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Independent Restrained k - Rainbow Dominating Function

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Abstract

Let G be a graph and let f be a function that assigns to each vertex a set of colors chosen from the set {1, 2…, k} that is f: V(G) → P [1, 2, …, k]. If for each vertex v ∈ V(G) such that f(v) = ∅ .we have $\bigcup_{u \in N[v]} f(u) = \{1, 2, .., k\}$ then f is called the k – Rainbow Dominating Function (KRDF) of G. A k – Rainbow Dominating Function is said to be Independent Restrained k - Rainbow Dominating function if (i)The set of vertices assigned with non – empty set is independent. (ii)The induced subgraph of G, by the vertices with label ∅ has no isolated vertices. The weight w(f) of a function f is defined as w(f) = $\sum_{v \in V(G)} |f(v)|$ .The Independent Restrained k – Rainbow Domination number is the minimum weight of G. In this paper we introduce Independent Restrained k – Rainbow Domination and find for some graphs

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1. Introduction

Domination in graphs originates from location problems in operations research. As a variation of domination in graphs, rainbow domination was introduced by Bresar et al. [2]. Shao et al. [7] gave bounds for the k – rainbow domination number on an arbitrary graph. Hao et al. [4] studied the k – rainbow domination number of directed graphs. Independent rainbow domination was introduced by Zehui Shao et al. [8]. Amjadi et al. [1] was investigated the rainbow restrained domination number. In this paper we introduce Independent Restrained k – Rainbow Domination and find for some graphs.

2. Preliminaries

A graph G consists of pair(V(G), E(G)) where V(G) is a non-empty finite set whose elements are called points or vertices and E(G) is a set of unordered pair of distinct elements of V(G). The elements of E(G) are called lines or edges of the graph G. For any vertex u in G, the open neighbourhood of u, is denoted by N(u) is the set of vertices adjacent to u and the closed neighbourhood of u, is denoted by N[u] = N(u) ∪ {u}. A set of vertices in a graph is said to be an independent set of vertices or simply an independent if no two vertices in the set are adjacent. The splitting graph of G is denoted by S[G]For each vertex v of a graph G, take a new vertex v' and join v' to all vertices adjacent to v in G.The corona product of two graphs G and H is defined as the graph obtained by taking one copy of G and $|V(G)|$ copies of H and joining the i^{th} vertex of G to every vertex in the i^{th} copy of H. It is denoted as $G \circ H$. Let G be a graph and let f be a function that assigns to each vertex a set of colors chosen from the set {1, 2, …, k} that is f : V(G) → P [1,2,…,k]. If for each vertex v∈ V(G) such that f(v) = ∅ .we have $\bigcup_{u\in N[v]} f(u) = \{1,2,..,k\}$ then f is called the k – Rainbow Dominating Function (KRDF) of G. A k – Rainbow dominating function is called independent k – Rainbow Domination if vertices assigned with non – empty sets are pairwise non-adjacent. A k – Rainbow dominating function is called Rainbow Restrained Domination function if vertices assigned with empty sets has no isolated vertex.

3. Main Results

Definition 3.1. Let G be a graph and let f be a function that assigns to each vertex a set of colors chosen from the set {1, 2, …, k} that is f : V(G) → P [1,2,…,K]. If for each vertex v∈ V(G) Such that f(v) = ∅ . we have $\bigcup_{u\in N[v]} f(u) = \{1,2,..,k\}$ then f is called the k – Rainbow Dominating Function (KRDF) of G. A k – Rainbow Dominating Function is said to be Independent Restrained k- Rainbow Dominating function if

i. The set of vertices assigned with non – empty set is independent.
ii. The induced subgraph of G, by the vertices with label ∅ has no isolated vertices.

The weight w(f) of a function f is defined as w(f) = $\sum_{v\in V(G)}|f(v)|$. The Independent Restained k – Rainbow Domination number is the minimum weight of G.

Observation 3.2.

1. Let u be a pendant vertex to v. If one of the vertex u and v is assigned {1, 2, … k} then the other may be assigned ∅. (i.e) if the end vertex is assigned {1, 2, … k} then the support vertex may be assigned ∅
2. Always $k \leq \gamma_{irkr} \leq nk$
3. If a graph G has a clique as a subgraph, then one of the vertices of the clique should be labelled {1, 2, … k} and all the remaining vertices are labelled ∅.

Theorem 3.3. Let G be a graph and let v be a full degree vertex in G. Suppose G – {v} has no isolated vertex then $\gamma_{irkr}(G) = k$

Proof:

Define f: V(G) → P [1,2,…,k] by

$$f(x) = \begin{cases} \{1,2,\dots,k\} if x = v \\ \emptyset \quad \text{otherwise} \end{cases}$$

Then clearly f is an independent restrained rainbow dominating function.

Further w(f) = k

As $\gamma_{irkr}(G) \geq k$, f is a minimum independent restrained k rainbow dominating function. Therefore, $\gamma_{irkr}(G) = k$

Corollary 3.4: Complete graph, Wheel graph, Fan graph, Flower graph all have independent restrained k rainbow domination number is equal to k.

Theorem 3.5: The Independent Restrained k- Rainbow Dominating function exists for path graph P_n if and only if $n \equiv 1\, mod\, 3$ and then $\gamma_{irkr}(P_n) = \left\lceil \frac{n}{3} \right\rceil$ k, where $n \equiv 1\, mod\, 3$

Proof: When $n \equiv 0\, or\, 2\;\; mod\, 3$, the graph G does not admit any Independent Restrained k Rainbow dominating function. Since any k Rainbow Dominating function f fails to satisfy either Independent or Restrained condition.

let $v_1, v_2, v_3, \dots v_n$ be the vertices of the path graph.

Define f: V(G) → P [1, 2, …, k] by

$$f(x) = \begin{cases} \{1,2,\dots,k\} if x = v_i \\ where i \equiv 1\, mod\, 3 \;\; and \;\; 0 \leq i \leq n \\ \emptyset \quad otherwise \end{cases}$$

Then clearly f is an Independent Restrained k Rainbow Dominating function.

Therefore, w(f) = $\sum|f(v_i)|$ = $|f(v_1)| + |f(v_4)| + |f(v_7)| + \cdots + |f(v_n)|$

Therefore, $\gamma_{irkr}(P_n) = \left\lceil \frac{n}{3} \right\rceil$ k

Theorem 3.6: The Independent Restrained k- Rainbow Dominating function exists for cycle graph C_n if and only if $n \equiv 0 \; mod\; 3$ and then $\gamma_{irkr}(C_n) = \frac{nk}{3}$, $n \equiv 0 \; mod\; 3$

Proof: When $n \equiv 1 \; or \; 2 \; mod\; 3$, the graph G does not admit any Independent Restrained k Rainbow dominating function. Since any k Rainbow Dominating function f fails to satisfy either Independent or Restrained condition.

let $v_1, v_2, v_3, \dots v_n$ be the vertices of the cycle graph.

Define f: V(G) → P [1, 2, …, k] by

$$f(x) = \begin{cases} \{1,2,\dots,k\} if x = v_i \\ where i \equiv 0 \; mod\; 3 \; and \; 0 \le i \le n \\ \emptyset \quad otherwise \end{cases}$$

Then clearly f is an Independent Restrained k Rainbow Dominating function.

Therefore, w(f) = $\sum |f(v_i)| = |f(v_3)| + |f(v_6)| + |f(v_9)| + \dots + |f(v_n)|$

Therefore, $\gamma_{irkr}(C_n) = \left\lceil \frac{n}{3} \right\rceil$ k

Theorem 3.7: Let H_n be the helm graph obtained from the wheel by attaching a pendant vertex to each rim vertex. Then $\gamma_{irkr}(H_n)$ = k + n - 2 for n ≥ 4

Proof: Let $v_1, v_2, v_3, \dots v_{n-1}$ be the rim vertex. Let u be the apex vertex. Let $w_1, w_2, \dots w_{n-1}$ such that w_i is adjacent to v_i for $1 \le i \le n-1$.

Define f: V(G) → P [1, 2, …, k] by

$$f(x) = \begin{cases} \{1\} if x = w_i \text{ where } 1 \le i \le n-1 \\ \{2,3,\dots,k\} if x = u \\ \emptyset \quad otherwise \end{cases}$$

Obviously, every vertex v with labelled $\emptyset$ satisfies the condition $\bigcup_{u \in N[v]} f(u) = \{1,2,\dots,k\}$

∴f is a k Rainbow Dominating Function.

Let S be the set of vertices assigned $\emptyset$ labelled then S = $\{v_1, v_2, v_3, \dots v_{n-1}\}$. Here <S> has no isolated vertices.

Let S' be the set of vertices assigned non empty labelled. Then S' = $\{u, w_1, w_2, \dots w_{n-1}\}$ is Independent.

Then clearly f is an Independent Restrained k Rainbow Dominating function.

∴ w(f) = $\sum (|f(u)| + |f(w_i)|)$ where i = 1 to n – 1

Therefore, $\gamma_{irkr}(H_n)$ = k + n – 2

Theorem 3.8: Let $(C_n \circ K_1)$be the crown graph obtained by joining a pendant edge to each vertex of C_n. Then for n ≥ 1, $\gamma_{i_{rkr}}(C_n \circ k_1)$ = nk

Proof: Let $v_1, v_2, v_3, \dots v_n$ be the vertices of the cycle. Let $w_1, w_2, w_3, \dots w_n$ be the set of end vertices of the crown graph, where $1 \le i \le n$.

Define f: V(G) → P [1, 2, …, k] by

$$f(x) = \begin{cases} \{1,2,\dots,k\} if x = w_i ; \; 1 \le i \le n \\ \emptyset \quad \text{otherwise} \end{cases}$$

Obviously, every vertex v with labelled $\emptyset$ satisfies the condition $\bigcup_{w\in N[v]} f(w) = \{1,2,..,k\}$

$\therefore$f is a k Rainbow Dominating Function.

By Observation 3.2(1), we assigned $\{1,2,\dots,k\}$ to the end vertices, which is independent and we assigned$\emptyset$ to all the support vertices. Then the induced subgraph of empty set is also connected.

Hence it satisfies both Independent and Restrained condition.

Then clearly f is an Independent Restrained k Rainbow Dominating function.

$\therefore$ w(f) = $\sum$ ($|f(w_i)|$) where i = 1 to n = $|f(w_1)| + |f(w_2)| + |f(w_3)| + \cdots + |f(w_n)| =$ nk

Therefore, $\gamma_{i_{rkr}}(C_n \circ k_1)$ = nk

Remark 3.9:

(i) In Observation 2.2 (2), Equality holds. Since $\gamma_{i_{rkr}}(K_n)$ = nk and $\gamma_{i_{rkr}}(C_n \circ k_1)$ = nk.

(ii) The inequality is also strict. Since $\gamma_{irkr}(H_n)$ = k + n – 2

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