On Forgotten Index of Stolarsky-3 Mean Graphs

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Abstract

The Forgotten index of a graph G is defined as F(G) = $\sum_{uv\in E(G)}({d_u}^2 + {d_v}^2)$ over all edges uv of G,where d_u ,d_v are the degrees of the vertices u and v in G, respectively. In this paper, we introduced Forgotten index of some standard Stolarsky-3 Mean Graphs.

Keywords: Forgotten index, Stolarsky-3 Mean Graphs.

AMS Subject Classification: 05C12[3]

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1. Introduction

Let G be a simple graph corresponding to a drug structure with vertex (atom) set V(G) and edge (bond) set E(G). The edge joining the vertices u and v is denoted by uv. Thus, if $u, v \in E(G)$ then u and v are adjacent in G. The degree of a vertex u, denoted by d(u), is the number of edge incident to u. Several topological indices such as Estrada index, Zagreb index, PI index, eccentric index, and wiener index have been introduced in the literature to study the chemical and pharmacological properties of molecules.
The forgotten topological index of a graph G is defined as the sum of weights ${d_u}^2 + {d_v}^2$over all edges uv of G,where d_u andd_v are the degrees of the vertices u and v in G, respectively. In this paper, we characterize the external properties of F-index (forgotten topological index). We first introduce some graph transformation which increase or decrease this index. Recently in 2015 Furtula and Gutman was introduced another topological index called index or F-index as F(G) = $\sum_{uv \in E(G)}({d_u}^2 + {d_v}^2)$. On the basis of this work, we introduce a new concept Forgotten index ofStolarsky3Meangraphs.Inthispaperweinvestigate Forgotten index of some standard graphs which admit Forgotten Mean graphs. We will provide a brief summary of definitions and other information which are necessary for our present investigation.

Definition:1.1 A graph G with p vertices and q edges is called a Stolarsky-3 Mean graph, if each vertex $x \in V$ with distinct labels $f(x)$ from $1,2, \ldots \ldots, q+1$ and eachedge $e = uv$ is assigned the distinct labels $f(e = uv) =$

$\left\lceil \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}} \right\rfloor$**t**hen the resulting edge labels are distinct. In this case f is called **Stolarsky-3 Mean labeling** of G.

Definition:1.2 Let G be a Stolarsky-3 Mean graph. The **Forgotten index** of a graph F(G) is defined by F(G) = $\sum_{uv \in E(G)}({d_u}^2 + {d_v}^2)$, where d(u) is the degree of vertex u in G.

Theorem 1.3: Any Path Pn is a Stolarsky-3 mean graph.

Theorem 1.4: Any Cycle Cn is a Stolarsky-3 mean graph.

Theorem 1.5: Any Comb $Pn \odot K1$is a Stolarsky 3 mean graph.

Theorem 1.6: The ladder graph $L_n$is a Stolarsky-3 mean graph.

Theorem 1.7: ATriangular Snake graph $T_n$is a Stolarsky-3 mean graph.

Theorem 1.8: AQuadrilateral Snake graph $Q_n$is a Stolarsky-3 mean graph.

Remark 1.9: If G is a Stolarsky 3 mean graph, then '1' must be a label of one of the vertices of G, Since, an edge should get label '1'.

Remark1.10: If u gets label '1', then any edge incident with must get label 1 (or) 2 (or) 3. Hence this vertex must have a degree ≤ 3.

2. Main results

Theorem 2.1: Let G = P_n be a Stolarsky-3 mean graph. Then the Forgotten index of a path P_n is $F(P_n)$ =5n+4.
Proof. Let G = P_n be a Stolarsky-3 mean graph.

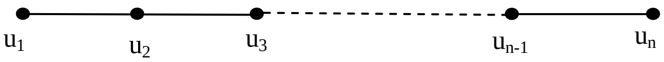

Figure: 1 Path P_n

We have $|V| = n$ and $|E| = n - 1$.
Therefore, by the definition of forgotten topological index, we obtain
F(G) = $\sum_{uv \in E(G)}(d_u{}^2 + d_v{}^2)$
= $[(d(u_1)^2 + d(u_2)^2) + (d(u_2)^2 + d(u_3)^2) + \cdots + (d(u_{n-1})^2 + d(u_n)^2)]$
= $(1^2 + 2^2) + (2^2 + 2^2) + \cdots + (2^2 + 1^2)$
= $(n-1) \times 2 + (n+2) \times 3 = 2n - 2 + 3n + 6$
F(G) = $5n + 4$

Example 2.2. Forgotten index of P_6 is given below

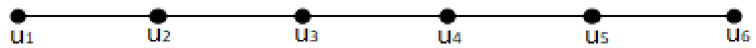

Figure: 2 Path P_6

$F(P_6)$ = $[(d(u_1)^2 + d(u_2)^2) + (d(u_2)^2 + d(u_3)^2) + (d(u_3)^2 + d(u_4)^2) + (d(u_4)^2 + d(u_5)^2) + (d(u_5)^2 + d(u_6)^2)] = (1^2 + 2^2) + (2^2 + 2^2) + (2^2 + 2^2) + (2^2 + 2^2) + (2^2 + 1^2)$
= $5 + 8 + 8 + 8 + 5$
= $5 \times 2 + 8 \times 3$
= 34

Theorem 2.3. The Forgotten index of cycle C_n is $F(C_n) = 8n$.
Proof. Let $G = C_n$ be a Stolarsky-3 mean graph

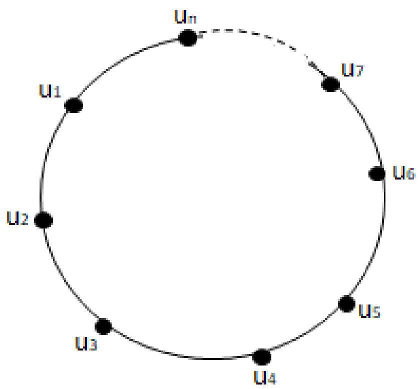

Figure: 3 $G = C_n$

We have $|V| = n$ and $|E| = n - 1$. Therefore, by the definition of forgotten topological index, we obtain

F(G) = $\sum_{uv \in E(G)}({d_u}^2 + {d_v}^2)$

= $[(d(u_1)^2 + d(u_2)^2) + (d(u_2)^2 + d(u_3)^2) + \cdots + (d(u_{n-1})^2 + d(u_n)^2) + (d(u_n)^2 + d(u_1)^2)]$

= $(2^2 + 2^2) + (2^2 + 2^2) + \cdots + (2^2 + 2^2) + (2^2 + 2^2)$

= $(8 + 8 + \cdots + n\ times)$

F(G) = $8n$

Example 2.4. Forgotten index of C_6 is given below

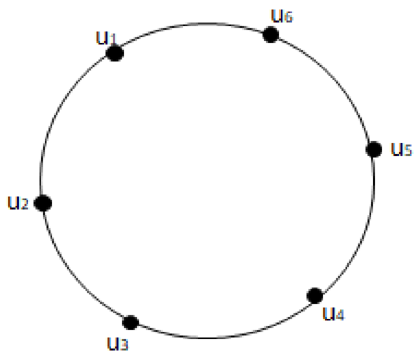

Figure: 4 $G = C_6$

F(C_6) = $[(d(u_1)^2 + d(u_2)^2) + (d(u_2)^2 + d(u_3)^2) + (d(u_3)^2 + d(u_4)^2) + (d(u_4)^2 + d(u_5)^2) + (d(u_5)^2 + d(u_6)^2) + (d(u_6)^2 + d(u_1)^2)]$

= $(2^2 + 2^2) + (2^2 + 2^2) + (2^2 + 2^2) + (2^2 + 2^2) + (2^2 + 2^2) + (2^2 + 2^2)$

= $8 \times 6 = 48$

Theorem 2.5. The Forgotten index of Comb graph $F(P_n \odot K_1) = 21n - 65$.

Proof. Let G = $P_n \odot K_1$ be a Stolarsky – 3 Mean graph.

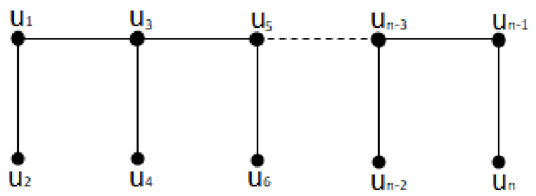

Figure: 5 Comb $P_n \odot K_1$

We have $|V| = n$ and $|E| = n - 1$. Therefore, by the definition of forgotten topological index, we obtain

F(G) = $\sum_{uv \in E(G)} (d_u{}^2 + d_v{}^2)$

$$= \left[\begin{array}{c} (d(u_1)^2 + d(u_2)^2) + (d(u_2)^2 + d(u_3)^2) + \cdots + (d(u_{n-1})^2 + d(u_n)^2) + (d(u_1)^2 + d(u_3)^2) \\ + \cdots + (d(u_{n-3})^2 + d(u_{n-1})^2) \end{array} \right]$$

$=[(2^2 + 1^2) + (3^2 + 1^2) + \cdots + (2^2 + 1^2) + (2^2 + 3^2) + (3^2 + 3^2) + \cdots + (3^2 + 2^2)]$

$= \ 18 \times 3 + 13 \times 2 + 10 \times 4 + 5 \times 2$

$= \ 9n + 4n + 6n + 16$

F(G) $= \ 19n + 16$

Example 2.6: Forgotten index of $P_6 \odot K_1$ is given below.

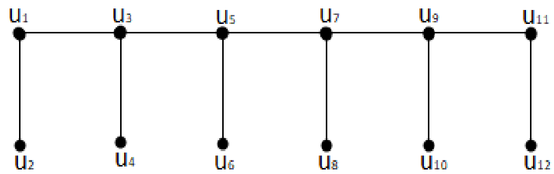

Figure: 6 Comb $P_6 \odot K_1$

F($P_6 \odot K_1$) $= \ [(2^2 + 1^2) + (2^2 + 3^2) + (3^2 + 1^2) + (3^2 + 3^2) + (3^2 + 1^2) + (3^2 + 3^2) + (3^2 + 1^2) + (3^2 + 3^2) + (3^2 + 1^2) + (3^2 + 2^2) + (2^2 + 1^2)]$

$= \ 18 \times 3 + 13 \times 2 + 10 \times 4 + 5 \times 2 = \ 130$

Theorem 2.7: Forgotten index of ladder graph L_n is $H(L_n) = \begin{cases} 8n & if \quad n = 2 \\ 32n + 12 & if \quad n > 2 \end{cases}$

Proof. Let G = $P_n \times P_2$ be a Stolarsky- 3 Mean graph

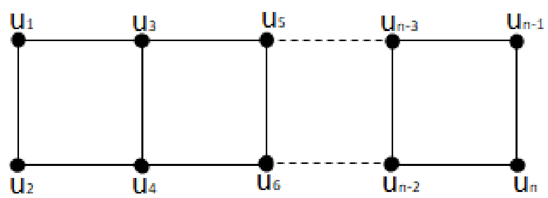

Figure: 7 G = $P_n \times P_2$

Case (i) if n = 2

F (L_2) $= [(d(u_1)^2 + d(u_2)^2) + (d(u_2)^2 + d(u_4)^2) + \cdots + (d(u_{n-1})^2 + d(u_n)^2)]$

$= (2^2 + 2^2) + (2^2 + 2^2) + \cdots + (2^2 + 2^2)$
$= 8 + 8 + \cdots + n\ times$
$= 8n$

Case (ii) if n>2

$F(L_4) = \begin{bmatrix} (d(u_1)^2 + d(u_2)^2) + \cdots + (d(u_{n-1})^2 + d(u_n)^2) + (d(u_1)^2 + d(u_3)^2) + \cdots + \\ (d(u_{n-3})^2 + d(u_{n-1})^2) + (d(u_2)^2 + d(u_4)^2) + \cdots + (d(u_{n-2})^2 + d(u_n)^2) \end{bmatrix}$
$= [(2^2 + 2^2) + (3^2 + 2^2) + \cdots + (2^2 + 2^2) + (2^2 + 3^2) + (3^2 + 3^2) + \cdots + (3^2 + 2^2) + (2^2 + 3^2) + \cdots + (3^2 + 2^2)]$
$= 8 \times 2 + 13 \times 4 + 18 \times 4$
$= 2n \times 2 + (3n + 1) \times 4 + (4n + 2) \times 4$
$= 4(8n + 3)$
$F\ (L_4) = 32n + 12$

Example 2.8. Forgotten index of L_4 is given below.

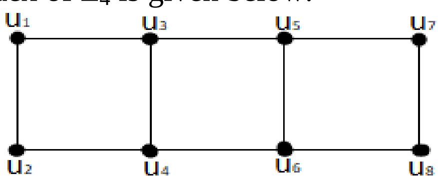

Figure: 8 L_4

$F(L_4) = \begin{bmatrix} (d(u_1)^2 + d(u_2)^2) + (d(u_3)^2 + d(u_4)^2) + \cdots + (d(u_7)^2 + d(u_8)^2) + \\ (d(u_1)^2 + d(u_3)^2) + \cdots + (d(u_5)^2 + d(u_7)^2) + (d(u_2)^2 + d(u_4)^2) + \cdots + (d(u_6)^2 + d(u_8)^2) \end{bmatrix}$
$= [(2^2 + 2^2) + (3^2 + 3^2) + (3^2 + 3^2) + (2^2 + 2^2) + (2^2 + 3^2) + (3^2 + 3^2) + (3^2 + 2^2) + (2^2 + 3^2) + (3^2 + 3^2) + (3^2 + 2^2)]$
$= 8 \times 2 + 13 \times 4 + 18 \times 4$
$= 16 + 52 + 72 = 140$

Theorem 2.9. Forgotten index of Triangular Snake graphT_n is 58n-6.

Proof. Let us consider a Stolarsky-3 Mean graph $G = T_n$ be a Stolarsky-3 mean graph.

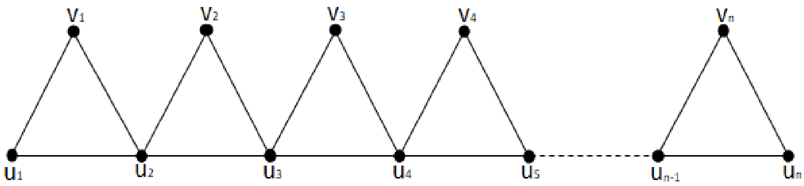

Figure: 9 $G = T_n$

$F(G) = \sum_{uv \in E(G)} ({d_u}^2 + {d_v}^2)$

$= [(d(u_1)^2 + d(u_2)^2) + (d(u_2)^2 + d(u_3)^2) + \cdots + (d(u_{n-1})^2 + d(u_n)^2) + (d(u_1)^2 + d(v_1)^2) + (d(u_2)^2 + d(v_2)^2) + \cdots + (d(u_n)^2 + d(v_n)^2) + (d(v_1)^2 + d(u_2)^2) + (d(v_2)^2 + d(u_3)^2) + \cdots (d(v_n)^2 + d(u_{n+1})^2)]$

$= [(2^2 + 4^2) + (4^2 + 4^2) + \cdots + (4^2 + 2^2) + (2^2 + 2^2) + (4^2 + 2^2) + \cdots + (2^2 + 4^2) + (2^2 + 4^2) + (2^2 + 4^2) + \cdots + (2^2 + 2^2)]$

$= 8 \times 2 + 20 \times 6 + 32$

$= (7n - 1) \times 6 + (3n - 1) \times 2 + 32$

$= 42n - 6 + 6n - 2 + 32$

F(G) $= 48n + 24$

Example 2.10. Forgotten index of T_3 is given below.

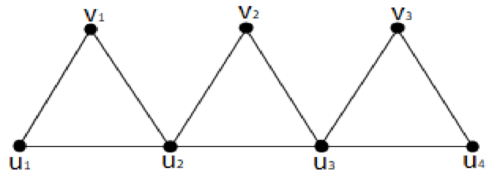

Figure: 10 $\mathrm{G} = \mathrm{T}_3$

F(T_3) $=[(d(u_1)^2 + d(u_2)^2) + (d(u_2)^2 + d(u_3)^2) + (d(u_3)^2 + d(u_4)^2) + (d(u_1)^2 + d(v_1)^2) + (d(u_2)^2 + d(v_2)^2) + (d(u_3)^2 + d(v_3)^2) + (d(v_1)^2 + d(u_2)^2) + (d(v_2)^2 + d(u_3)^2) + (d(v_3)^2 + d(u_4)^2)]$

$=[(2^2 + 4^2) + (4^2 + 4^2) + (4^2 + 2^2) + (2^2 + 2^2) + (4^2 + 2^2) + (2^2 + 4^2) + (2^2 + 4^2) + (2^2 + 4^2) + (2^2 + 2^2)]$

$= 8 \times 2 + 20 \times 6 + 32$ = 120+16+32

= 168

Theorem 2.11: Forgotten index of Quadrilateral Snake graph Q_n is 80n-48.

Proof. Consider $\mathrm{G} = Q_n$ be a Stolarsky-3 mean graph.

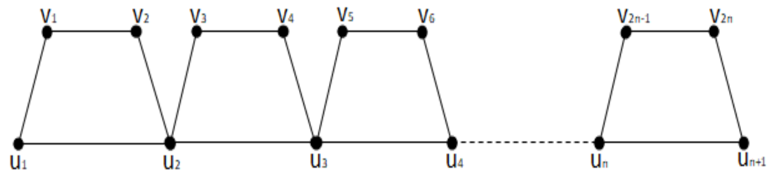

Figure:11G $= Q_n$

F (G)= $\sum_{uv \in E(G)} (d_u{}^2 + d_v{}^2)$

$= [(d(u_1)^2 + d(u_2)^2) + (d(u_2)^2 + d(u_3)^2) + \cdots + (d(u_n)^2 + d(u_{n+1})^2) + (d(u_1)^2 + d(v_1)^2) + (d(u_2)^2 + d(v_3)^2) + \cdots + (d(u_n)^2 + d(v_{2n-1})^2) + (d(u_2)^2 + d(v_2)^2) + (d(u_3)^2 + d(v_4)^2) + \cdots + (d(u_{n+1})^2 + d(v_{2n})^2) + (d(v_1)^2 + d(v_2)^2) + (d(v_3)^2 + d(v_4)^2) + \cdots + (d(v_{2n-1})^2 + d(v_{2n})^2)$

$= [(2^2 + 4^2) + (4^2 + 4^2) + \cdots + (4^2 + 2^2) + (2^2 + 2^2) + (4^2 + 2^2) + \cdots + (4^2 + 2^2) + (4^2 + 2^2) + (4^2 + 2^2) + \cdots + (2^2 + 2^2) + (2^2 + 2^2) + \cdots + (2^2 + 2^2)]$

$= 6(6n + 2) + 5(2n + 2) + (n - 2)(9n + 5)$

$= 36n + 12 + 10n + 10 + 9n^2 + 5n - 18n - 10$
F (G)= $9n^2 + 33n + 12$

Example 2.12. Forgotten index of Q_3 is given below.

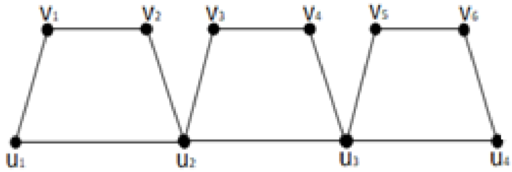

Figure: 12G $= Q_3$

F(Q_3) $= [(d(u_1)^2 + d(u_2)^2) + (d(u_2)^2 + d(u_3)^2) + (d(u_3)^2 + d(u_4)^2) + (d(u_1)^2 + d(v_1)^2) + (d(u_2)^2 + d(v_3)^2) + (d(u_3)^2 + d(v_5)^2) + (d(u_2)^2 + d(v_2)^2) + (d(u_3)^2 + d(v_4)^2) + (d(u_4)^2 + d(v_6)^2) + (d(v_1)^2 + d(v_2)^2) + (d(v_3)^2 + d(v_4)^2) + (d(v_5)^2 + d(v_6)^2)]$
$= [(2^2 + 4^2) + (4^2 + 4^2) + (4^2 + 2^2) + (2^2 + 2^2) + (4^2 + 2^2) + (4^2 + 2^2) + (4^2 + 2^2) + (4^2 + 2^2) + (2^2 + 2^2) + (2^2 + 2^2) + (2^2 + 2^2) + (2^2 + 2^2)]$
$= [(20 \times 6) + (8 \times 5) + 32 \times 1$
$= 120 + 40 + 32$
$= 192$

Theorem 2.13: Forgotten index of Crown graph $C_n \odot K_1$ is 28n.
Proof. Consider G $= C_n \odot K_1$ be a Stolarsky-3 mean graph.
F (G) $= \sum_{uv \in E(G)} (d_u{}^2 + d_v{}^2)$
$= [(d(u_1)^2 + d(u_2)^2) + (d(u_2)^2 + d(u_3)^2) + \cdots + (d(u_{n-1})^2 + d(u_n)^2) + (d(u_n)^2 + d(u_1)^2) + (d(u_1)^2 + d(v_1)^2) + (d(u_2)^2 + d(v_2)^2) + \cdots + (d(u_n)^2 + d(v_n)^2)]$
$= [(3^2 + 3^2) + (3^2 + 3^2) + \cdots + (3^2 + 3^2) + (3^2 + 1^2) + \cdots + (3^2 + 1^2)]$
$= [(18 + 18 + \cdots + 18) + (10 + 10 + \cdots + 10)$
$= 18n + 10n$
F (G)= $28n$

Example 2.14. Forgotten index of $C_n \odot K_1$ is given below.

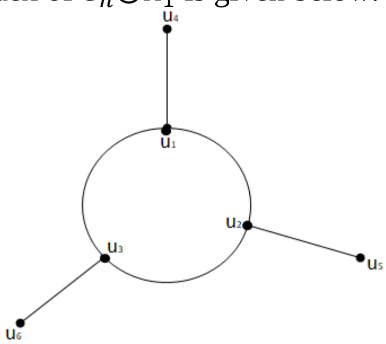

Figure: 13G $= C_n \odot K_1$

F $(C_n \odot K_1)$ = $[(3^2+3^2)+(3^2+3^2)+(3^2+3^2)+(3^2+1^2)+(3^2+1^2)+(3^2+1^2)]$
$= [(18 \times 3) + (10 \times 3)]$
$= 54 + 30$
$= 84$

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