Ratio Mathematica Volume 40, 2021, pp. 47-66

Analysis of classical retrial queue with differentiated vacation and state dependent arrival rate

Poonam Gupta*
Naveen Kumar#

Abstract

In the present paper, we have introduced the concept of differentiated vacations in a retrial queueing model with state-dependent arrival rates of customers. The arrival rate of customers is different in various states of the server. The vacation types are differentiated by means of their durations as well as the previous state of the server. In type I vacation, the server goes just after providing service to at least one customer whereas in type II, it comes after remaining free for some time. In a steady state, we have obtained the system size probabilities and other system performance measures. Finally, sensitivity and cost analysis of the proposed model is also performed. The probability generating function technique, parabolic method and MATLAB is used for this purpose.

Keywords: Retrial queue; Markov process; differentiated vacations; exponential distribution etc.

2010 AMS subject classification: **60K25, 60K30**

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* Baba Mastnath University, Asthal Bohar Rohtak India; poonammittal2207@gmail.com
# Baba Mastnath University, Asthal Bohar Rohtak India; naveenkapilrtk@gmail.com
[1]Received on May 4th 2021. Accepted on June 24th 2021. Published on June 30th 2021. doi: 10.23755/rm.v40i1.619. ISSN: 1592-7415. eISSN: 2282-8214.

1.Introduction

Retrial queues have wide applications in communication system, production system, computer networking system, telecommunication etc. Retrial queues are characterized by the fact that arriving customers on finding the server busy, leave the system and join the retrial group to complete their request for service after a random time period. A good survey on retrial queues have been done by Falin,Templeton [5] and Artalejo,Gomez-coral [1].

In queueing theory, many situations occur where arrival rate of customers depends upon the different states of server such as busy state, idle state or on vacation state etc. Singh et al.[14] studied M/G/1 queueing model with state dependent arrival of customers. Batch arrival queueing system under retrial policy with state dependent admission is analysed by Bagyam and chandrika [2]. Niranjan et al. [12] did the pioneer work on state dependent arrival in bulk retrial queues with Bernoulli feedback and multiple vacations.

Nowadays, Retrial queueing system with server vacation has become increasingly important due to wide applications in research area. In queueing system with vacation, server becomes unavailable from service station for random period of time due to some reasons like server breakdown, maintenance of server, service provided by server in secondary service station when primary station is empty or simply going for break etc. The time period during which the server is not available for primary customers is known as vacation. In single vacation queueing model, server goes for vacation of random time duration whenever there is no customer in the system and returns to the system after vacation completion. The idea of queueing system with server vacation was first discussed by Levy and Yechiali [9]. Doshi [3] had performed good survey on queueing model with vacation. Later on Takagi [16], Tian and Zhang [18] did the pioneer work on vacation queueing system.

In multiple vacation system, if server finds no customer in system on returning from vacation, then server immediately goes for another vacation otherwise server will serve the customers. Servi and Finn [13] introduced the concept of working vacation queueing system in which server works at slow rate during vacation period rather than completely stopping the service during vacation. In queueing literature, lot of work have been done on queueing model with working vacation by many researchers [8,23]. Li and Tian [10] analysed M/M/1 queueing model with working vacation and interruption. Retrial queueing model with working vacation was first studied by Do [4]. Later on Li et al. and Tao et al. [11,17] did pioneer work on retrial queueing model with working vacation and interruption.

In differentiated vacation queueing model, server takes vacation I i.e. vacation of longer duration after serving all the customers in system and vacation II i.e. vacation of shorter duration will be taken by server if there is no customer in system after completing the type I vacation. The concept of differentiated vacations in queuing literature was first introduced by Ibe and Isijola [6]. In this paper they considered two types of vacations with different durations. Further they extended their model by introducing the concept of vacation interruption [7]. M/M/1 single server queue with m kinds of differentiated working vacations was analyzed by Zhang and Zhou [22]. Vijayashree and Janani [21] performed transient solution of M/M/1 queueing system with differentiated vacation. Suranga Sampath and Liu [15] studied the customer’s impatience behaviour on M/M/1 queueing system subject to differentiated vacation. Unni and Mary [19] studied queueing system with multiple servers under differentiated vacations. Further they extended their work by introducing differentiated working vacation [20].

In this paper, we have extended the concept of differentiated vacations to queueing system under classical retrial policy considering the state dependent arrival of customers. The organization of rest of the paper into different sections is as follows. The model description is given in section 2. Section 3 is devoted to steady state equations and solutions. The closed form expressions for some of the performance measures are derived in section 4. Section 5 represents the effect of various parameters on some important system performance measures graphically. Conclusion and future scope is discussed in section 6.

2. Model description

The model is outlined as follows.

1. Customers arrive according to Poisson process but with different rates depending on the present state of the server. The different arrival rates of customers are λ, α, γ, δ in busy, free, vacation I, vacation II states of the server, respectively.
2. The arriving customers are served on FCFS basis. If server is free in active period, the arriving customer is immediately served otherwise due to unavailability of waiting space in service area, he has to join a free pool of

infinite capacity known as orbit to wait for the service. From the orbit, customers retry for their turn with classical rate β. For convenience, the service time is supposed to follow exponential distribution with parameter μ.

3. As soon as the last customer is served i.e. system gets empty, the server leaves for type I vacation. At the end of type I vacation, if system is still empty, the server goes on type II vacation otherwise returns to active state to serve the waiting customers. On completion of vacation II, if there is a customer waiting in the system, server returns to free state in normal active period otherwise again goes on vacation II repeatedly. The vacation I is assumed to be of longer duration than vacation II. The time period of both vacations is assumed to follow exponential distribution with parameters $v_{1,}v_2$ respectively.

3. Steady state equations and solution

Denoting the probability of n customers in state k of the server by $p_{n\,k}$ and server states at time t by S(t) were

$$S(t)=\begin{cases} 1, server\ is\ busy\ in\ active\ period \\ 2, server\ is\ free\ in\ active\ period \\ 3, server\ is\ on\ type\ I\ vacation \\ 4, server\ is\ on\ type\ II\ vacation \end{cases}$$

Let N(t) be the number of customers in the orbit at time t. Then the quasi birth-death process is a Markovian process represented by {N(t),S(t)} with state space {(n, k), n ≥ 0, k=1,3,4} U {(n, 2), n ≥1}.

Using Markov Process, the differential difference equations for the proposed model are

$$\frac{d}{dt}p_{0\,1}(t) = \beta p_{1\,2}(t) - (\lambda+\mu)p_{0\,1}(t) \tag{1}$$

$$\frac{d}{dt}p_{n\,1}(t) = \lambda p_{n-1\,1}(t) + (n+1)\beta p_{n+1\,2}(t) + \alpha p_{n\,2}(t) \\ - (\lambda+\mu)p_{n\,1}(t), \quad n \geq 1 \tag{2}$$

$$\frac{d}{dt}p_{n\,2}(t) = v_1 p_{n\,3}(t) + v_2 p_{n\,4}(t) + \mu p_{n\,1}(t) - (\alpha + n\beta)p_{n\,2}(t), \ \ n \geq 1 \tag{3}$$

$$\frac{d}{dt}p_{0\,3}(t) = \mu p_{0\,1}(t) - (\gamma + v_1)p_{0\,3}(t) \tag{4}$$
$$\frac{d}{dt}p_{n\,3}(t) = \gamma p_{n-1\,3}(t) - (\gamma + v_1)p_{n\,3}(t), \qquad n \geq 1 \tag{5}$$
$$\frac{d}{dt}p_{0\,4}(t) = v_1 p_{0\,3}(t) - \delta p_{0\,4}(t) \tag{6}$$
$$\frac{d}{dt}p_{n\,4}(t) = \delta p_{n-1\,4}(t) - (\delta + v_2)p_{n\,4}(t), \qquad n \geq 1 \tag{7}$$

To obtain steady state equations, taking limit t → ∞ and using

$$\left.\begin{aligned} \lim_{t\to\infty} p_{n\,i}(\mathrm{t}) &= p_{n\,i} \\ \lim_{t\to\infty} \frac{d}{dt} p_{n\,i}(t) &= 0 \end{aligned}\right\} \quad i = 1,2,3,4$$

The steady state equations are

$$(\lambda + \mu)p_{0\,1} = \beta p_{1\,2} \tag{8}$$
$$(\lambda + \mu)p_{n\,1} = \lambda p_{n-1\,1} + (n+1)\beta p_{n+1\,2} + \alpha p_{n\,2}, \qquad n \geq 1 \tag{9}$$
$$(\alpha + n\beta)p_{n\,2} = v_1 p_{n\,3} + v_2 p_{n\,4} + \mu p_{n\,1}, \qquad n \geq 1 \tag{10}$$
$$(\gamma + v_1)p_{0\,3} = \mu p_{0\,1} \tag{11}$$
$$(\gamma + v_1)p_{n\,3} = \gamma p_{n-1\,3}, \qquad n \geq 1 \tag{12}$$
$$\delta p_{0\,4} = v_1 p_{0\,3} \tag{13}$$
$$(\delta + v_2)p_{n\,4} = \delta p_{n-1\,4}\ , \qquad n \geq 1 \tag{14}$$

Defining the probability generating functions as

$$P_i(z) = \sum_{n=0}^{\infty} p_{n\,i} z^n\,, \qquad i = 1,3,4 \tag{15}$$

$$P_2(z) = \sum_{n=1}^{\infty} p_{n\,2} z^n \tag{16}$$

Using equations (10), (11), (13)and P.G.Fs defined in(15) and (16), we get

$$z\beta P_2'(z) + \alpha P_2(z) = v_1 P_3(z) + v_2 P_4(z) + \mu P_1(z) - \left(\gamma + 2v_1 + \frac{v_1 v_2}{\delta}\right) p_{0\,3} \tag{17}$$

From equations (8), (9), (15) and (16) we obtain

$$(\lambda+\mu-\lambda z)P_1(z) = \beta P_2'(z)+\alpha P_2(z) \qquad (18)$$

Similarly using equations (11) and (12) along with (15), we get

$$(\gamma+v_1-\gamma z)P_3(z) = (\gamma+v_1)p_{0\,3}$$

$$P_3(z) = \frac{(\gamma+v_1)p_{0\,3}}{(\gamma+v_1-\gamma z)} \qquad (19)$$

On similar steps from equations (13), (14) and (15) we obtain

$$\mathrm{P}_4(\mathrm{z}) = \frac{\mathrm{v}_1(\delta+\mathrm{v}_2)}{\delta(\delta+\mathrm{v}_2-\delta \mathrm{z})}\mathrm{p}_{0\,3} \qquad (20)$$

Taking z=1 in equation (20),we obtain

$$P_4(1) = \frac{v_1(\delta+v_2)}{\delta v_2}p_{0\,3} \qquad (21)$$

From equation (17)

$$z\beta P_2'(z)+\alpha P_2(z) = v_1P_3(z)+v_2P_4(z)+\mu P_1(z)-Ap_{0\,3} \qquad (22)$$

where $\mathrm{A} = \left(\gamma+2v_1+\frac{v_1v_2}{\delta}\right)$

Using equations (18), (22) together, after some rearrangement of terms we obtain

$$P_2'(z)+\frac{\alpha\lambda}{\beta(\lambda z-\mu)}P_2(z) = \frac{(\lambda+\mu-\lambda z)}{\beta(1-z)(\lambda z-\mu)}\left(v_1P_3(z)+v_2P_4(z)-Ap_{0\,3}\right) \qquad (23)$$

To solve the differential equation (23)

Taking $\mathrm{I.F} = (\lambda \mathrm{z}-\mu)^{\frac{\alpha}{\beta}}$

$$\mathrm{P}_2(\mathrm{z}) = (\lambda \mathrm{z}-\mu)^{\frac{-\alpha}{\beta}}\int_0^{\mathrm{z}}(\lambda \mathrm{x}-\mu)^{\frac{\alpha}{\beta}}\frac{(\lambda+\mu-\lambda \mathrm{x})}{\beta(1-\mathrm{x})(\lambda \mathrm{x}-\mu)}\left(\mathrm{v}_1\mathrm{P}_3(\mathrm{x})+\mathrm{v}_2\mathrm{P}_4(\mathrm{x})-\mathrm{Ap}_{0\,3}\right)\mathrm{dx} \qquad (24)$$

Substituting value of $P_2'(z)$ in equation (18) and solving, we get

$$P_1(z) = \frac{(\alpha - \alpha z)P_2(z) - v_1 P_3(z) - v_2 P_4(z) + Ap_{0\,3}}{(\mu - z(\lambda + \mu - \lambda z))} \tag{25}$$

On differentiating equations (19),(20) we get

$$P_3'(z) = \frac{\gamma(\gamma+v_1)p_{0\,3}}{(\gamma + v_1 - \gamma z)^2} \tag{26}$$

$$P_4'(z) = \frac{v_1(\delta + v_2)}{\delta(\delta + v_2 - \delta z)^2} p_{0\,3} \tag{27}$$

Again, differentiating equations (26) and (27), we get

$$P_3''(z) = \frac{2\gamma^2(\gamma+v_1)p_{0\,3}}{(\gamma + v_1 - \gamma z)^3} \tag{28}$$

$$P_4''(z) = \frac{2\delta v_1(\delta + v_2)}{(\delta + v_2 - \delta z)^3} p_{0\,3} \tag{29}$$

Also, from equation (18), we get

$$P_2'(z) = \frac{(\lambda + \mu - \lambda z)P_1(z) - \alpha P_2(z)}{\beta} \tag{30}$$

Taking limit $z \to 1$in equations (19), (20), (24), (26), (27), (28) and (29) we get

$$P_3(1) = \frac{(\gamma+v_1)p_{0\,3}}{v_1} \tag{31}$$

$$P_4(1) = \frac{v_1(\delta + v_2)}{\delta v_2} p_{0\,3} \tag{32}$$

$$P_2(1) = (\lambda - \mu)^{\frac{-\alpha}{\beta}} \int_0^1 (\lambda x - \mu)^{\frac{\alpha}{\beta}} \frac{(\lambda + \mu - \lambda x)}{\beta(1 - x)(\lambda x - \mu)} (v_1 P_3(x) + v_2 P_4(x) - Ap_{0\,3})\, dx \tag{33}$$

$$P_3'(1) = \frac{\gamma(\gamma+v_1)}{(v_1)^2} p_{0\,3} \tag{34}$$

$$P_4'(1) = \frac{v_1(\delta + v_2)}{{v_2}^2} p_{0\,3} \tag{35}$$

$$P_3''(1) = \frac{2\gamma^2(\gamma+v_1)}{{v_1}^3} p_{0\,3} \tag{36}$$

$$P_4''(1) = \frac{2\delta v_1(\delta + v_2)}{{v_2}^3} p_{0\,3} \tag{37}$$

Taking limit $z \to 1$ in equation (25) and using L-Hospital rule, we get

$$P_1(1) = \frac{\alpha P_2(1) + v_1 P_3'(1) + v_2 P_4'(1)}{\mu - \lambda} \tag{38}$$

Taking limit $z \to 1$ in equation (30)

$$P_2'(1) = \frac{\mu P_1(1) - \alpha P_2(1)}{\beta} \tag{39}$$

On differentiating equation (25) and taking limit $z \to 1$ we get

$$P_1'(1) = \frac{\left(2\alpha P_2'(1) + v_1 P_3''(1) + v_2 P_4''(1)\right)(\mu - \lambda) + 2\lambda\left(\alpha P_2(1) + v_1 P_3'(1) + v_2 P_4'(1)\right)}{2(\mu - \lambda)^2} \tag{40}$$

All the P.G. F's are expressed in terms of $p_{0\,3}$ which is obtained by using normalization condition

$$\sum_{i=1}^{4} P_i(1) = 1 \tag{41}$$

It follows that,

$$p_{0\,3}\left[\left(\frac{\alpha + \mu - \lambda}{\mu - \lambda}\right)(\lambda - \mu)^{\frac{-\alpha}{\beta}} \int_0^1 (\lambda - \mu)^{\frac{\alpha}{\beta}} \frac{(\lambda + \mu - \lambda z)}{\beta(1 - z)(\lambda z - \mu)} \left\{v_1\left(\frac{\gamma + v_1}{\gamma + v_1 - \gamma z}\right) + \frac{v_1 v_2}{\delta}\left(\frac{\delta + v_2}{\delta + v_2 - \delta z}\right) - A\right\} dz + \frac{\gamma + v_1}{v_1} + \frac{\gamma(\gamma + v_1)}{v_1(\mu - \lambda)} + \frac{v_1(\delta + v_2)}{v_2(\mu - \lambda)} + \frac{v_1(\delta + v_2)}{v_2 \delta}\right] = 1 \tag{42}$$

$$p_{0\,3} = \left[\left(\frac{\alpha+\mu-\lambda}{\mu-\lambda}\right)(\lambda-\mu)^{\frac{-\alpha}{\beta}}\int_0^1 (\lambda-\mu)^{\frac{\alpha}{\beta}}\frac{(\lambda+\mu-\lambda z)}{\beta(1-z)(\lambda z-\mu)}\left\{v_1\left(\frac{\gamma+v_1}{\gamma+v_1-\gamma z}\right)+\frac{v_1v_2}{\delta}\left(\frac{\delta+v_2}{\delta+v_2-\delta z}\right)-A\right\}dz+\frac{\gamma+v_1}{v_1}+\frac{\gamma(\gamma+v_1)}{v_1(\mu-\lambda)}+\frac{v_1(\delta+v_2)}{v_2(\mu-\lambda)}+\frac{v_1(\delta+v_2)}{v_2\delta}\right]^{-1} \tag{43}$$

4. Important performance measures

In this section, we present some of the important performance measures of the system as follows.

The expected number of customers in the orbit is

$$\mathrm{E}[L_0] = \sum_{i=1}^{4} P_i'(1) \tag{44}$$

The expected number of customers in the system is

$$\mathrm{E}[L_s] = \mathrm{E}[L_0] + P_1(1) \tag{45}$$

Probability of server in type I vacation

$$Pr_{V1} = P_3(1)$$

$$= \sum_{n=0}^{\infty} p_{n\,3}$$

$$= \frac{(\gamma+v_1)p_{0\,3}}{v_1} \tag{46}$$

Probability of server in type II vacation

$$Pr_{V2} = P_4(1)$$

$$= \sum_{n=0}^{\infty} p_{n\,4}$$

$$= \frac{v_1(\delta + v_2)}{\delta v_2} p_{0\,3} \tag{47}$$

Probability of server on vacations

$$Pr_V = Pr_{V1} + Pr_{V2}$$

$$= \frac{(\gamma + v_1) p_{0\,3}}{v_1} + \frac{v_1(\delta + v_2)}{\delta v_2} p_{0\,3} \tag{48}$$

Probability of server in working (active) state

$$Pr_N = P_1(1) + P_2(1)$$

$$= \sum_{n=0}^{\infty} p_{n\,1} + \sum_{n=1}^{\infty} p_{n\,2} \tag{49}$$

5. Graphical results

In this section, we illustrate the effect of various parameters on some of the performance measures of system. We have also optimized the cost with respect to service rate.

In the below graphs, we have set λ=1.2, μ=3, β=2, γ=0.6, α=1, $v_1 = 0.6$, $v_2 = 1$, δ=0.8 unless they are varied in the graphs.

5.1 Sensitivity analysis

For qualitative analysis of the proposed model, we represent some of the numerical results graphically.

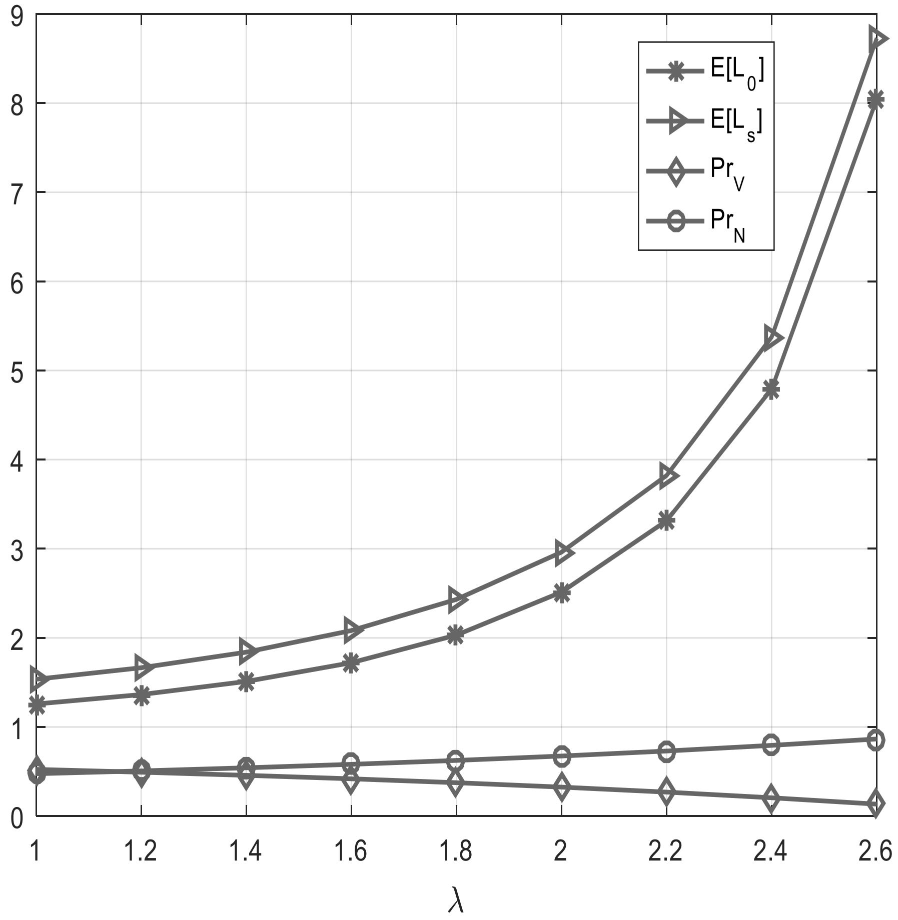

Figure1. Effect of active state arrival rate (λ) on system performance measures.

From figure1, we observe that with the increase in arrival rate λ, expected orbit length, system length and probability of normal state increase, whereas the probability of vacation decreases. This is explained by the fact that with the increase in arrival rate, the number of customers increases in orbit and in system. Hence, the probability of normal state increases and thereby, the probability of vacation decreases.

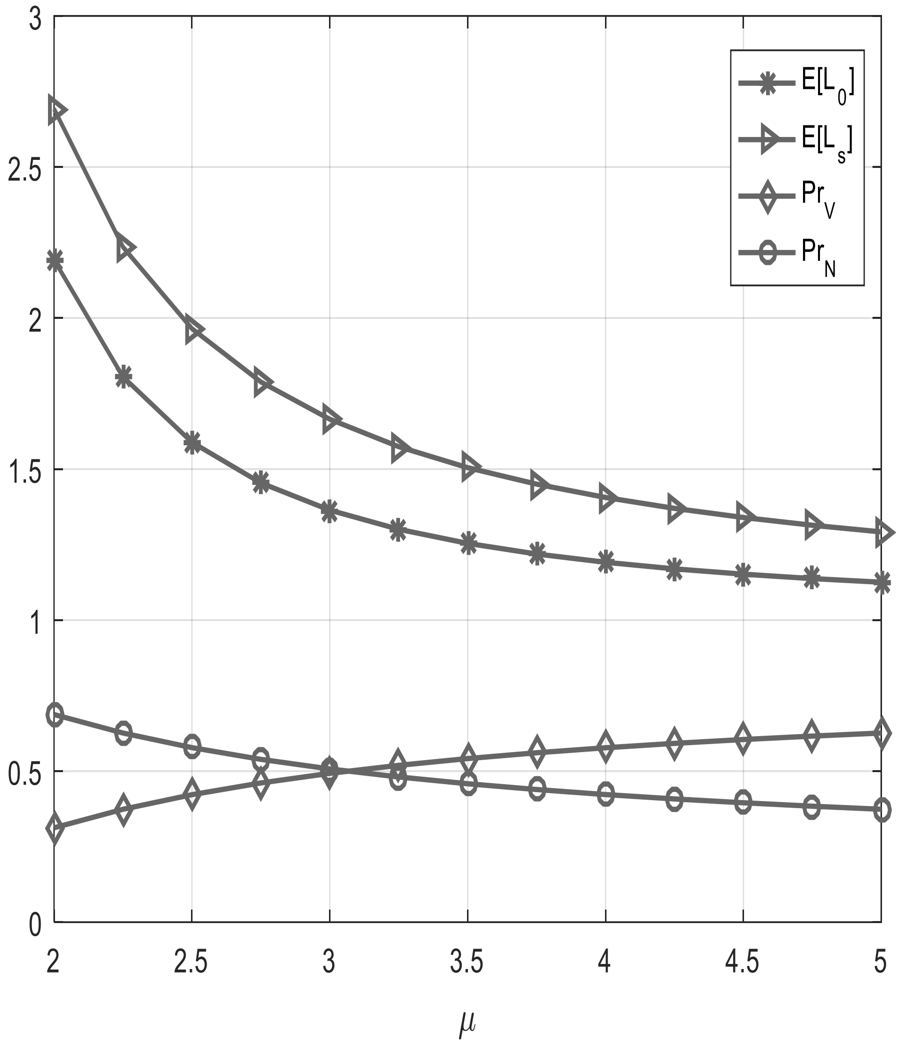

Figure 2. Effect of service rate (μ) on system performance measures.

Figure 2 reveals that the expected length of orbit, system and probability of normal (active) state decrease, but the probability of vacation state increases with an increase in service rate μ. The reason being that with the increase in μ, the customers will be served fasterand this reduces the number of customers in orbit and hence in the system. Also, due to faster service, the probability of normal period decreases and this increases the probability of a vacation period.

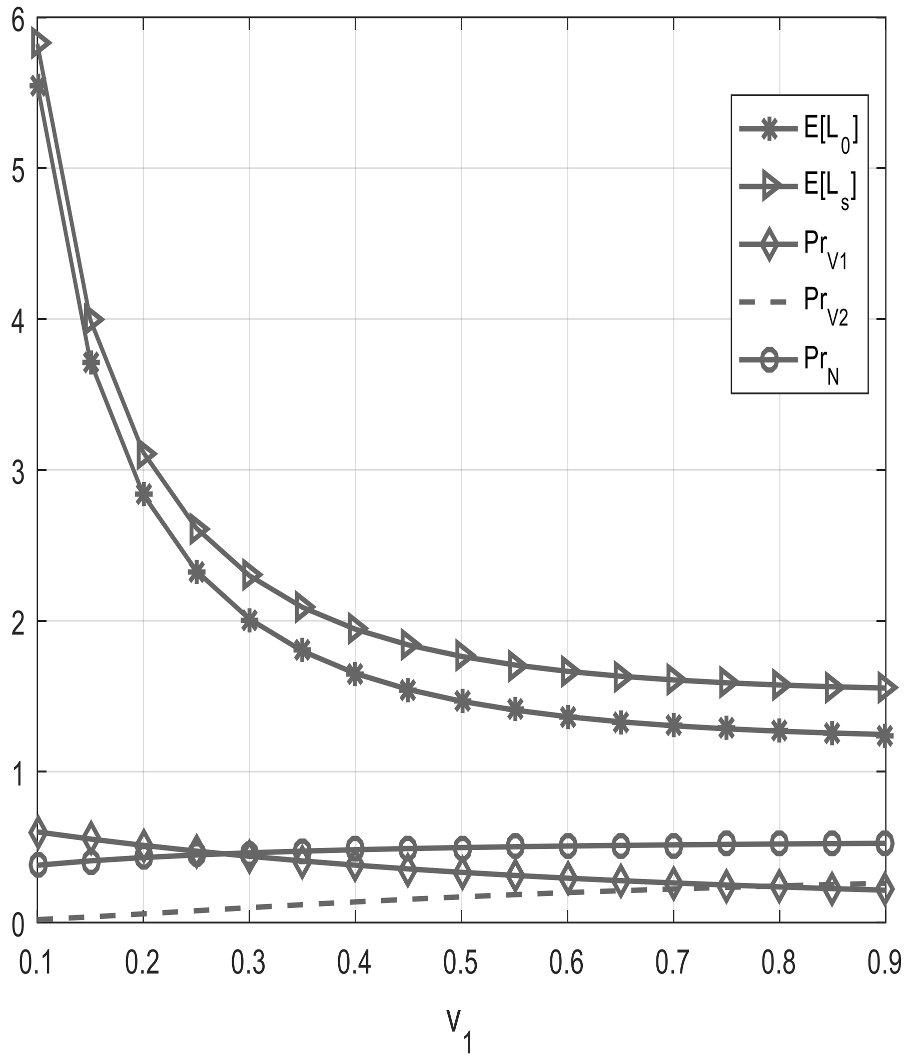

Figure3.Effect of rate of type I vacation (v_1) on system performance measures.

From figure 3, we see that as the type I vacation rate increases, the expected length of orbit, expected length of system and probability of type I vacation decrease but the probability of type II vacation and probability of normal (active) state increase. The fact behind the observation is that with the increase in type I vacation rate, the duration of type I vacation decreases and this causes increase in probability of normal state and the probability of type II vacation. Due to which the expected number of customers in orbit and that in the system decrease.

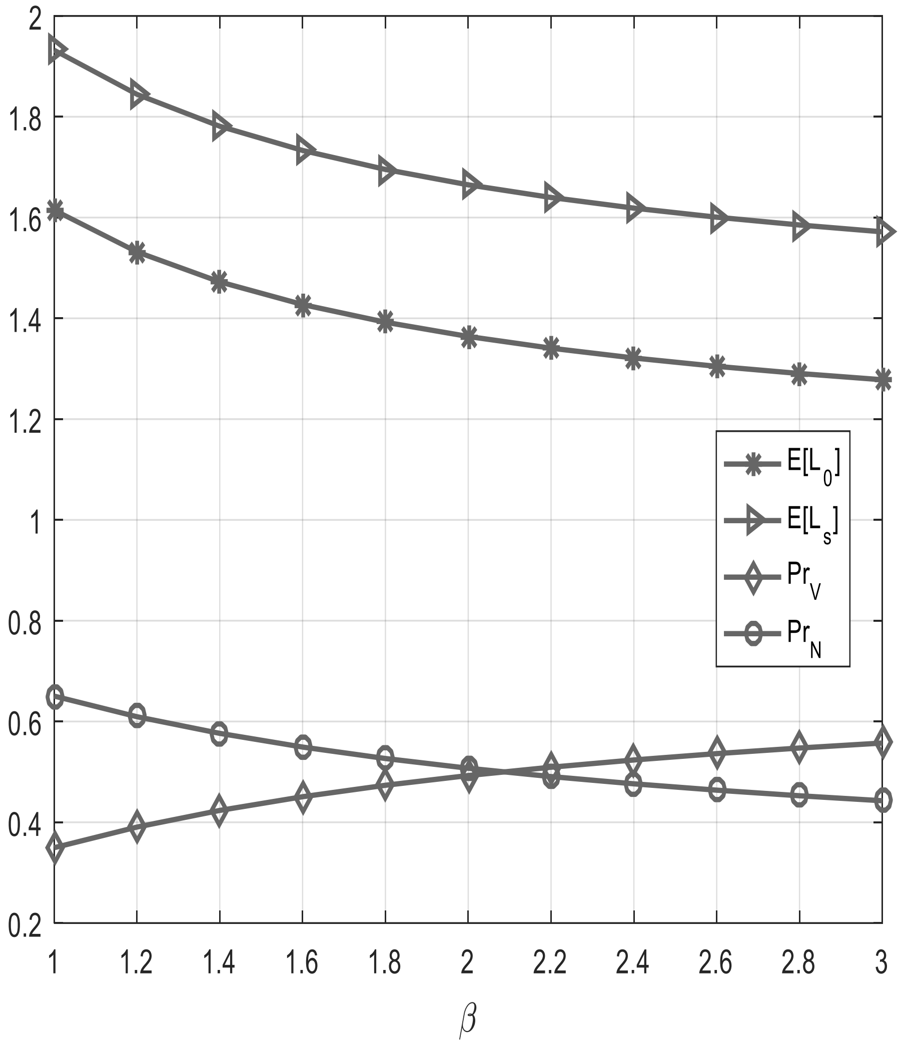

Figure 4. Effect of variation in retrial rate (β) on system performance measures.

Figure 4 shows the effect of change in retrial rate on expected orbit length, system length, probability of vacation and active server states. The graphical results obtained here matches the intuitive expectations.

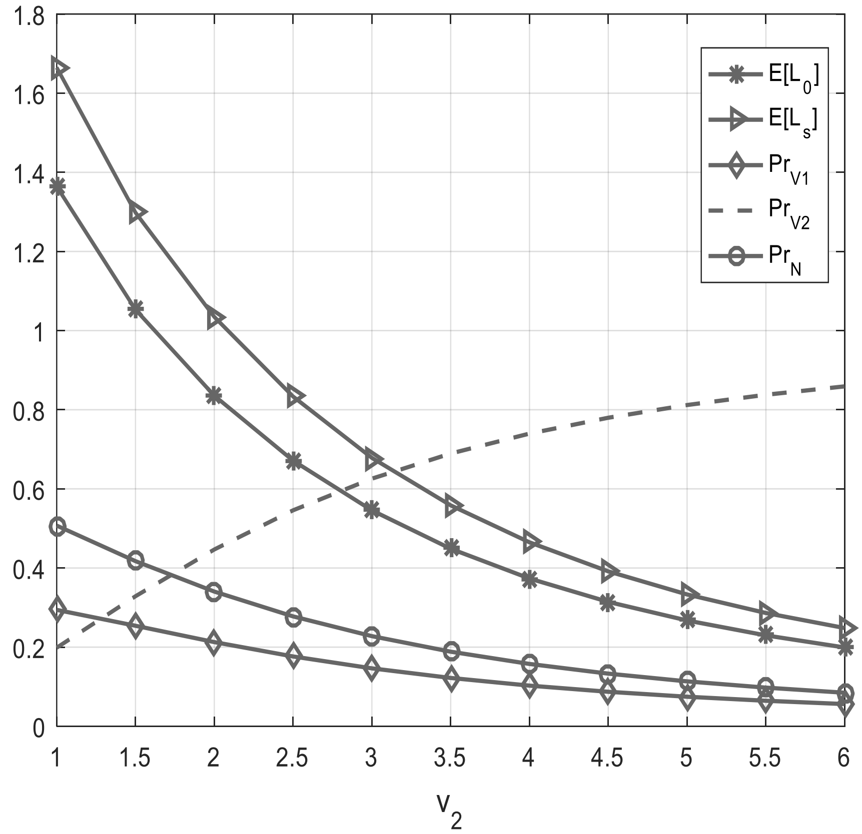

Figure 5. Effect of variation in rate of type II vacation (v_2) on system performance measures.

Figure5 represents that expected orbit length,expected system length, probability of server in normal state and probability of type I vacation decrease as the rate of type II vacation increase. As the type II vacation rate increases, the duration of type II vacation decreases: hence, the expected queue length and system length decrease.

5.2 Cost analysis

In this subsection, we optimize the operating cost function with respect to service rate in working state. To obtain the optimal value ofμ, some cost elements are taken as

C_L = Cost per unit time for each customer present in the orbit.

C_μ = Cost per unit time for service in working state.

C_{v1}= Cost per unit time in type I vacation.

C_{v2}= Cost per unit time in type II vacation.

The corresponding cost function per unit time is defined as

F(μ) = $C_L E[L_0]$ + μC_μ + $v_1 C_{v1}$+ $v_2 C_{v2}$

We takeC_L= 20, C_μ=28, C_θ=10, C_ϕ=8 in the parabolic method for obtaining optimal cost F(x) and the corresponding value of x. Parabolic-method works by generating quadratic function through calculated points in every iteration to which the function F(x) can be approximated. The point at which F(x) is optimum in three- point pattern $\{x_1, x_2, x_3\}$ is given by

$$x_L = \frac{0.5(F(x_1)({x_2}^2 - {x_3}^2) +\ F(x_2)({x_3}^2 - {x_1}^2) +\ F(x_3)({x_1}^2 - {x_2}^2))}{F(x_1)(x_2 - x_3) + F(x_2)(x_3 - x_1) + F(x_3)(x_1 - x_2)}$$

The new value obtained replaces one of the three points to improve the current three-point pattern. The process is repeatedly applied until optimum value is obtained up to the desired degree of accuracy.

Table 1 shows that optimum value $F(\mu)$ =112.83101corresponding to μ= 2.15566 with the permissible error of 10^{-4}, which is verified by Figure 6.

Table 1. Optimization of cost by parabolic method

x_1	x_2	x_3	$F(x_1)$	$F(x_2)$	$F(x_3)$	x_L
1.70	2.00	2.50	127.37253	113.85024	115.77571	2.21852
2.00	2.21852	2.50	113.85024	112.95905	115.77571	2.18165
2.00	2.18165	2.21852	113.85024	112.85383	112.95905	2.16269
2.00	2.16269	2.18165	113.85024	112.83273	112.85383	2.15844
2.00	2.15844	2.16269	113.85024	112.83128	112.83273	2.15648
2.00	2.15648	2.15844	113.85024	112.83103	112.83128	2.15594
2.00	2.15594	2.15648	113.85024	112.83101	112.83103	2.15572
2.00	2.15572	2.15594	113.85024	112.83100	112.83101	2.15566

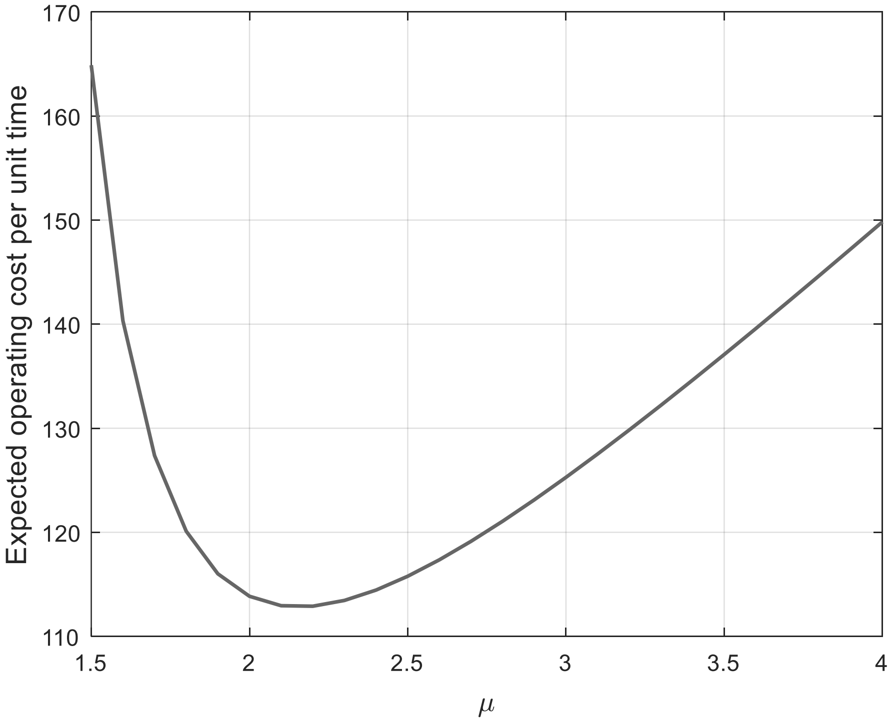

Figure 6. Variation in expected operating cost per unit time with service rate (μ)

6. Conclusion and future scope

In this paper, we have analyzed single server Markovian queueing model with state dependent arrival rates of customers under differentiated vacations and classical retrial policy. The closed form expressions for various performance measures are derived with the help of probability generating functions. The performance of the proposed model is represented graphically using MATLAB software. The operating cost of the queueing system is optimized with respect to service rate of the server. The model can be extended to multiple servers.

Conflicts of interests

The authors declare that there is no conflict of interests.

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