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A new form of continuity in fuzzy soft topological spaces

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Abstract

The current work introduces a new class of fuzzy soft b continuous functions such as slightly b continuous, semi b continuous, pre b continuous functions and their relation with the existing fuzzy soft continuous functions in fuzzy soft topological spaces. Further optimal definitions of totally b continuous functions have also been brought out in the paper. A new space such as fuzzy soft b compact space is also initiated.

Keywords: fuzzy soft (fs) open set, fs semi-open set, fs pre-open set, fs b-open set, fs continuous functions.
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1 Introduction

Topology is significant and a significant zone of arithmetic, and it can give numerous connections between logical regions and numerical models. Both mathematicians and computer scientists have concentrated on fuzzy set theory, and numerous utilizations of these have emerged throughout the long term. The soft set hypothesis has been applied to various fields with incredible achievement and rich potential for application in every engineering and sciences gambit. The idea of fuzzy soft sets is presented as a comprehensive numerical tool for managing vulnerability.

In the past years, issues in the field of Engineering, physics, social sciences, and medical sciences etc., in recent times involving uncertainties, cannot be dealt with crisp data. Zadeh [30] in 1965 introduced a general mathematical device recognized as a "fuzzy set" to address uncertainties. The topological structure of fuzzy sets was introduced by Chang [4]. To overcome the existing difficulties in fuzzy set theory, soft sets were introduced by Molodtsov [9] in 1999. This theory of soft sets can be successfully applied in many directions such as Game theory, Riemann integration, Smoothness of functions, Probability theory etc. Maji et al. [8] introduced the merger of fuzzy set and soft set known as a fuzzy soft set. The notion of the topological structure of fuzzy soft (fs) set was introduced by Tanay and Kandemir [26] in 2011 and studied further by Varol and Aygun [29], Roy and Samanta [16] and Pradeep[14]. Continuity is the core of any topological space. The authors Patil et. al. [11] and Missier and Rodrigo[12] many others contributed significantly to the continuous functions in topology. Kharal and Ahmad [7] studied mappings of fuzzy soft classes. The concept of a fuzzy soft semi-open set was introduced by A.Kandil et al. [6].

Fuzzy soft pre-open and regular open sets were introduced by Sabir Hussain in 2016 [19]. Sabir Hussain[20] has also proposed fuzzy soft semi-open and semi-permanent functions in fuzzy soft topological spaces. The concept of fs b-open sets was introduced by Anil P. N. [2] in 2016. Anil P. N et al. [3] has also introduced fs strongly b-continuous and perfectly b-continuous functions. Fuzzy soft pre continuous functions were introduced and studied by Ponselvakumari and Selvi [13]. Sabir Hussain [21] has also introduced fs locally connected spaces and the concept of fs semi pre-open set. The idea of generalized fs b open set and fs gb continuous functions were initiated by Sandhya and Anil P. N. [23]. Fuzzy soft connectedness through fs b open set was formed by Rodyna [15]. Further Abbas et al. [1] and Ruth and Selvam [18] contributed to the concept of fuzzy soft connectedness in 2018. Ibedou and Abbas [5] defined a fuzzy soft net consisting of fuzzy soft points and their convergence. This powerful tool called net is applied to study some important properties of fuzzy soft topological spaces by Rui Gao and Jianrong Wu [17] in 2018. Sabir Hussain [22] introduced compactness and locally compactness in fuzzy soft topological spaces. Smitha and Sindhu [24] introduced gb-closed and gb-open sets in intuitionistic fuzzy soft topological spaces in 2019. Tingshui Ping [27] investigated a few mappings on fuzzy soft topological spaces. Alkouri [28] introduced a new mathematical tool called complex generalised fuzzy soft set, a combination of generalised fuzzy soft set and complex fuzzy set. Parimala and Karthika [10] reviewed fuzzy soft topological spaces and neutrosophic soft topological spaces in 2020. Smitha and Sindhu [25] studied gb-continuous functions in intuitionistic fuzzy soft topological spaces in 2021. Zhi Kong and Lifu Wang [31] applied a fuzzy soft set in decision-making problems based on grey theory in 2021.

In this work, a new class of $fs\, b$ continuous functions known as fs slightly b continuous, fs semi b continuous functions, fs pre b continuous, and fs totally b continuous mappings in fs topological spaces are introduced, and some of their properties are studied. Further, the concept of $fs\, b$ compact spaces is initiated.

2 Preliminaries

Definition 2.1. *[8] Let U be the initial universe and K be the set of parameters. I^U be the set of all fuzzy sets on U. Let $A \subseteq K$ and a mapping $f : A \to I^U$. A pair (f, A) is called fuzzy soft (fs) set over U. It is also denoted by f_A. That is for each $a \in A$, $f(a) = f_a : U \to I$ is a fuzzy set on U.*

Example 2.1. *[8] Let the set of shirts be U, and the set of parameters be K. A fs set describes the attractiveness of shirts with respect to the given parameters. The set of all fuzzy sets of U is I^U $X = \{x_1, x_2, x_3\}$ and $K = \{e_1, e_2, e_3, e_4, e_5\}$. Let $E = \{e_1, e_2, e_3\}$ be the subset of K.A fs set is denoted by (F, E) or f_E . Where e_1= colourful, e_2= bright, e_3= reasonable price, e_4= good quality, e_5=modern $(F, E) = \{\{0.5/x_1, 0.9/x_2, 1/x_3\}, \{0.3/x_1, 0.6/x_2, 0/x_3\},$
$\{0.2/x_1, 0.9/x_2, 1/x_3\}\}$ describes three shirts with respect to parameters e_1, e_2, and e_3. The shirt x_1 w.r.t e_1 = colouful has a graded value 0.5 out of 1. Similarly,x_2 with respect to e_1 has a graded value of 0.9, and x_3 has 1 out of 1... so on.*

Definition 2.2. *[26] Let τ be a collection of all fs sets over a universe U and K be a fixed parameter set. A triplet (U, τ, K) is called fuzzy soft topological space [fsts] if the following hypotheses are satisfied:*

i. $\widetilde{O}_K, \widetilde{I}_K \in \tau$

ii. *Arbitrary union of members of τ is a member of τ.*

iii. *Finite intersection of members of τ is a member of τ.*

Each member of τ is called fs open set. If $f_K \in \tau$ then $1 - f_K$ is known as fs closed set.

Definition 2.3. *If (U, τ, K) is $fsts$, then a fs set f_K in U is called a*

i. *fs semi-open [6] if $f_K \leq fsclfsint(f_K)$, fs semi-closed if $fsintfscl(f_K) \leq f_K$.*

ii. *fs pre-open[19] if $f_K \leq fsintfscl(f_K)$, fs pre-closed if $fsclfsint(f_K) \leq f_K$.*

iii. *fs b-open [2] if $f_K \leq fsclfsint(f_K) \vee fsintfscl(f_K)$. The complement of the fs b open set is fs b closed. A fs set that is both b open and b closed is called fs b clopen. And fs b open set is referred to as $fsbo$.*

iv. *fs semi pre-open [20] if $f_K \leq fsclfsintfscl(f_K)$, fs semi pre-closed $fsclfsintfscl(f_K) \leq f_K$.*

v. *fs generalised b open[23] if $fsbint(f_K) \geq g_K$ whenever $(f_K) \geq g_K$ and g_K is fs closed set in U.*

Example 2.2. *Consider a fsts (U, τ, K) and $K = \{e_1, e_2\}$ where $U = \{a, b\}$, $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K), (F_3, K)\}$ $(F_1, K) = \{\{0.6/a, 0.8/b\}, \{0.7/a, 1/b\}\}$*
$(F_2, K) = \{\{0.4/a, 0.4/b\}, \{0.3/a, 0.1/b\}\}$
$(F_3, K) = \{\{0.3/a, 0.3/b\}, \{0.2/a, 0.1/b\}\}$.
In (U, τ, K), $(G_1, K) = \{\{0.5/a, 0.6/b\}, \{0.4/a, 0.3/b\}\}$, is fs semi open.
$(G_2, K) = \{\{0.4/a, 0.3/b\}, \{0.3/a, 0.1/b\}\}$, is fs pre open, fs b open and also fs gb open. $(G_3, K) = \{\{0.6/a, 0.4/b\}, \{0.3/a, 0.2/b\}\}$, is fs semi pre open.

Definition 2.4. *[8] If f_K is a fs set, then*

i. *the intersection of all fs closed supersets of f_K is fs closure of f_K.*

ii. *the union of all fs open subsets of f_K is called fs interior of f_K.*

Definition 2.5. *[2] If f_K is a fs set, then*

i. *the intersection of all fs b closed supersets of f_K is fs b closure (fsbcl) of f_K.*

ii. *the union of all fs b open subsets of f_K is called fs b interior (fsbint) of f_K.*

Definition 2.6. *Let (U, τ, K) and (V, σ, K) be fsts and f be a function from U to V. Then f is said to be a*

i. *fs continuous [7] if the inverse of every fs open set in V is fs open in U.*

ii. *fs semi-continuous(resp.fs pre continuous) [13] if the inverse of every fs open set in V is fs semi-open (respfs pre-open)in U.*

iii. *fs semi pre continuous [21] if the inverse of every fs open set in V is fs semi pre-open in U.*

iv. *fs b-continuous [23] if the inverse of every fs open set in V is fs b open in U.*

v. *fs b-irresolute [23] if the inverse of every fs b open set in V is fs b open in U.*

vi. *fs contra b continuous [3] if the inverse of every fs open set in V is fs b closed in U.*

vii. *fs strongly continuous [3] if the inverse of every fs set in V is fs clopen in U.*

viii. *fs perfectly continuous [3] if the inverse of every fs open set in V is fs clopen in U.*

ix. *fs strongly b-continuous [3] if the inverse of each fs b open set in V is fs open set in U.*

x. *fs perfectly b-continuous [3] if for each fs b-open set in V its inverse is fs clopen in U.*

xi. *fs gb continuous [23] if for each fs open set in V its inverse is fs gb open set in U.*

Definition 2.7. *Any fsts (U, τ, K) is called*

i. *fs discrete space [23] if every fs set is fs open in τ.*

ii. *fs locally indiscrete space[21] if every fs open set is closed in τ .*

iii. *fs $bT_{1/2}$ space [3] if every fs b open set is fs open.*

iv. *fs b connected [15] if there are no fs b separations of $\widetilde{1}_K$, otherwise (U, τ, K) is said to be fs b disconnected space.*

Definition 2.8. *[15] Let (U, τ, K) be a fsts. An fs b separation on $\widetilde{1}_K$ is a pair of non-null proper fs b open sets f_K and g_K where $f_K \cap g_K = \widetilde{0}_K$, $\widetilde{1}_K = f_K \cup g_K$.*

3 Fuzzy soft slightly b-continuous functions

Consider two fsts (U, τ, K), (V, σ, K) and f is a function from U to V and K is the set of parameters throughout this section.

Definition 3.1. *A function f is said to be fs slightly continuous (fssc) if the inverse of each fs clopen set in V is fs open in U.*

Definition 3.2. *A function f is said to be fs slightly b continuous (fssbc) if the inverse of each fs clopen set in V is fs b open (fsbo) in U.*

Example 3.1. *Suppose f is an identity map and $U = \{a, b\}$ $V = \{c, d\}$, $K = \{e_1, e_2\}$, $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$ and*
$\sigma = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$ $(F_1, K) = \{\{1/a, 0.9/b\}, \{0.8/a, 0.8/b\}\}$
$(F_2, K) = \{\{0/a, 0.1/b\}, \{0.2/a, 0.2/b\}\}$
$(G_1, K) = \{\{0.7/c, 0.6/d\}, \{0.5/c, 0.6/d\}\}$
$(G_2, K) = \{\{0.3/c, 0.4/d\}, \{0.5/c, 0.4/d\}\}$. *The inverse images of (G_1, K) and (G_2, K) are fs b open sets. Therefore f is fs slightly b continuous.*

Theorem 3.1. *Every fs slightly continuous function is fs slightly b continuous.*
Proof: *Let f be fs slightly continuous. Let (G, K) be fs clopen set in V, then $f^{-1}(G, K)$ is fs open and hence fs b-open in U. Hence f is fs slightly b-continuous.*
Converse need not be confirmed, as seen from the below example.
In example 3.1, $f^{-1}(G_1, K)$ and $f^{-1}(G_2, K)$ are fs b-open sets but not fs open in U. Therefore, f is slightly b continuous but not fs slightly continuous.

Theorem 3.2. *Every fs contra b continuous function is fs slightly b-continuous.*
Proof: *If f is fs contra b continuous map and (G, K) is fs clopen set in V, then $f^{-1}(G, K)$ is fs b open in U. Hence the theorem.*
The reverse implication is not valid.

Example 3.2. *Let f be an fs identity map. Let $U = \{a, b\}$, $V = \{c, d\}$ and $K = \{e_1, e_2\}$ and $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$ and $\sigma = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$, where $(F_1, K) = \{\{1/a, 0.9/b\}, \{0.8/a, 0.8/b\}\}$*
$(F_2, K) = \{\{0/a, 0.1/b\}, \{0.2/a, 0.2/b\}\}$
$(G_1, K) = \{\{0.7/c, 0.6/d\}, \{0.5/c, 0.6/d\}\}$
$(G_2, K) = \{\{0.3/c, 0.4/d\}, \{0.5/c, 0.4/d\}\}$
$(G_3, K) = \{\{0.2/c, 0.3/d\}, \{0.4/c, 0.3/d\}\}$. *It is verified that $f^{-1}(G_1, K)$ and $f^{-1}(G_2, K)$ are fs b open sets but $f^{-1}(G_3, K)$ is not fs b closed in U. Thus f is fs slightly b-continuous but not fs contra b continuous.*

Theorem 3.3. *Every fs b continuous function is fs slightly b continuous.*
Proof: *Let (G, K) be fs clopen set in V and f be fs b continuous. Then $f^{-1}(G, K)$ is fs b clopen in U. Hence f is fs slightly b continuous.*
The reverse implication need not be true in general.

Example 3.3. *Consider fs identity map. Let $U = \{a, b\}$, $V = \{c, d\}$, $K = \{e_1, e_2\}$, $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$ and $\sigma = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$, where*
$(F_1, K) = \{\{0.5/a, 0.4/b\}, \{0.3/a, 0.4/b\}\}$
$(F_2, K) = \{\{0.3/a, 0.3/b\}, \{0.2/a, 0.3/b\}\}$
$(G_1, K) = \{\{0.4/c, 0.5/d\}, \{0.4/c, 0.6/d\}\}$
$(G_2, K) = \{\{0.6/c, 0.5/d\}, \{0.6/c, 0.4/d\}\}$
$(G_3, K) = \{\{0.4/c, 0.5/d\}, \{0.3/c, 0.3/d\}\}$. *Since the inverse of fs clopen sets (G_1, K) and (G_2, K) are fs b open sets in U, but f is fs slightly b continuous and $f^{-1}(G_3, K)$ is not fs b open in U. Hence, f is not fs b continuous.*

Theorem 3.4. *Composition of fs slightly b continuous functions need not be fs slightly b continuous.*

Example 3.4. *Let $f : (U, \tau, K) \to (V, \tau', K)$ and $g : (V, \tau', K) \to (w, \sigma, K)$ be fs identity mappings. So, $g \circ f : (U, \tau, K) \to (w, \sigma, K)$ is also fs identity map. Let $U = \{a, b\}$, $V = \{c, d\}$, $w = \{g, h\}$, $K = \{e_1, e_2\}$, $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$, $\tau' = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$ and*
$\sigma = \{\widetilde{O}, \widetilde{1}, (H_1, K), (H_2, K)\}$ be fuzzy soft topological spaces. Here, $(F_1, K) = \{\{0.4/a, 0.3/b\}, \{0.4/a, 0.3/b\}\}$
$(F_2, K) = \{\{0.5/a, 0.4/b\}, \{0.4/a, 0.4/b\}\}$
$(G_1, K) = \{\{0.5/c, 0.4/d\}, \{0.5/c, 0.5/d\}\}$
$(G_2, K) = \{\{0.5/c, 0.6/d\}, \{0.5/c, 0.5/d\}\}$
$(H_1, K) = \{\{0.7/g, 0.8/h\}, \{0.6/g, 0.5/h\}\}$
$(H_2, K) = \{\{0.3/g, 0.2/h\}, \{0.4/g, 0.5/h\}\}$. *Then $f^{-1}(G_1, K)$, $f^{-1}(G_2, K)$ in U and $g^{-1}(H_1, K)$, $g^{-1}(H_2, K)$ in V are fs b open sets but $(g \circ f)^{-1}(H_2, K)$ is not fs b open in U.*

Theorem 3.5. *Let $f : (U, \tau, K) \to (V, \tau', K)$ and $g : (V, \tau', K) \to (w, \sigma, K)$ be two fs mappings, then*

i. *If f is fs b-irresolute and g is fssbc, then $g \circ f$ is fssbc .*

ii. *If f is fs b-irresolute and g is fs b-continuous, then $g \circ f$ is fssbc.*

iii. *If f is fs b-irresolute and g is fssc, then $g \circ f$ is fs b continuous .*

iv. *If f is fs b-continuous and g is fssc, then $g \circ f$ is fssbc.*

v. *If f is strongly b continuous and g is fssbc, then $g \circ f$ is fssc.*

vi. *If f is fssbc and g is fs perfectly b continuous, then $g \circ f$ is fs b-irresolute.*

vii. *If f is fssbc and g is fs contra continuous, then $g \circ f$ is fssbc.*

viii. *If f is fs b irresolute and g is fs contra b continuous, then $g \circ f$ is fssbc.*

Proof:

i. *Let (H, K) be fs clopen set in W, since g is fssbc, $g^{-1}(H, K)$ is fs b open in V and f is fs b irresolute $(g \circ f)^{-1}(H, K) = f^{-1}(g^{-1}(H, K))$ is fs b open in U. Thus $g \circ f$ is fssbc.*

ii. *Let (H, K) be fs clopen set in W, since g is fs b continuous, $g^{-1}(H, K)$ is fs b open in V and f is fs b irresolute, $(g \circ f)^{-1}(H, K) = f^{-1}(g^{-1}(H, K))$ is fs b open in U. Therefore $g \circ f$ is fssbc.*

iii. *Let (H, K) be fs clopen set in W, since g is fssc, $g^{-1}(H, K)$ is fs open set in V and also fs b open in V since f is fs b irresolute, $(g \circ f)^{-1}(H, K) = f^{-1}(g^{-1}(H, K))$ is fs b-open. Therefore, $g \circ f$ is fssbc.*

iv. *Let (H, K) be fs clopen set in W, since g is fs slightly continuous, $g^{-1}(H, K)$ is fs b open set in V, since f is fs b continuous, $(g \circ f)^{-1}(H, K) = f^{-1}(g^{-1}(H, K))$ is fs b-open in U and hence $g \circ f$ is fssbc.*

v. *Let (H, K) be fs clopen set in W. Since g is fs slightly continuous, $g^{-1}(H, K)$ is fs b open in V and f is fs strongly b continuous. $(g \circ f)^{-1}(H, K) = f^{-1}(g^{-1}(H, K))$ Is fs open in U. Consequently $g \circ f$ is fssc.*

vi. *Let (H, K) be fs b open set in W, since g is fs perfectly b continuous, $g^{-1}(H, K)$ is fs open. fs closed in V since f is fssbc, $(g \circ f)^{-1}(H, K) = f^{-1}(g^{-1}(H, K))$ is fs b open in U. Accordingly $g \circ f$ is fs b irresolute.*

vii. *Let (H, K) be fs clopen set in W, since g is fs contra continuous, $g^{-1}(H, K)$ is fs open and fs closed in V since, f is fssbc, $(g \circ f)^{-1}(H, K) = f^{-1}(g^{-1}(H, K))$ is fs b open in U. Hence $g \circ f$ is fssbc.*

viii. *Let (H, K) be fs b clopen set in W, since g is fs contra b continuous, $g^{-1}(H, K)$ is fs b open and fs b closed in V. Since f is fs b irresolute, $(g \circ f)^{-1}(H, K) = f^{-1}(g^{-1}(H, K))$ is fs b open, and fs b closed in U. So $g \circ f$ is fssbc.*

Theorem 3.6. *If f is fssbc and U is fs $bT_{1/2}$ topological space, then f is fssc.*
Proof: *Let (H, K) be fs clopen set in W. Since f is fssbc, $f^{-1}(H, K)$ is fs b open in the space U and U is fs $bT_{1/2}$ space, so $f^{-1}(H, K)$ is fs open in U. Hence f is fssc.*

Theorem 3.7. *If f is fssbc and U is fs b connected space, then V is not fs discrete space.*
Proof: *Let us assume V as fs discrete space. Let (H, K) be a proper non-empty fs open subset of V. Since, f is fs slightly b continuous, so $f^{-1}(H, K)$ is proper non-empty fs b clopen subset of U, which contradicts that U is fs b connected. Therefore V is not fs discrete space.*

Theorem 3.8. *If f is fssbc and V is fs locally indiscrete space, then f is fs b continuous.*
Proof: *Let (H, K) be fs open set in V and V is locally indiscrete space with (H, K) is fs closed in V. And function is fs slightly b continuous, $f^{-1}(H, K)$ is fs b open in U. Hence f is fs b continuous.*

Remark 3.1. *From the above observations of stronger and weaker forms of fs slightly b continuous functions in fsts we have the following implications.*

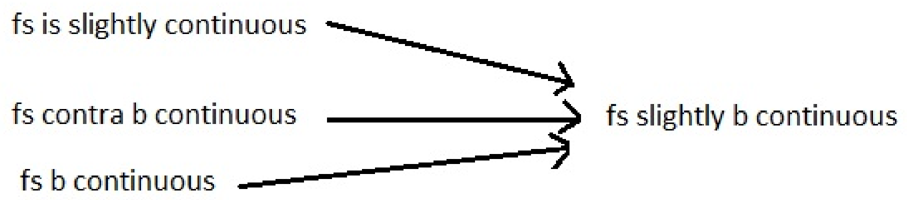

Figure 1:

4 Fuzzy soft semi b continuous functions

Throughout this section, (U, τ, K) and (V, σ, K) be any two fsts where K is the set of parameters and f be a mapping from U to V

Definition 4.1. *A function f is fs semi b continuous if the inverse of every fs b open (fsbo) set is fs semi-open. The family of all fs semi b continuous functions is denoted by fssmbc.*

Theorem 4.1. *If f is a member of fssmbc then it is fs semi-continuous.*
Proof: *If $f \in fssmbc$ and (G, K) is fs open set in V, since every fs open set is fsbo, $f^{-1}(G, K)$ is fs semi-open in U. Hence f is fs semi-continuous.*
The converse of the this theorem need not be true in general.

Example 4.1. *Consider the fs identity map f from U to V.*
Let $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$, $\sigma = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$ be fsts. Let $U = \{a, b\}$, $V = \{c, d\}$, $K = \{e_1, e_2\}$, where
$(F_1, K) = \{\{0.5/a, 0.3/b\}, \{0.2/a, 0.4/b\}\}$
$(F_2, K) = \{\{0.3/a, 0.1/b\}, \{0.2/a, 0.3/b\}\}$,
$(G_1, K) = \{\{0.5/c, 0.7/d\}, \{0.3/c, 0.5/d\}\}$,
$(G_2, K) = \{\{0.4/c, 0.5/d\}, \{0.2/c, 0.3/d\}\}$.
Consider $(H, K) = \{\{0.3/c, 0.4/d\}, \{0.3/c, 0.1/d\}\}$ a fsbo set in V. Since $f^{-1}(H, K)$ it is not fs semi-open in U, f does not belong to fssmbc. But it is fs semi-continuous.

Theorem 4.2. *If $f \in fssmbc$ then f is fs b continuous.*
Proof: *If f is in fssmbc, the inverse of every fsbo set is fs semi-open. Consider fs open set (G, K) in V, $f^{-1}(G, K)$ is fs semi-open and hence fsbo in U, f is fs b continuous.*
But the converse is not as seen from the above example 4.1, $f^{-1}(G_1, K)$ and $f^{-1}(G_2, K)$ are fs b-open sets in U. Therefore, f is fs b continuous. But $f^{-1}(H, K)$ is not fs semi-open in U, Hence $f \notin fssmbc$.

Theorem 4.3. *If $f \in fssmbc$ then f is fsgb continuous.*
Proof: *For an fs semi b-continuous function, the inverse image of a fsbo set is fs semi-open. Each fs semi-open set is fs gb open. Hence f is fsgb continuous. With the counter example, we can prove that converse is not valid.*

Example 4.2. *Let f be fs identity mapping and $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$, $\sigma = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$ be fsts.*
Let $U = \{a, b\}$, $V = \{c, d\}$, $K = \{e_1, e_2\}$, where
$(F_1, K) = \{\{0.3/a, 0.4/b\}, \{0.5/a, 0.6/b\}\}$,
$(F_2, K) = \{\{0.4/a, 0.4/b\}, \{0.6/a, 0.6/b\}\}$,

$(G_1, K) = \{\{0.5/c, 0.4/d\}, \{0.1/c, 0.8/d\}\}$,
$(G_2, K) = \{\{0.4/c, 0.3/d\}, \{0.1/c, 0.6/d\}\}$.
Consider fs b open set, $(H, K) = \{\{0.4/c, 0.2/d\}, \{0.5/c, 0.4/d\}\}$ in V. Then $f^{-1}(G_1, K)$ and $f^{-1}(G_2, K)$ are fs gb-open sets in U but $f^{-1}(H, K)$ is not fs semi available in U. Thus f is fs gb continuous but $f \notin fssmbc$.

Theorem 4.4. *If f is a member of fssmbc then it is fs semi pre continuous.*
Proof: *Every fs semi-open set is fs semi pre-open proof is evident.*

Theorem 4.5. *If $\theta \in fssmbc$ and U is fs $bT_{1/2}$ space, then θ is fs continuous.*
Proof: *Since θ is fs semi b continuous function, for any fs open set (G, K) in V, $\theta^{-1}(G, K)$ is fs semi-open in U. And every fs semi-open set is fsbo and hence fs open in fs $bT_{1/2}$ space, θ is fs continuous.*
The converse is not true.

Example 4.3. *Let $\theta : (U, \tau, K) \to (V, \sigma, K)$ be a fs mapping defined by $\theta(a) = d$ and $\theta(b) = c$. Let $U = \{a, b\}$, $V = \{c, d\}$ and $K = \{e_1, e_2\}$.*
Let $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$ and $\sigma = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$ be fsts, Where $(F_1, K) = \{\{0.6/a, 0.5/b\}, \{0.4/a, 0.5/b\}\}$,
$(F_2, K) = \{\{0.5/a, 0.4/b\}, \{0.3/a, 0.4/b\}\}$,
$(G_1, K) = \{\{0.5/c, 0.6/d\}, \{0.5/c, 0.4/d\}\}$,
$(G_2, K) = \{\{0.4/c, 0.5/d\}, \{0.4/c, 0.3/d\}\}$. *Consider fs b open set $(H, K) = \{\{0.6/c, 0.5/d\}, \{0.7/c, 0.7/d\}\}$ in V and $\theta^{-1}(G_1, K)$, $\theta^{-1}(G_2, K)$ are fs open sets but $\theta^{-1}(H, K)$ is not fs semi-open in U. Thus θ is fs continuous but $\theta \notin fssmbc$.*

Theorem 4.6. *If $\alpha : (U, \tau, K) \to (V, \tau', K)$ is fs semi b continuous and $\beta : (V, \tau', K) \to (W, \sigma, K)$ is fs b continuous, then $\beta \circ \alpha : (U, \tau, K) \to (W, \sigma, K)$ is fs gb continuous.*
Proof: *Let (H, K) be fs open set in W, since β is fs b continuous, $\beta^{-1}(H, K)$ is fsbo in V and α is fs semi b continuous, so $(\beta \circ \alpha)^{-1}(H, K) = \alpha^{-1}(\beta^{-1}(H, K))$ is fs semi-open and hence fs gb open in U. Thus $(\beta \circ \alpha)$ is fs gb continuous.*

Theorem 4.7. *If $\alpha : (U, \tau, K) \to (V, \tau', K)$ is fs semi b continuous and $\beta : (V, \tau', K) \to (W, \sigma, K)$ is fs semi-continuous, then $\beta \circ \alpha : (U, \tau, K) \to (W, \sigma, K)$ is fs semi pre continuous.*
Proof: *Let (H, K) be fs open set in W, since β is fs semi-continuous, $\beta^{-1}(H, K)$ is fs semi-open and also fsbo in V. Since α is fs semi b continuous $(\beta \circ \alpha)^{-1}(H, K) = \alpha^{-1}(\beta^{-1}(H, K))$, is fs semi-open in U. Every fs semi-open set is fs semi pre-open. Hence $(\beta \circ \alpha)$ is fs semi pre continuous.*

Remark 4.1. *The relations of stronger and weaker forms of fs semi-continuous functions in fsts is represented as :*

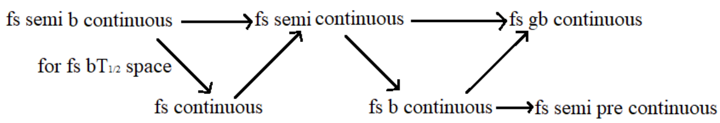

Figure 2:

5 Fuzzy soft pre b continuous functions

In this section $\eta : (U, \tau, K) \to (V, \tau', K)$ is defined as fs mapping, and parameter set be K where U and V are fsts.

Definition 5.1. *A function η is said to be fs pre b continuous if the inverse of each fsbo in V is fs pre-open in U. The family of fs pre b continuous functions is denoted by fspbc.*

Theorem 5.1. *Every fs pre b continuous function is fs pre continuous.*
Proof: *Let η be fs pre b continuous mapping, (G, K) be fs open set in V. Since every fs open set is fsbo, $\eta^{-1}(G, K)$ is fs pre-open in U. Hence η is fs pre continuous. But converse need not be true in general.*

Example 5.1. *Let $\eta : (U, \tau, K) \to (V, \tau', K)$ be a function defined by $\eta(x_1) = y_2$ and $\eta(x_2) = y_1$ where $U = \{x_1, x_2\}$, $V = \{y_1, y_2\}$, $K = \{e_1, e_2\}$.*
Let $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$ and $\tau' = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$ be fsts.
$(F_1, K) = \{\{0.5/x_1, 0.6/x_2\}, \{0.3/x_1, 0.4/x_2\}\}$
$(F_2, K) = \{\{0.4/x_1, 0.3/x_2\}, \{0.2/x_1, 0.4/x_2\}\}$
$(G_1, K) = \{\{0.3/y_1, 0.3/y_2\}, \{0.2/y_1, 0.2/y_2\}\}$
$(G_2, K) = \{\{0.5/y_1, 0.3/y_2\}, \{0.3/y_1, 0.3/y_2\}\}$.
Consider $(H, K) = \{\{0.4/y_1, 0.1/y_2\}, \{0.3/y_1, 0.2/y_2\}\}$ which is a fsbo in V. $\eta^{-1}(G_1, K)$ and $\eta^{-1}(G_2, K)$ are fs pre-open sets in U, But $\eta^{-1}(H, K)$ is not fs pre-open in U. Therefore, η is fs pre-continuous but not fs pre b continuous.

Theorem 5.2. *If $\eta \in fspbc$ then η is fs b continuous.*
Proof: *Let η be fs pre b-continuous function. So the inverse of every fsbo set is fs pre-open and each fs pre-open set is fsbo. Hence η is fs b continuous.*
Converse of the above theorem need not be accurate.

Example 5.2. *Let $\eta : (U, \tau, K) \to (V, \tau', K)$ be fs identity map, where $U = \{x_1, x_2\}$, $V = \{y_1, y_2\}$ and $K = \{e_1, e_2\}$.*
Let $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$ and $\tau' = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$ be fuzzy soft topological spaces. $(F_1, K) = \{\{0.3/x_1, 0.2/x_2\}, \{0.2/x_1, 0.3/x_2\}\}$
$(F_2, K) = \{\{0.3/x_1, 0.3/x_2\}, \{0.8/x_1, 0.5/x_2\}\}$
$(G_1, K) = \{\{0.5/y_1, 0.6/y_2\}, \{0.2/y_1, 0.3/y_2\}\}$

$(G_2, K) = \{\{0.4/y_1, 0.3/y_2\}, \{0.2/y_1, 0.3/y_2\}\}$.
Consider $(H, K) = \{\{0.5/y_1, 0.6/y_2\}, \{0.3/y_1, 0.3/y_2\}\}$ *which is a fsbo set in* V. $\eta^{-1}(H, K)$ *is not* fs *pre-open in* U. *Hence* $\eta \notin fspbc$. *But it is* $fs\ b$ *continuous.*

Theorem 5.3. *If* $\eta \in fspbc$ *then* η *is fsgb continuous.*
Proof: *Let* $\eta : (U, \tau, K) \to (V, \tau', K)$ *be* fs *pre* b*- continuous function and* (G, K) *be* fs *open set in* V *since every* fs *open set is fsbo and* η *is* fs *pre-b-continuous,* $\eta^{-1}(G, K)$ *is* fs *pre-open, and hence* fs *gb open in* U.
Converse is not be true.

Example 5.3. *Let* $\eta : (U, \tau, K) \to (V, \tau', K)$ *be* fs *identity mapping, where* $U = \{x_1, x_2\}$, $V = \{y_1, y_2\}$ *and* $K = \{e_1\}$.
Let $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$ *and* $\tau' = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$ *be fuzzy soft topological spaces.* $(F_1, K) = \{\{0.3/x_1, 0.2/x_2\}\}$ $(F_2, K) = \{\{0.3/x_1, 0.3/x_2\}\}$ $(G_1, K) = \{\{0.5/y_1, 0.6/y_2\}\}$ $(G_2, K) = \{\{0.4/y_1, 0.3/y_2\}\}$.
Consider $fs\ b$ *open set* $(H, K) = \{\{0.8/y_1, 0.7/y_2\}\}$ *in* V. *Since,* $\eta^{-1}(G_1, K)$ *and* $\eta^{-1}(G_2, K)$ *are* fs *gb-open but* $\eta^{-1}(H, K)$ *is not* fs *pre-open in* U, η *is* fs *gb-continuous but not* fs *pre* b*- continuous.*

Theorem 5.4. *If* $\eta \in fspbc$ *then it is* fs *semi pre continuous.*
Proof: *Let* $\eta : (U, \tau, K) \to (V, \tau', K)$ *be* fs *pre* b*- continuous function. Let* (G, K) *be* fs *open set in* V. *Hence* $fs\ b$ *open and* $\eta^{-1}(G, K)$ *is* fs *pre-open in* U. *Every* fs *pre-open set is* fs *semi pre-open,* η *is* fs *semi pre continuous.*
Converse need not be accurate as seen from the example 6.1, $\eta^{-1}(G_1, K)$ *and* $\eta^{-1}(G_2, K)$ *are* fs *semi pre-open sets in* U. *Therefore* η *is* fs *semi pre continuous. But* $\eta^{-1}(H, K)$ *is not* fs *pre-open in* U. *Hence* η *is not* fs *pre* b*- continuous.*

Theorem 5.5. *If* η *is* fs *pre* b *continuous and* (U, τ, K) *is* $fs\ bT_{1/2}$ *space, then* η *is* fs *continuous.*
Proof: *Let* $\eta : (U, \tau, K) \to (V, \tau', K)$ *be* fs *pre* b*- continuous function. So the inverse of each fsbo set is* fs *pre-open and hence fsbo in* U. *But* U *is* $fs\ bT_{1/2}$ *space each* $fs\ b$ *open set is* fs *open,* η *is* fs *continuous.*

Example 5.4. *Let* $\eta : (U, \tau, K) \to (V, \tau', K)$ *be defined by* $\eta(x_1) = y_2$ *and* $\eta(x_2) = y_1$, *where* $U = \{x_1, x_2\}$, $V = \{y_1, y_2\}$, $K = \{e_1, e_2\}$
$\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$*and* $\tau' = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$ *be fuzzy soft topological spaces.*
$(F_1, K) = \{\{0.6/x_1, 0.5/x_2\}, \{0.4/x_1, 0.5/x_2\}\}$
$(F_2, K) = \{\{0.5/x_1, 0.4/x_2\}, \{0.3/x_1, 0.4/x_2\}\}$
$(G_1, K) = \{\{0.5/y_1, 0.6/y_2\}, \{0.5/y_1, 0.4/y_2\}\}$
$(G_2, K) = \{\{0.4/y_1, 0.5/y_2\}, \{0.4/y_1, 0.3/y_2\}\}$.
Consider $fs\ b$ *open set* $(H, K) = \{\{0.6/y_1, 0.5/y_2\}, \{0.6/y_1, 0.7/y_2\}\}$ *in* V.
Since $\eta^{-1}(G_1, K)$ *and* $\eta^{-1}(G_2, K)$ *are* fs *open sets and* $\eta^{-1}(H, K)$ *is not* fs *pre-open in* U, η *is* fs *continuous but not* fs *pre* b *continuous.*

Theorem 5.6. *If $\eta : (U, \tau, K) \to (V, \tau', K)$ is fs pre b continuous and $\mu : (V, \tau', K) \to (W, \sigma, K)$ is fs b continuous, then $\mu \circ \eta : (U, \tau, K) \to (W, \sigma, K)$ is fs gb continuous.*
Proof: *Let (H, K) be fs open set in W, since g is fs b continuous, $\mu^{-1}(H, K)$ is fsbo in V and η is fs pre b continuous $(\mu \circ \eta)^{-1}(H, K) = \eta^{-1}(\mu^{-1}(H, K))$ is fs pre-open in U. Every fs pre-open set is fs gb open, $(\mu \circ \eta)$ is fs gb continuous.*

Remark 5.1. *From the above observations we have the following implication:*

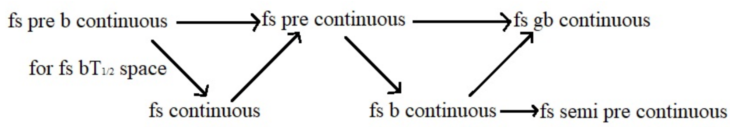

Figure 3:

6 Fuzzy soft totally b continuous functions

Definition 6.1. *A function $\psi : (U, \tau, K) \to (V, \tau', K)$ is fs totally continuous if the inverse of every fs open set is fs clopen.*

Definition 6.2. *A function $\psi : (U, \tau, K) \to (V, \tau', K)$ is fs totally b continuous if inverse image of every fs open set in V is fs b clopen in U.*

Theorem 6.1. *Every fs totally continuous function is fs totally b continuous.*
Proof: *Since every fs open(closed) set is fs b open (b closed), it was evident that every fs totally continuous function is fs totally b-continuous.*
But converse need not be true.

Example 6.1. *Let $\psi : (U, \tau, K) \to (V, \tau', K)$ be defined by $\psi(x_1) = y_2$ and $\psi(x_2) = y_1$. Let $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$, $\tau' = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$, be fuzzy soft topological spaces. Let $U = \{x_1, x_2\}$, $V = \{y_1, y_2\}$ and $K = \{e_1, e_2\}$*
$(F_1, K) = \{\{1/x_1, 0.9/x_2\}, \{0.8/x_1, 0.8/x_2\}\}$
$(F_2, K) = \{\{0/x_1, 0.1/x_2\}, \{0.2/x_1, 0.2/x_2\}\}$
$(G_1, K) = \{\{0.7/x_1, 0.6/x_2\}, \{0.5/x_1, 0.6/x_2\}\}$
$(G_2, K) = \{\{0.3/x_1, 0.4/x_2\}, \{0.5/x_1, 0.4/x_2\}\}$. *Then $\psi^{-1}(G_1, K)$ and $\psi^{-1}(G_2, K)$ are neither fs open, nor fs closed sets in U. But they are fs b clopen sets in U. Therefore ψ, fs totally b continuous but not fs totally continuous.*

Theorem 6.2. *Every fs perfectly b-continuous function is fs totally b continuous.*
Proof: *Let $\psi : (U, \tau, K) \to (V, \tau', K)$ be fs perfectly b continuous function. Consider an fs open set f_K in V. Since f_K is fs b open and ψ is fs perfectly b*

continuous. $\psi^{-1}(f_K)$ *is* fs *open and* fs *closed in* U*. Every* fs *open (closed) set is fsbo (closed), thus* ψ *is* fs *totally* b *continuous.*
But the converse is not valid.
In example 6.1, $\psi^{-1}(G_1, K)$ *and* $\psi^{-1}(G_2, K)$ *are* fs b *open, and* b *closed sets in* U*. Therefore* ψ *is* fs *totally b-continuous. Consider a fsbo set* $(H, K) = \{\{0.2/y_1, 0.3/y_2\}, \{0.3/y_1, 0.4/y_2\}\}$*. But* $\psi^{-1}(H, K)$ *it is neither* fs *open nor* fs *closed in* U*. Therefore* ψ *is not perfectly* b *continuous.*

Remark 6.1. *The concepts of* fs *strongly b-continuous function and* fs *totally* b *continuous functions are independent of each other.*
In Example 6.1, $\psi^{-1}(G_1, K)$ *and* $\psi^{-1}(G_2, K)$ *are* fs b *open, and* b *closed sets in* U*. Therefore* ψ *is* fs *totally b-continuous. Consider a fsbo set* $(H, K) = \{\{0.2/y_1, 0.3/y_2\}, \{0.3/y_1, 0.4/y_2\}\}$ *in* V*,* $\psi^{-1}(H, K)$ *is not* fs *open in* U*. Therefore* ψ *is not strongly b-continuous.*

Theorem 6.3. *Every* fs *totally b-continuous function is* fs b *continuous.*
Proof: *Let* $\psi : (U, \tau, K) \to (V, \tau', K)$ *be* fs *totally* b *continuous function. Then inverse of each* fs *open set is fsbo, and* fs b *closed in* U*. So* ψ *is* fs b *continuous.*
Following example shows that, fs b *continuous function need not be* fs *totally* b *continuous.*

Example 6.2. *Let* $\psi : (U, \tau, K) \to (V, \tau', K)$ *be a* fs *identity map.* $U = \{x_1, x_2\}$*,* $V = \{y_1, y_2\}$ *and* $K = \{e_1, e_2\}$*,*
Let $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K), (F_3, K), (F_4, K), (F_5, K), (F_6, K), (F_7, K)\}$*,*
$\tau' = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$ *be fsts.*

$$(F_1, K) = \left\{\left\{\frac{1/2}{x_1}, \frac{1/3}{x_2}\right\}, \left\{\frac{1/4}{x_1}, \frac{2/3}{x_2}\right\}\right\} \quad (F_2, K) = \left\{\left\{\frac{1/3}{x_1}, \frac{1/4}{x_2}\right\}, \left\{\frac{0}{x_1}, \frac{1/6}{x_2}\right\}\right\}$$

$$(F_3, K) = \left\{\left\{\frac{1/2}{x_1}, \frac{1}{x_2}\right\}, \left\{\frac{2/3}{x_1}, \frac{1/6}{x_2}\right\}\right\} \quad (F_4, K) = \left\{\left\{\frac{1/5}{x_1}, \frac{1/3}{x_2}\right\}, \left\{\frac{1/4}{x_1}, \frac{1/6}{x_2}\right\}\right\}$$

$$(F_5, K) = \left\{\left\{\frac{1/5}{x_1}, \frac{1/4}{x_2}\right\}, \left\{\frac{0}{x_1}, \frac{1/6}{x_2}\right\}\right\} \quad (F_6, K) = \left\{\left\{\frac{1/2}{x_1}, \frac{1}{x_2}\right\}, \left\{\frac{2/3}{x_1}, \frac{2/3}{x_2}\right\}\right\}$$

$$(F_7, K) = \left\{\left\{\frac{1/3}{x_1}, \frac{1/3}{x_2}\right\}, \left\{\frac{1/4}{x_1}, \frac{1/6}{x_2}\right\}\right\}$$

$$(G_1, K) = \left\{\left\{\frac{1/2}{y_1}, \frac{1/4}{y_2}\right\}, \left\{\frac{1/5}{y_1}, \frac{0}{y_2}\right\}\right\} \quad (G_2, K) = \left\{\left\{\frac{1/4}{y_1}, \frac{1/5}{y_2}\right\}, \left\{\frac{1/6}{y_1}, \frac{0}{y_2}\right\}\right\}$$

Then $\psi^{-1}(G_1, K)$ *and* $\psi^{-1}(G_2, K)$ *are fsbo sets, but they are not* fs b *closed sets in* U*. Therefore* ψ *is* fs *b-continuous but not* fs *totally* b *continuous.*

Theorem 6.4. *Every* fs *totally b-continuous function is fsgb continuous.*
Proof: *Let* $\psi : (U, \tau, K) \to (V, \tau', K)$ *be* fs *totally b-continuous function. Then*

inverse of fs open set is fs b-open, and fs b-closed in U. Since the inverse of every fsbo set is fs gb-open, ψ is fs gb-continuous. Example 6.2, give the converse is not true, $\psi^{-1}(G_1, K)$ and $\psi^{-1}(G_2, K)$ are fs gb open sets but not fs b closed sets in U. Therefore ψ, fsgb continuous but not fs totally b-continuous.

Theorem 6.5. *Every fs totally b continuous function is fs semi pre continuous.*
Proof: *Let $\psi : (U, \tau, K) \to (V, \tau', K)$ be fs totally b- continuous function. Let (G, K) be fs open set in V, then $\psi^{-1}(G, K)$ is fs b open and fs b closed in U and every fs b open set is fs semi pre-open, ψ is fs semi pre continuous.*
In example 6.2, $\psi^{-1}(G_1, K)$ and $\psi^{-1}(G_2, K)$ are fs semi pre-open sets but not fs b open, and fs b closed sets in U. Therefore ψ, fs semi pre continuous but not fs totally b continuous. Hence converse of this is not true in general.

Theorem 6.6. *If $f : (U, \tau, K) \to (V, \tau', K)$ is fs totally b continuous and $\lambda : (V, \tau', K) \to (W, \sigma, K)$ is fs b continuous, then $\lambda \circ f : (U, \tau, K) \to (W, \sigma, K)$ is fs gb continuous.*
Proof: *Let (H, K) be fs open set in W, since λ is fs b continuous, $\lambda^{-1}(H, K)$ is fsbo in V and f is fs totally b continuous, $(\lambda \circ f)^{-1}(H, K) = f^{-1}(\lambda^{-1}(H, K))$ is fs b open and fs b closed in U. Every fsbo is fs gb open, $(\lambda \circ f)$ is fsgb continuous.*

Theorem 6.7. *If $f : (U, \tau, K) \to (V, \tau', K)$ is fs totally b continuous and $\lambda : (V, \tau', K) \to (W, \sigma, K)$ is fs b continuous, then $\lambda \circ f : (U, \tau, K) \to (W, \sigma, K)$ is fs semi pre continuous.*
Proof: *Let (H, K) be fs open set in W, since λ is fs b continuous, $\lambda^{-1}(H, K)$ is fsbo in V and f is fs totally b continuous, $(\lambda \circ f)^{-1}(H, K) = f^{-1}(\lambda^{-1}(H, K))$ is fs b open and fs b closed in U. Since every fs b open set is fs semi pre-open, $(\lambda \circ f)$ is fs semi pre continuous.*

Remark 6.2. *The concept of fs pre b continuous and totally b continuous functions in fsts are independent of each other.*

Example 6.3. *Suppose $\psi : (U, \tau, K) \to (V, \tau', K)$ is defined by $\lambda(x_1) = y_2$ and $\lambda(x_2) = y_1$ and $\tau = \{\widetilde{O}, \widetilde{1}, (F_1, K), (F_2, K)\}$, $\tau' = \{\widetilde{O}, \widetilde{1}, (G_1, K), (G_2, K)\}$, be any two fsts. Let $U = \{x_1, x_2\}$, $V = \{y_1, y_2\}$ and $K = \{e_1, e_2\}$,*

$$(F_1, K) = \left\{ \left\{ \frac{1}{x_1}, \frac{0.9}{x_2} \right\}, \left\{ \frac{0.8}{x_1}, \frac{0.8}{x_2} \right\} \right\} \ (F_2, K) = \left\{ \left\{ \frac{0}{x_1}, \frac{0.1}{x_2} \right\}, \left\{ \frac{0.2}{x_1}, \frac{0.2}{x_2} \right\} \right\}$$

$$(G_1, K) = \left\{ \left\{ \frac{0.7}{y_1}, \frac{0.6}{y_2} \right\}, \left\{ \frac{0.5}{y_1}, \frac{0.6}{y_2} \right\} \right\} \ (G_2, K) = \left\{ \left\{ \frac{0.3}{y_1}, \frac{0.4}{y_2} \right\}, \left\{ \frac{0.5}{y_1}, \frac{0.4}{y_2} \right\} \right\}$$

Here, $\lambda^{-1}(G_1, K)$ and $\lambda^{-1}(G_2, K)$ are fs b open, and b closed sets in U. Thus λ is fs totally b continuous. Consider an fs b open set $(H, K) = \{\{0.2/y_1, 0.3/y_2\}, \{0.3/y_1, 0.4/y_2\}\}$. Since $\lambda^{-1}(H, K)$ it is not fs pre-open in U, λ it is not fs pre b continuous.

Remark 6.3. *From the above observations, we have the following :*

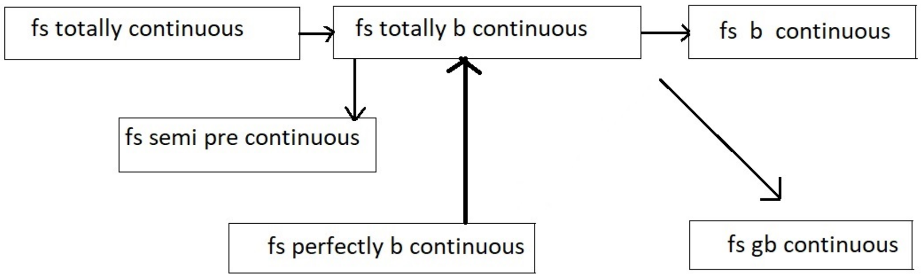

Figure 4:

7 Fuzzy soft b compact spaces

Definition 7.1. *Let (U, τ, K) be fsts and $f_K \in fss(U, \tau, K)$ the set of all fs sets. An fs set f_K is called b compact if each fs b open cover of f_K has a finite subcover. Also (U, τ, K) is called fs b compact if each fs open cover of $\widetilde{1_K}$ has a finite subcover.*

Remark 7.1. *A fsts is fs b compact if U is finite.*

Example 7.1. *Let (U, τ, K) and (V, σ, K) be two fsts and $\tau \subset \sigma$. Then fsts (U, τ, K) is fs b compact if (V, σ, K) is fs b compact.*

Definition 7.2. *A fsts (U, τ, K) is called a*

i. strongly compact if and only if every fs pre-open cover of U has a finite subcover.

ii. semi-compact if and only if every fs semi-open cover of U has a finite subcover.

iii. semi pre compact if and only if every fs semi pre-open cover of U has a finite subcover.

iv. S-closed if and only if every fs semi-open cover of U has a finite subcollection whose closures cover U.

Remark 7.2. *Each fs semi-open and fs pre-open sets implies fs b open sets. Every fs b compact space means each of fs strongly compact and fs semi-compact spaces. Also, since fs b-open set implies fs semi pre-open set, it is clear that fs semi pre compact space means fs b compact space. The finite intersection property for fs b compact spaces is provided as follows.*

Definition 7.3. *A family ψ of fs b open sets has the finite intersection property if the intersection of members of each finite subfamily of ψ is not the null fs set.*

Theorem 7.1. *A fsts U is fs b compact if and only if each family of fs b closed sets with the finite intersection property has a non-null intersection.*
Proof: *Let ψ be an arbitrary family of fs b closed sets with the finite intersection property. We assume that $\bigcap\limits_{i\in I}\{(f_i, K) : (f_i, K) \in \psi\}$ is non-null, that is $\bigcap\limits_{i\in I}(f_i, K) = \widetilde{O}_K$. Then $(\bigcap\limits_{i\in I}(f_i, K))^c = \bigcup\limits_{i\in I}(f_i, K)^c = \widetilde{1}_K$. Since each (fi, K) is b closed, the family $\{(f_i, K)^c : i \in I\}$ is fs b open cover of U. But U is fs b compact, therefore $\bigcup\limits_{i\in I}(f_i, K) = \widetilde{1}_K$. Thus we have $K = (\bigcap\limits_{i\in I}(f_i, K))^c \bigcap\limits_{i\in I}(f_i, K) = \widetilde{O}_K$ a contradiction to assumption.*
Suppose U is such that each family of fs b closed sets with the finite intersection property has a non-null intersection. Let $\psi = \{(f_i, K) : i \in I\}$ be a family of fs b open sets. Let ψ has a finite subfamily that also covers U. Assume that $\bigcup\limits_{i\in J}(f_i, K) = \widetilde{1}_K$ for any finite $J < I$.
Then $\bigcap\limits_{i\in J}(f_i, K)^c = (\bigcup\limits_{i\in J}(f_i, K))^c \neq \widetilde{0}_K$, since J is finite. Thus $\{(f_i, K)^c : i \in I\}$ has finite intersection property. By assumption $\bigcap\limits_{i\in I}(f_i, K)^c \neq \widetilde{O}_K$, and we have $\bigcup\limits_{i\in I}(f_i, K) \neq \widetilde{1}_K$. This is a contradiction. Thus U is fs b compact.

Theorem 7.2. *Let g_K be fs closed set in fs b compact space (U, τ, K). Then g_K is also fs b compact.*
Proof: *Let $\psi = \{(h_i, K) : i \in I\}$ be fs b open cover of g_K. Then $\widetilde{1}_K \subseteq \{(\bigcup\limits_{i\in I}(h_i, K)) \cup (g, K)^c\}$. Therefore there exists a finite sub covering $(h_1, K), (h_2, K), (h_3, K)\ldots..(g, K)^c$. Hence we get $\widetilde{1}_K \subseteq (h_1, K) \cup (h_2, K) \cup (h_3, K) \cup \ldots ..... \cup (h_n, K) \cup (g, K)^c$. Therefore $(g, K) \subseteq (h_1, K) \cup (h_2, K) \cup (h_3, K) \cup \ldots ..... \cup (h_n, K) \cup (g, K)^c$, which implies $(g, K) \subseteq (h_1, K) \cup (h_2, K) \cup (h_3, K) \cup \ldots ..... \cup (h_n, K)$ since $(g, K) \cap (g, K)^c = \widetilde{0}_K$. Hence (g, K) has a finite subcover thus g_K is fs b compact.*

8 Conclusion

The present work zeroed in on introducing slightly b, semi b, pre b and totally b continuous mappings in fuzzy soft topological spaces. The correlation with the existing fs continuous functions are studied, established and compared. It is proved that every fs is slightly continuous, fs contra b continuous, and fs b continuous function is fs slightly b continuous. In contrast, the composition of fs slightly b continuous function need not be fs slightly b continuous. In fs bT1/2 space fs slightly b continuous function becomes fs slightly continuous. Counter examples have been shown to illustrate and evidence that the reverse implications do not imply either. It is deduced that every fs pre b continuous and fs semi b continuous function is fs b continuous, fs gb continuous, and fs semi pre continuous function. It is also enumerated that the converse is not valid with evidence using a counter example. In addition, it is implicated that fs totally b continuous function is also fs b continuous, fs gb continuous, and fs semi pre continuous with clear inferences that the reverse implication is not true. Further, it is also concluded that fs pre b continuous and fs totally b continuous, fs strongly b continuous and fs totally b continuous functions are independent of each other. A new form of topological space such as fs b compact space is also introduced. The current word forms the basis for further work to be computed, emphasising fuzzy soft topology as the fulcrum of computations.

Declaration of interests

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