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Odd prime labeling for some arrow related graphs

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Abstract

In a graph $\mathcal{G}$ a mapping g is known as odd prime labeling , if g is a bijection from $\mathcal{V}$ to $\{1, 3, 5, ..., 2|\mathcal{V}| - 1\}$ satisfying the condition that for each line xy in $\mathcal{G}$ the gcd of the labels of end points $(g(x), g(y))$ is one. In this article we prove that some new arrow related graphs such as A_y^2, A_y^3,A_y^5, are all odd prime graphs. Also we prove that double arrow graphs, $\mathcal{D}A_y^2$ and $\mathcal{D}A_y^3$ are odd prime graphs.

Keywords: Prime graph, Odd prime graph, Arrow graphs.
2020 AMS subject classifications: 05C78 [1]

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1 Introduction

In this article by a graph $\mathcal{G} = \langle V(\mathcal{G}), E(\mathcal{G}) \rangle$ we mean a simple graph. For graph theoretical notations we refer J.A.Bondy and U .S. R.Murthy [1976] .

Graph labeling has been introduced in mid 1960. For entire survey of graph labeling we refer Gallian [2015].

The concept of prime labeling was established by Roger Entringer and was discussed in a article by Deretsky et al. [1991], Tout et al. [1982]. A graph $\mathcal{G}$ of order p is known as prime graph if it's points can be labeled with distinct positive integers $\{1, 2, 3, \cdot, p\}$ such that the labels of any two adjacent points are relatively prime Meena and Vaithilingam [2013]. Meena and Kavitha [2014] investigated prime labeling for some butterfly related graphs. Meena et al. [2021] investigated odd prime labeling for some new classes of graph.

The notion of odd prime labeling was established by Prajapati and Shah [2018] and many researchers. Arrow graph was introduced by Kaneria et al. [2015]. Motivated by this study, in this article investigate the existence of odd prime labeling of some graphs related to arrow graphs.

Definition 1.1. *Let $\mathcal{H} = \langle \mathcal{V}(\mathcal{H}), \mathcal{E}(\mathcal{H}) \rangle$ be a graph. A bijection $g : \mathcal{V}(\mathcal{H}) \to O_{|V|}$ is know as odd prime labeling if for each line $xy \in \mathcal{E}$, greatest common divisor $\langle g(x), g(y) \rangle = 1$. A graph is know as odd prime graph if its admits odd prime labeling .*

Definition 1.2. *Let $\mathcal{H}_1 = (P_1, Q_1)$ and $\mathcal{H}_2 = (P_2, Q_2)$ be two graphs with $P_1 \cap P_2 = \phi$. The cartesian product $\mathcal{H}_1 \times \mathcal{H}_2$ is defined as a graph having $P = P_1 \times P_2$ and $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are adjacent if $x_1 = y_1$ and x_2 is adjacent to y_2 in $\mathcal{H}_2$ or x_1 is adjacent to y_1 in $\mathcal{H}_1$ and $x_2 = y_2$. The cartesian product of two paths P_m and P_n denoted as $P_m \times P_n$ is known as a grid graph on nm points and $2nm - (n + m)$.*

Definition 1.3. *In rectangular grid $P_m \times P_n$ on mn points the n points $v_{1,1}, v_{2,1}, v_{3,1} \ldots v_{m,n}$ and points $v_{1,n}, v_{2,n}, v_{3,n} \ldots v_{m,n}$ are called an superior points from both the ends.*

Definition 1.4. *An arrow graph A_y^x with width x and length y is got by connecting a point v with superior points of $P_x \times P_y$ by new edges from one end.*

Definition 1.5. *A double arrow graph $\mathcal{D}A_y^x$ with width x and length y is got by conecting two points v and w with superior points of $P_m \times P_y$ by $x + x$ new edges from both the end.*

2 Main Results

Theorem 2.1. *A_y^2 is an odd prime graph where $y \geq 2$.*

Proof. Let $\mathcal{G}$ = A_y^2 be an arrow graph got by connecting a point $g(u_0)$ with superior points of $P_2 \times P_y$ by new lines.
Let $\mathcal{V}(\mathcal{G}) = \{u_l/0 \leq l \leq y\} \cup \{v_l/1 \leq l \leq y\}$
$\mathcal{E}(\mathcal{G}) = \{u_l u_{l+1}/1 \leq l \leq y-1\} \cup \{u_0 v_1\} \cup \{u_0 u_1\}$
$\cup \{v_l v_{l+1}/1 \leq l \leq y-1\} \cup \{u_l v_l/1 \leq l \leq y\}$.
Now $|\mathcal{V}(\mathcal{G})|$ = 2y+1 and $|\mathcal{E}(\mathcal{G})|$ = 3y
Define a Mapping $f : \mathcal{V} \to O_{2y}$ as follows
$g(u_0)$ = 1
$g(u_l)$ = $4l - 1$ for $1 \leq l \leq y$
$g(v_l)$ = $4l + 1$ for $1 \leq l \leq y$
Clearly point labels are distinct.
For each $e \in E$, if $gcd(g(u), g(v)) = 1$
(i) $e = u_0 u_l$, $gcd(g(u_0), g(u_l)) = gcd(1, 3) = 1$
(ii) $e = u_0 v_1$, $gcd(g(u_0), g(v_l))$ = $gcd(1, 5) = 1$
(iii)$e = u_l v_l$, $gcd(g(u_l), g(v_l))$ = $gcd(4l - 1, 4l + 1) = 1$
for $1 \leq l \leq y$
(iv) $e = u_l u_{l+1}$, $gcd(g(u_l), g(u_{l+1}))$= $gcd(4l - 1, 4l + 3) = 1$
for $1 \leq l \leq y - 1$
(v) $e = v_i v_{l+1}$, $gcd(g(v_l)), g(v_{l+1}) = gcd(4l + 1, 4l + 5) = 1$
for $1 \leq l \leq y - 1$
Hence A_y^2 is an odd prime graph. □

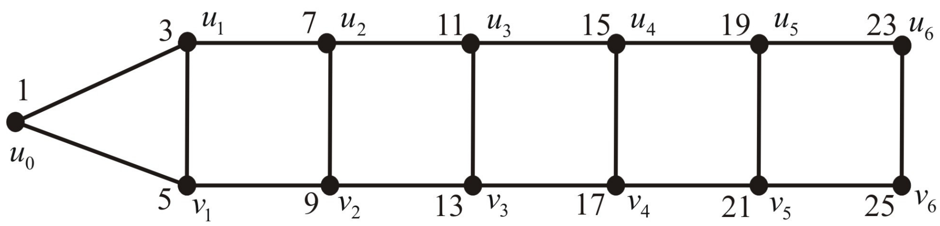

Figure 1: Arrow graph A_y^2 and its odd prime labeling

Theorem 2.2. *A_y^3 is an odd prime graph where $y \geq 2$.*

Proof. Let $G = A_y^3$ be an arrow graph got by connecting a point $g(u_0)$ with superior points of $P_3 \times P_2$ by 3 new lines.
$\mathcal{V}(\mathcal{G}) = \{u_l, v_l, w_l, /1 \leq l \leq y\} \cup \{u_0\}$
$\mathcal{E}(\mathcal{G}) = \{u_l u_{l+1}, v_l v_{l+1}, w_l w_{l+1}/1 \leq l \leq y-1\} \cup \{v_l w_l, u_l v_l/1 \leq l \leq y\}$
$\cup \{u_o u_1\} \cup \{u_0 v_1\} \cup \{u_0 w_1\}$

Now $|\mathcal{V}(\mathcal{G})| = 3y + 1$ and $|\mathcal{E}(\mathcal{G})| = 5y - 1$
Define a mapping $f : \mathcal{V} \to O_{2y}$ as follows
$g(u_0) = 1$
$g(u_l) = 6l - 3$ for $1 \leq l \leq y$, l is odd
$g(u_l) = 6l - 1$ for $1 \leq l \leq y$, l is even
$g(v_l) = 6l - 1$ for $1 \leq l \leq y$, l is odd
$g(v_l) = 6l - 3$ for $1 \leq l \leq y$, l is even
$g(w_l) = 6l + 1$ for $1 \leq l \leq y$
Clearly all the point labels are distinct. With this labeling for each $e = uv \in E$ if
(i) $e = u_0u_1, gcd(g(u_0), g(u_1)) = gcd(1, 3) = 1$ for $1 \leq l \leq y$
(ii) $e = u_0w_1, gcd(g(u_0), g(w_1)) = gcd(1, 7) = 1$ for $1 \leq l \leq y$
(iii)$e = u_0v_1, gcd(g(u_0), g(v_1)) = gcd(1, 5) = 1$ for $1 \leq l \leq y$
(iv)$e = u_lv_l, gcd(g(u_l), g(v_l) = gcd(6l - 3, 6l - 1) = 1$ for $1 \leq l \leq y$ $l \not\equiv 0(mod2)$
(v)$e = u_lv_l, gcd(g(u_l), g(v_l)) = gcd(6l - 3, 6l - 1) = 1$ for $1 \leq l \leq y$ $l \equiv 0(mod2)$
(vi)$e = v_lw_l, gcd(g(v_l), g(w_l)) = gcd(6l - 1, 6l + 1) = 1$ for $1 \leq l \leq y$ $l \not\equiv 0(mod2)$
(vii)$e = v_lw_l, gcd(g(u_l), g(v_l)) = gcd(6l - 1, 6l - 3) = 1$ for $1 \leq l \leq y$ $l \equiv 0(mod2)$
(viii)$e = u_lu_{l+1}, gcd(g(u_l), g(u_{l+1})) = gcd(6l - 3, 6l - 5) = 1$ for $1 \leq l \leq y$ $l \not\equiv 0(mod2)$
(ix)$e = u_lu_{l+1}, gcd(g(u_l), g(u_{l+1})) = gcd(6l - 1, 6l + 3) = 1$ for $1 \leq l \leq y$ $l \equiv 0(mod2)$
(x)$e = v_lvl + 1, gcd(g(v_l), g(v_{l+1})) = gcd(6l - 3, 6l - 1) = 1$ for $1 \leq l \leq y$ $l \equiv 0(mod2)$
(xi) $e = v_lvl + 1, gcd(g(v_l), g(v_{l+1})) = gcd(6l - 1, 6l - 3) = 1$ for $1 \leq l \leq y$ $l \not\equiv 0(mod2)$
(xii) $e = w_lwl + 1, gcd(g(w_l), g(w_{l+1})) = gcd(6l + 1, 6l + 7) = 1$ for $1 \leq l \leq y$
Hence A_y^3 is an odd prime graph . □

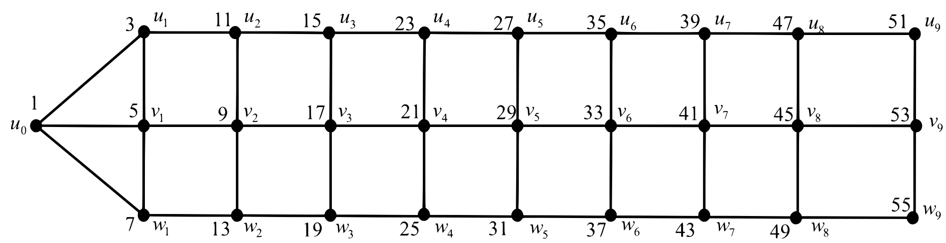

Figure 2: Arrow graph A_y^3 and its odd prime labeling

Theorem 2.3. *A_y^5 is an odd prime graph where $y \geq 5$.*

Proof. Let $\mathcal{G}$ = A_y^5 be an arrow graph got by connecting a point v with superior points $P_5 \times P_y$ by 5 new lines.
$\mathcal{V}(\mathcal{G}) = \{u_l, v_l, w_l/1 \leq l \leq y\} \cup \{u_0\}$
$\mathcal{E}(\mathcal{G}) = \{u_l v_l, v_l w_l/1 \leq l \leq y\} \cup \{(u_l u_{l+1}), (v_l v_{i+1}), (w_l w_{l+1}/1 \leq l \leq y-1\}$
Now $|\mathcal{V}(\mathcal{G})| = 5y+1$ and $|\mathcal{E}(\mathcal{G})| = 9y$
Define a mapping $f : \mathcal{V} \to O_y$ as follows
$g(u_0) = 1$
$g(u_l) = 6l-3$ for $1 \leq l \leq l$, l is odd
$g(u_l) = 6l-1$ for $1 \leq l \leq l$, l is even
$g(v_l) = 6l-1$ for $1 \leq l \leq l$, l is odd
$g(v_l) = 6l-3$ for $1 \leq l \leq l$, l is even
$g(w_l) = 6l+1$ for $1 \leq l \leq l$,
Clearly all the point labels are distinct. With this labeling for each $e \in E$ if $gcd(g(u), g(v)) = 1$
(i) $e = u_0 u_1, gcd(g(u_0), g(u_1)) = gcd(1, 3) = 1$
(ii) $e = u_0 u_{l+1}, gcd(g(u_0), g(u_{l+1})) = gcd(1, 6l-3) = 1$ for $1 \leq l \leq y$
(iii)$e = u_l v_l, gcd(g(u_l), g(v_l) = gcd(6l-3, 6l-1) = 1$ for $1 \leq l \leq y$ $l \not\equiv 0(mod2)$
(iv) $e = u_l v_l, gcd(g(u_l), g(v_l)) = gcd(6l-1, 6l-3) = 1$ for $1 \leq l \leq y$ $l \equiv 0(mod2)$
(v) $e = v_l w_l, gcd(g(v_l), g(w_l)) = gcd(6l-1, 6l+1) = 1$ for $1 \leq l \leq y$ $l \not\equiv 0(mod2)$
(vi) $e = v_l w_l, gcd(g(v_l), g(w_l)) = gcd(6l-3, 6l+1) = 1$ for $1 \leq l \leq y$ $l \equiv 0(mod2)$
(vii) $e = u_l u_{l+1}, gcd(g(u_l), g(u_{l+1})) = gcd(6l-3, 6l+5) = 1$ for $1 \leq l \leq y$ $l \not\equiv 0(mod2)$
(viii) $e = u_l u_{l+1}, gcd(g(u_l), g(u_{l+1})) = gcd(6l-1, 6l-3) = 1$ for $1 \leq l \leq y$ $l \equiv 0(mod2)$
(ix) $e = v_l v_{l+1}, gcd(g(v_l), g(v_{l+1})) = gcd(6l-1, 6l+3) = 1$ for $1 \leq l \leq y$ $l \not\equiv 0(mod2)$
(x) $e = v_i v_{i+1}, gcd(g(v_l), g(v_{l+1})) = gcd(6l-3, 6l+5) = 1$ for $1 \leq l \leq y$ $l \equiv 0(mod2)$
(xi) $e = w_l w_{l+1}, gcd(g(w_l), g(w_{l+1})) = gcd(6l+1, 6l+7) = 1$ for $1 \leq l \leq y$
Hence A_y^5 is an odd prime graph . □

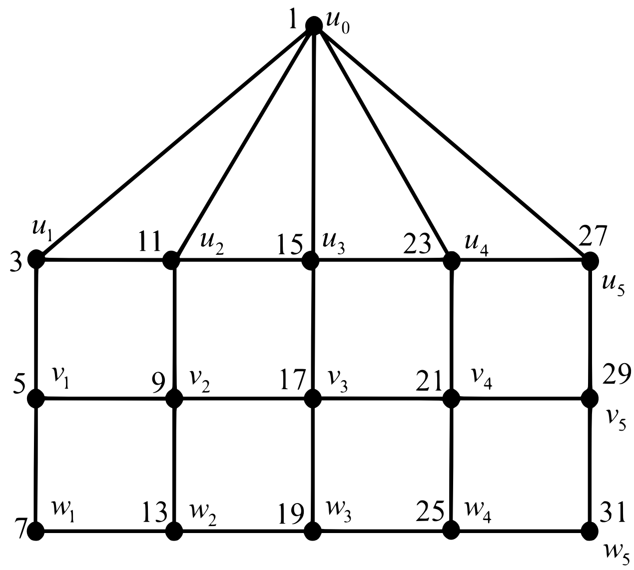

Figure 3: Arrow graph A_y^5 and its odd prime labeling

Theorem 2.4. *$\mathcal{D}A_y^2$ is an odd prime graph where $y \geq 2$.*

Proof. Let $\mathcal{G}$ = $\mathcal{D}A_y^2$ be a double arrow graph got by connecting two points u, v with superior points from both the ends of $P_2 \times P_y$ by 2+2 new lines.
Let $\mathcal{V}(\mathcal{G}) = \{u_l v_l/1 \leq l \leq y\} \cup \{v, v_0\}$
$\mathcal{E}(\mathcal{G}) = \{(u_l u_{l+1}), (v_l v_{l+1}), 1 \leq l \leq y-1\} \cup \{v_l u_l/1 \leq l \leq y\} \cup \{vv_1\} \cup \{vu_1\}$
$\cup \{u_y v_0\} \cup \{v_y v_0\}$
Now $|\mathcal{V}(\mathcal{G})|$ = 2y+2 and $|\mathcal{E}(\mathcal{G})|$ = 3y+4
Define a mapping $f : \mathcal{V} \to O_{2y}$ as follows
$g(v)$ = 1
$g(u_i)$ = $4l-1$ for $1 \leq l \leq y$
$g(v_i)$ = $4l+1$ for $1 \leq l \leq y$
$g(v_0)$= $4y+3$
Clearly point labels are distinct.
For every $e = uv \in E$, if $gcd(g(u), g(v)) = 1$
(i) $e = vu_1$, $gcd(g(v), g(u_1)) = gcd(1, 3) = 1$
(ii) $e = vv_1$, $gcd(g(v), g(v_1))$ = $gcd(1, 5) = 1$
(iii) $e = u_l u_{l+1}$, $gcd(g(u_l), g(u_{l+1}))$= $gcd(4l-1, 4l+3) = 1$ for $1 \leq l \leq y-1$
(iv) $e = v_l v_{l+1}$, $gcd(g(v_l)), g(v_{l+1}) = gcd(4l+1, 4l+5) = 1$ for $1 \leq l \leq y-1$
(v) $e = v_l u_l$, $gcd(g(v_l), g(u_l))$ = $gcd(4l+1, 4l-1) = 1$ for $1 \leq l \leq y$
(vi) $e = v_y w$, $gcd(g(v_y), g(w))$ = $gcd(4y+1, 4y+3) = 1$
(vii)$e = u_y w$, $gcd(g(u_y), g(w))$ = $gcd(4y-1, 4y+3) = 1$
Hence $\mathcal{D}A_y^2$ is an odd prime graph. □

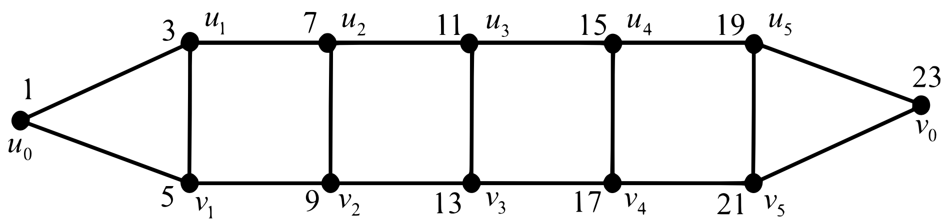

Figure 4: Arrow graph $\mathcal{D}A_y^2$ and its odd prime labeling

Theorem 2.5. *$\mathcal{D}A_y^3$ is an odd prime graph where $y \geq 3$.*

Proof. Let $\mathcal{D}$ = $\mathcal{D}A_y^3$ be an arrow graph got by connecting two point set u_0 and z_0 with superior pointss from both the ends of $P_3 \times P_2$ by 3+3 new lines.
$\mathcal{V}(\mathcal{G}) = \{u_l, v_l, w_l, /1 \leq l \leq y\} \cup \{u_0\} \cup \{z_0\}$
$\mathcal{E}(\mathcal{G}) = \{u_l u_{l+1}, v_l v_{l+1}, w_l w_{l+1}/1 \leq l \leq y-1\} \cup \{w_l v_l, v_l u_l/1 \leq l \leq y\} \cup$
$\{u_0 u_1, u_0 v_1, u_0 w_1, z_0 u_y, z_0 v_y, z_0 w_y\}$
Now $|\mathcal{V}(\mathcal{G})| = 3y+2$ and $|\mathcal{E}(\mathcal{G})| = 5y+3$
Define a mapping $f : \mathcal{V} \to O_y$ as follows
$g(u_0) = 1$
$g(u_l) = 6l-3$ for $1 \leq l \leq y$, l is odd
$g(u_l) = 6l-1$ for $1 \leq l \leq y$, l is even
$g(v_l) = 6l-1$ for $1 \leq l \leq y$, l is odd
$g(v_l) = 6l-3$ for $1 \leq l \leq y$, l is even
$g(w_l) = 6l+1$ for $1 \leq l \leq y$
$g(z_0) = 6y+3$ for $1 \leq i \leq y$
Clearly all the point values are different. With this labeling for each $e \in E$ if
(i) $e = u_0 u_1, gcd(g(u_0), g(u_1)) = gcd(1,3) = 1$
(ii) $e = u_0 v_1, gcd(g(u_0), g(v_1)) = gcd(1,5) = 1$
(iii) $e = u_0 w_1, gcd(g(u_0), g(w_1) = gcd(1,7) = 1$
(iv) $e = u_l v_l, gcd(g(u_l), g(v_l)) = gcd(6l-3, 6l-1) = 1$ for $1 \leq l \leq y$ $l \not\equiv 0(mod 2)$
(v) $e = u_l v_l, gcd(g(u_l), g(v_l)) = gcd(6l-1, 6l-3) = 1$ for $1 \leq l \leq y$ $l \equiv 0(mod 2)$;
(vi) $e = v_l w_l, gcd(g(v_l), g(w_l)) = gcd(6l-1, 6l+1) = 1$ for $1 \leq l \leq y$ $l \not\equiv 0(mod 2)$
(vii) $e = v_l w_l, gcd(g(v_l), g(w_l)) = gcd(6l-3, 6l-1) = 1$ for $1 \leq l \leq y$ $l \equiv 0(mod 2)$;
(viii) $e = u_l u_{l+1}, gcd(g(u_l), g(u_{l+1})) = gcd(6l-3, 6l+5) = 1$ for $1 \leq l \leq y-1$ $l \not\equiv 0(mod 2)$;
(ix) $e = u_l u_{l+1}, gcd(g(u_l), g(u_{l+1})) = gcd(6l-1, 6l+3) = 1$ for $1 \leq l \leq y-1$ $l \equiv 0(mod 2)$;
Hence $\mathcal{D}A_y^3$ is an odd prime graph . □

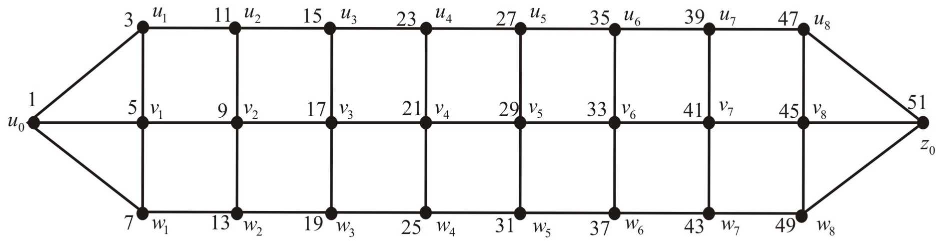

Figure 5: Arrow graph DA_n^3 and its odd prime labeling

3 Conclusions

The odd Prime labeling of various classes of graphs such as A_y^2 where $y \in N$, A_y^3, A_y^5, where $y \geq 2$ are odd prime graph and double arrow graphs $\mathcal{D}A_y^2$,$\mathcal{D}A_y^3$ are proved. To derive similar results for other graph families is an open area of research.

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