}{\sqrt{2}}(\alpha_1 + \alpha_2); \psi_i \rightarrow \alpha_2\psi_{f_2}, \quad (6.3)$$

or, in general:

$$\sum_n c_n \alpha_n; \psi_i \rightarrow \alpha_k \psi_{f_k}. \quad (6.4)$$

This process is entirely different from the linear evolution predicted by “standard” quantum mechanics. Until we observe it, the system follows the expected linear evolution; when we perform a measurement, returning the outcome  $A_k$ , the wavefunction is reduced to the state  $\alpha_k$ . When the measurement ends, the system goes back to the Schrödinger dynamics, causing it to return in a state of superposition. However, if we performed a second measurement immediately after the first one, we would undoubtedly get  $A_k$ . The reduction postulate ensures that repeated measurements yield to the same result.

Standard quantum mechanics predicts a chain of superposition: the microscopic object passes his superposition state on the macroscopic measurement apparatus, which in turn passes it on the observer’s sensory organs, to his brain and so on. The reduction postulate asserts that at some point the chain breaks and the superposition is lost. How the reduction process exactly works is debated, so there are several interpretations. The most accredited hypothesis is that it happens as we pass from a microscopic object to a macroscopic one. This assumption somewhat reminds of Bohr’s “complementarity” [10], that consists of refusing to assign a wavefunction to a

macroscopic system. In the same way, as Bohr does, we are posing an arbitrary cut between the microscopic and the macroscopic world. For this reason, it can not be regarded as a rigorous solution to the measurement problem, but it has shown to be an efficient practical workaround.

In Von Neumann's [12] original formulation, the reduction process instead happens at the level of the observer's "mind", that interrupts the concatenations of superposition thanks to its introspection abilities. Von Neumann's theory also presents an interactionist aspect: the observation alters the wavefunction, thus shaping the physical reality.

We should bear in mind that, if we admit the existence of a reduction process, we are stating that the wavefunction is subject to two separate evolutions: the Schrödinger dynamics and the reduction process. The strongest criticism to this interpretation is that a measurement is not a fundamental process, but it is made up of simpler interactions described by the Schrödinger equation, so it can not be regarded as a primitive notion of a theory. Moreover, what constitutes a measurement is vaguely defined, although the naïve approach [8] ("a measurement is that thing an experimenter does in the laboratory with scales, rulers, spectrometers and such") is a good starting point from a pragmatical perspective.

The reduction postulate is not the only possible solution to the measurement problem. Despite what we have said up to now, many theories assume that the superposition state (4.6) can represent a macroscopic outcome of a measurement, denying the necessity of a reduction process. In a theory pioneered by Everett [5], known as **many world interpretation**, any possible outcome of a measurement happens simultaneously in parallel universes that do not communicate with each other. When we perform a measurement, the physical reality branches, forming a parallel universe for any possible outcome. In this way, there is no need for a wavefunction reduction, since any possible state of the superposition occurs in a different world. Everett's theory has internal consistency and is supported by many authors. One of its flaws is that it fails to rigorously discriminate what should be considered a measurement and what not. Another drawback is that it is not clear how "deep" the branching of the different realities is: if we consider an observable with a continuous eigenstate spectrum, we are not able to say how distant two outcomes should be to generate two different realities.

Another remarkable theory is the Ghirardi-Rimini-Weber theory [7], also known as the **spontaneous reduction theory**. It consists of a correction on the Schrödinger equation itself, throwing in a non-linear term, which causes a process analogous to reduction. The difference is that this process, which we may call a "spontaneous reduction", does not happen when a measurement is made but can happen at any time. The probability of a spontaneous reduction is proportional to the size of the system, so it is very likely to happen for

macroscopic objects. In fact, in the macroscopic world, this probability is so big that a superposition of states cannot exist for more than an infinitesimal fraction of a second. Conversely, the occurrence of a reduction on a microscopic scale is so small that a quantum system can retain its state of superposition virtually forever. This theory has been modified many times since its formulation, mainly to take relativity into account and to fix several incongruences.

Some other theories, like **ensemble theories**, consider the superposition state with a statistical approach, as a collection of alternatives that occurs on a set of identically prepared system; the wavefunction is not suitable for describing a single system, such as a single particle, and indeed it has no physical meaning, since it does not represent a physical property of the particle but is an abstract statistical function.

Another approach is that of non-local hidden variables theories, such as the **theory of incomplete measurements** [1], postulating that there exist some hidden quantities we cannot access that regulate the measurement process. We can think of it as a realist point of view on the measurement, in contrast to the orthodox position, according to which the wavefunction is the complete information on a system.

**Decoherence theories** state that, when a quantum system is thermodynamically paired with the environment, it loses information; this happens because the surrounding interferes with the phase of the wavefunction causing a mixed state, a statistical mixture of pure states. That can explain the observation of the wavefunction reduction, which is an effect of the loss of coherence, while the hypothetical “universal wavefunction” (i.e., the wavefunction describing the entire universe) always remains coherent.

Recently, interpretations focused around quantum information became increasingly popular. For example, according to the **“it from bit” theory** [14], quantum mechanics describes the observer’s experience of reality, but not reality itself: in this interpretation, physical reality is an effect (perhaps a consequence) of information and not the other way around.

## 7 Conclusions

We saw how the laws of standard quantum mechanics unavoidably lead to the measurement problem. The possible solution ideas presented above are only some of the many possible interpretations of quantum mechanics and the role of the measurement, each with its strengths and weaknesses. So far, the question is quite open. Nonetheless, it should be noted that there is no experimental way to discriminate between the interpretations. For this reason, the question seems to be rather philosophical or ontological than physical. From a scientific perspective, is it meaningful to search for an answer that we cannot empirically prove? Most importantly, such a conjecture would not be

disprovable either, because there is no way to distinguish between the alternatives.

Even if, throughout this paper, we have used the words “interpretation” and “theory” interchangeably, they really are different, if we want to be careful. The ideas presented as possible solutions to the measurement problem are indeed interpretations of the measurement theory, and not theories themselves since the prerogative of a scientific theory is that it can be disproved by empirical facts. These interpretations aim to provide a “mental picture” to explain the same consequences. They all come to the same conclusions: they agree that a wavefunction reduction (apparent or real that is) exists, the wavefunction has a statistical interpretation (whether the wavefunction itself is a real physical property or just a mathematical tool) and so on. We could say that, if a scientific theory asks “how?”, those interpretations pursue the *why*. That surely is an ontological argument, trying to point out where our perception of reality ends and where the real thing starts, if such a thing even exists.

Therefore, many do not bother much about it and gladly embrace a pragmatic instrumental position: the statistical nature of the wavefunction and the measurement outcomes are cold facts, and we do not care about why that is so. We know how to calculate probabilities from the wavefunction, we know how a wavefunction evolves, we know what to expect from a measurement, and this is everything we can hope to know. Quantum measurement theory is successful in describing the experimental result we get in the laboratory. We cannot know why, but there is no point in trying to ask this question. What we know is what we need to work with quantum mechanics properly. It is possible that in the future a new theory could entirely replace quantum mechanics, making those questions meaningless; at the end of the day, quantum theory is still the most logical description of nature we have, so we should not feel stuck but pave the way for research and progress one step at a time. As Bell said [2]:

“This progress is made in spite of the fundamental obscurity in quantum mechanics. Our theorists’ stride through that obscurity unimpeded... sleepwalking? The progress so made is immensely impressive. If it is made by sleepwalkers, is it wise to shout 'wake up'? I am not sure that it is. So I speak now in a very low voice.”

## **References**

- [1] Aerts D. (1986). A possible explanation for the probabilities of quantum mechanics, *Journal of Mathematical Physics*, 27.
- [2] Bell J. S. (1987). *Speakable and Unspeakable in Quantum Mechanics*, Cambridge: Cambridge Univ. Press.
- [3] Bohm D. (1952). A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden Variables' I, *Physical Review*, 85.
- [4] Cohen T. C., Diu B., Laloë F. (1997). *Quantum Mechanics: I*, Weinheim: Wiley-VCH.
- [5] De Witt B. S., Graham N. (1973). *The Many-Worlds Interpretation of Quantum Mechanics*, New Jersey: Princeton University Press.
- [6] Dirac P. A. M. (1974). *The principles of Quantum Mechanics*, 2nd edition, Oxford: Clarendon Press.
- [7] Ghirardi G. C., Rimini A., Weber T. (1986). Unified dynamics for microscopic and macroscopic systems, *Physical Review D*, 24.
- [8] Griffiths D. J. (1994). *Introduction to Quantum Mechanics*, New Jersey: Prentice Hall.
- [9] Home D., Whitaker M. A. B. (1997). A conceptual analysis of Quantum Zeno; paradox, measurement, and experiment, *Annals of Physics* 258, 237-285.
- [10] Landau L. D., Lifshitz E. M. (1958). *Quantum Mechanics. Non-Relativistic Theory*, Oxford: Pergamon.
- [11] Peres A. (1993). *Quantum Theory: Concepts and Methods*, Dordrecht: Kluwer.
- [12] von Neumann J. (1955). *The Mathematical Foundations of Quantum Mechanics*, New Jersey: Princeton University Press.
- [13] Wigner E. P. (1967). *Remarks on the Mind-Body Question, Symmetries and Reflections*, Indiana: Indiana University Press.

Alessio Giuseppe Ferraioli and Canio Noce

[14] Zurek W. H. (1990). Complexity, Entropy, and the Physics of Information, Massachusetts: Addison-Wesley.