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On a class of sets between a-open sets and gδ-open sets

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Abstract

In this paper, a new class of sets called Da-open sets are introduced and investigated with the help of gδ-open and δ-closed sets. Relationships between this new class and other related classes of sets are established and as an application Da-continuous and almost Da-continuous functions have been defined to study its properties in terms of Da-open sets. Finally, some properties of Da-closed graph and (D,a)-closed graph are investigated.

Keywords:a-open set,δ-open set,gδ-open,Da-open set,Da-closed set.
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1 Introduction

The concept of generalized open sets introduced by Levine[Levine, 1970] plays a significant role in General Topology. The study of generalized open sets and its properties found to be useful in computer science and digital topology[Khalimsky et al., 1990, Kovalevsky, 1994, Smyth, 1995]. Since Professor El- Naschie has recently shown in [El Naschie, 1998, 2000, 2005] that the notion of fuzzy topology may be relevant to quantum particle physics in connection with string theory and ϵ^{∞} theory.So,the fuzzy topological version of the notions and results introduced in this paper are very important. Recently, Ekici [Ekici, 2008] introduced the notion of a-open sets as a continuation of research done by Velicko [Velicko, 1968] on the notion of δ-open sets.Dontchev et al., introduced gδ-closed sets and gδ-continuity.In this paper,new generalizations of a-open sets by using $g\delta$-open and δ-closed sets called Da-open sets are presented. Also Da-continuous functions,almost Da-continuous functions,Da-closed graphs and (D,a)-closed graphs have been defined to study its properties in terms of Da-open sets.

2 Prerequisites, Definitions and Theorems

In what follows,spaces always mean topological spaces on which no separation axioms are assumed unless explicitly stated and f:(X,τ) $\rightarrow$ (Y,η) or simply f:X $\rightarrow$Y denotes a function f of a space (X,τ) into a space (Y,η). The δ-closure of a subset A of X is the intersection of all δ-closed sets containing A and is denoted by Cl$_\delta$(A).

Definition 2.1. *In (X,τ),let N $\subset$ X.Then N is called:*
(i)regular closed[Stone, 1937] (resp.,a-closed[Ekici, 2008], δ-preclosed[Raychaudhuri and Mukherjee, 1993], e^*-closed[Ekici, 2009], δ-semiclosed[Park et al., 1997], β-closed[Abd El-Monsef, 1983], semiclosed[Levine, 1963], preclosed[Mashhour, 1982]) if N = Cl(Int(N)) (resp., Cl(Int(Cl$_\delta$(N))) $\subset$ N, Cl(Int$_\delta$(N)) $\subset$ N, Int(Cl(Int$_\delta$(N)) $\subset$ N, Int(Cl$_\delta$(N)) $\subset$ N, Int(Cl(Int(N)) $\subset$ N, Int(Cl(N)) $\subset$ N, Cl(Int(N)) $\subset$ N).
(ii) δ-closed [Velicko, 1968] if N = Cl$_\delta$(N)
where Cl$_\delta$(N) = $\{p\in X$:Int(Cl(O))$\cap N\neq\phi$,O$\in\tau$ and $p\in O\}$.
(iii)generalized δ-closed (briefly,gδ-closed)[Dontchev et al., 2000] if Cl(N)) $\subset$ G whenever N $\subset$ G and G is δ-open in X.
(iv)generalized closed (briefly,g-closed)[Levine, 1970] if Cl(N)) $\subset$ G whenever N $\subset$ G and G is open in X.
The complements of the above mentioned closed sets are their respective open sets.

The set of all regular open (resp.,δ-open, β-open, δ-preopen, preopen, semiopen, δ-semiopen,e^*-open,gδ-open and a-open) sets of (X,τ) is denoted by RO(X) (resp.δO(X), βO(X), δPO(X), PO(X), SO(X), δSO(X), e^*O(X), GδO(X) and aO(X)).

The a-closure[Ekici, 2008](resp, gδ-closure,δ-closure) of a set N is the intersection of all a-closed(resp, gδ-closed,δ-closed) sets containing N and is denoted by a-Cl(N) (resp., $\mathrm{Cl}_{g\delta}$(N),Cl_{δ}(N)). The a-interior[Ekici, 2008](resp,gδ-interior,δ-interior) of a set N is the union of all a-open(resp, gδ-open,δ-open) sets contained in M and is denoted by a-Int(M)(resp, $\mathrm{Int}_{g\delta}$(M),Int_{δ}(M))

Definition 2.2. *[Ekici, 2005] A topological space (X,τ) is said to be:*
(1) r-T_1 if for each pair of distinct points x and y of X, there exist regular open sets U and V such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.
(2) r-T_2 if for each pair of distinct points x and y of X, there exist regular open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \phi$.

Theorem 2.1. *Let C and D be subsets of a topological space (X,τ).Then*
(i)If C is gδ-closed,then $Cl_{g\delta}(C) = C$.
(ii) If $C \subset D$,then $Cl_{g\delta}(C) \subset Cl_{g\delta}(D)$.
(iv) $x \in Cl_{g\delta}(C)$ if and only if for each gδ-open set O containing x,$O \cap C \neq \phi$,
(v)$Cl_{g\delta}(C) \cup Cl_{g\delta}(D) \subset Cl_{g\delta}(A \cup D)$.
(vi)$Cl_{g\delta}(C \cap D) \subseteq Cl_{g\delta}(C) \cap Cl_{g\delta}(D)$.

3 Da-Open Sets.

Definition 3.1. *A subset M of a topological space (X,τ) is said to be:*
(1) Da-open if $M \subset Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M))$,
(2) Da-closed if $Cl_{g\delta}(Int_{\delta}(Cl_{g\delta}(M)) \subset M$.
The collection of all Da-open(resp,Da-closed) sets in (X,τ) is denoted by DaO(X) (resp,DaC(X)).

Theorem 3.1. *Let (X,τ) be a space.Then for any $N \subset X$,*
(i) $N \in \delta O(X)$ implies $N \in aO(X)$[Ekici, 2008].
(ii) $N \in \delta O(X)$ implies $N \in G\delta O(X)$[Dontchev et al., 2000].
(iii)$N \in GO(X)$ implies $N \in G\delta O(X)$[Dontchev et al., 2000].
(iv) $N \in aO(X)$ implies $N \in DaO(X)$.
(v) $N \in G\delta O(X)$ implies $N \in DaO(X)$.
Proof: *(iv) Since $\delta O(X) \subset G\delta O(X)$, $Int_{\delta}(N) \subset Int_{g\delta}(N)$.*
Now,let $N \in aO(X)$, then $N \subset Int(Cl(Int_{\delta}(N))$. Therefore,
$N \subset Int(Cl(Int_{\delta}(N))=Int_{\delta}(Cl(Int_{\delta}(N)) \subset Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(N))$. Hence $N \in DaO(X)$.
(v) Suppose N is gδ-open. Then $Int_{g\delta}(N)=N$.

Therefore, $Int_{g\delta}(N) \subset Cl_{\delta}(Int_{g\delta}(N)$*.Then*
$N = Int_{g\delta}(N) = Int_{g\delta}(Int_{g\delta}(N)) \subset Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(N))$*. Hence* $N \in DaO(X)$*.*

Remark 3.1. *The following diagram holds for any subset of a space* (X,τ)*.*

$$\begin{tikzcd}
\text{open set} \arrow[dd] & \delta\text{-open set} \arrow[l] \arrow[r] \arrow[dd] & a\text{-open set} \arrow[d] \\
 & & Da\text{-open set} \\
g\text{-open set} \arrow[r] & g\delta\text{-open set} \arrow[ur] &
\end{tikzcd}$$

None of these implications is reversible

Example 3.1. *Let* $X=\{p,q,r,s\}$ *and* $\tau=\{X,\phi,\{p\},\{q\},\{p,q\},\{p,r\}.\{p,q,r\}\}$*,then*
$aO(X)=\{X,\phi,\{q\},\{p,r\},\{p,q,r\}\}$
$G\delta O(X)=\{X, \phi,\{p\},\{q\},\{r\},\{p,q\},\{p,r\}\{q,r\},\{p,q,r\}\}$.
$DaO(X)=\{X, \phi,\{p\},\{q\},\{r\},\{p,q\},\{p,r\},\{q,r\}\{p,q,r\}\{p,q,s\},\{q,r,s\}\}$.
Therefore, $\{q,r,s\} \in DaO(X)$ *but* $\{q,r,s\} \notin aO(X)$ *and* $\{q,r,s\} \notin g\delta O(X)$*.*

Lemma 3.1. *If there exists a* $M \in G\delta O(X)$ *such that* $M \subset N \subset Int_{g\delta}(Cl_{\delta}(M))$*,then N is Da-open.*
Proof: *Since M is* $g\delta$*-open,* $Int_{\delta g}(M)=M$*. Therefore,*
$Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(N)) \supset Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M)) = Int_{g\delta}(Cl_{\delta}(M)) \supset N$.
Hence N is Da-open.

Converse of the Lemma 3.1 is not true as shown in Example 3.1.

Example 3.2. *In Example 3.1,* $\{p,q,r\} \in DaO(X)$ *and* $\{p,r\} \in G\delta O(X)$ *but* $\{p,r\} \subseteq \{p,q,r\} \nsubseteq Int_{g\delta}(Cl_{\delta}(\{p,r\}))=\{p,r\}$ *.*

Lemma 3.2. *For a family* $\{ B_{\lambda}:\lambda \in \wedge\}$ *of subsets of a space* (X,τ)*,the following hold:*
(1) $Cl_{g\delta}(\bigcap\{B_{\lambda}:\lambda\in\wedge\}) \subset \bigcap\{Cl_{g\delta}(B_{\lambda}):\lambda\in\wedge\}$.
(2) $Cl_{g\delta}(\bigcup\{V_{\lambda}:\lambda\in\wedge\}) \supset \bigcup\{Cl_{g\delta}(B_{\lambda}):\lambda\in\wedge\}$.
(3) $Cl_{\delta}(\bigcap\{B_{\lambda}:\lambda\in\wedge\}) \subset \bigcap \{Cl_{\delta}(B_{\lambda}):\lambda\in\wedge\}$.
(4) $Cl_{\delta}(\bigcup\{B_{\lambda}:\lambda\in\wedge\}) \supset \bigcup\{Cl_{\delta}(B_{\lambda}):\lambda\in\wedge\}$

Theorem 3.2. *If* $\{G_{\alpha}:\lambda\in\wedge\}$ *is a collection of Da-open sets in a space* (X,τ)*,then* $\bigcup_{\alpha\in\wedge} G_{\alpha}$ *is a Da-open set in* (X,τ) *:*
Proof: *Since each* G_{α}*is Da-open,* $G_{\alpha} \subset Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(G_{\alpha}))$ *for each* $\alpha\in\wedge$ *and hence* $\bigcup_{\alpha\in\wedge} G_{\alpha} \subset \bigcup_{\alpha\in\wedge} Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(G_{\alpha})) \subset Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(\bigcup_{\alpha\in\wedge} G_{\alpha}))$*. Thus* $\bigcup_{\alpha\in\wedge} G_{\alpha}$ *is Da-open.*

Corolary 3.1. *If $\{F_\alpha:\alpha\in\wedge\}$ is a collection of Da-closed sets in a space (X,τ),then $\bigcap\limits_{\alpha\in\wedge} F_\alpha$ is a Da-closed set in (X,τ)*

Remark 3.2. *M and N $\in$ DaO(X) $\not\Rightarrow$ M $\cap$ N $\in$ DaO(X) as seen from Example 3.1, where both M = $\{q,r,s\}$ and N = $\{p,q,s\}$ $\in$ DaO(X) but M $\cap$ N = $\{q,s\}$ $\notin$ DaO(X).*

Corolary 3.2. *If M$\in$ DaO(X) and B$\in$aO(X),then M$\cup$B$\in$ DaO(X).*
Proof:*Follows from Theorem 3.1(iv) and Theorem 3.2*

Corolary 3.3. *If M$\in$ DaO(X) and B$\in$GδO(X),then M$\cup$B$\in$ DaO(X).*
Proof:*Follows from Theorem 3.1(v) and Theorem 3.2*

Definition 3.2. *In (X,τ),let M $\subset$ X.*
(1)The Da-interior of M, denoted by $Int_a^D(M)$ is defined as
$Int_a^D(M)=\bigcup\{G:G\subseteq M$ and $M\in DaO(X)\}$;
(2)The Da-closure of M, denoted by $Cl_a^D(M)$ is defined as
$Cl_a^D(A)=\bigcap\{F:M\subseteq F$ and $F\in DaC(X)\}$.

Theorem 3.3. *In (X,τ),let M, N,F $\subset$ X.Then:*
(1)M $\subset Cl_a^D(M)\subset aCl(M)$, $Cl_a^D(M)\subset Cl_{g\delta}(M)$.
(2) $Cl_a^D(M)$ is a Da-closed set.
(3) If F is a Da-closed set, and F $\supset$ M,then F $\supset Cl_a^D(M)$.
i.e.,$Cl_a^D(M)$ is the smallest Da-closed set containing M.
(4)M is Da-closed set if and only if $Cl_a^D(M)$=M.
(5) $Cl_a^D(Cl_a^D(M)) = Cl_a^D(M)$.
(6)M $\subseteq$ N implies $Cl_a^D(M) \subseteq Cl_a^D(N)$.
(7)p $\in Cl_a^D(M)$ if and only if for each Da-open set V containing p,V $\cap$ M$\neq\phi$.
(8) $Cl_a^D(M) \cup Cl_a^D(N) \subset Cl_a^D(M \cup N)$.
(9) $Cl_a^D(M \cap N) \subset Cl_a^D(M) \cap Cl_a^D(N)$.
Proof: *(1)It follows from Theorem 3.1(iv) and (v)*
(2)It follows from Definition 3.2 and Corollary 3.1
(3)Let F be a Da-closed set,containing M.$Cl_a^D(M)$ is the intersection of Da-closed sets containing M, and F is one among these;hence F $\supset Cl_a^D(M)$.
(4) Let M be Da-closed,then by Definition 3.2(2),$Cl_a^D(M)$=M.
Conversely,let $Cl_a^D(M)$=M. Then by (2) above,M is Da-closed.
(5)It follows from (2) and (4).
(6) Obvious.
(7)

$$\begin{aligned} p \notin Cl_a^D(M) &\Leftrightarrow (\exists\, G\in DaC(X))(M\subset G)(p \notin G)\\ &\Leftrightarrow (\exists\, G\in DaC(X))(M\subset G)(p \in G^c)\\ &\Leftrightarrow (\exists\, G^c\in DaO(X))(M\cap G^c=\phi)(p \in G^c)\\ &\Leftrightarrow (\exists\, G^c\in DaO(X,p))(M\cap G^c=\phi)\end{aligned}$$

i.e.,$(\exists\ U(=G^c)\in DaO(X,p))(M\cap U=\phi)$
(8) and (9) follows from (6).

Remark 3.3. *(1)* $Cl_a^D(M)\cup Cl_a^D(N)\neq Cl_a^D(M\cup N)$*, in general, as seen from Example 3.1 where* $M=\{p\}$, $N=\{r\}$ *and* $M\cup N=\{p,r\}$*.Then* $Cl_a^D(M)=\{p\}$, $Cl_a^D(N)=\{r\}$,$Cl_a^D(M)\cup Cl_a^D(N)=\{p,r\}$ *and* $Cl_a^D(M\cup N)=\{p,r,s\}$;
(2) $Cl_a^D(M\cap N)\neq Cl_a^D(M)\cap Cl_a^D(N)$*, in general,as seen from Example 3.1 where,* $M=\{p,q,r\}$, $N=\{s\}$ *and* $M\cap N=\phi$*.Then* $Cl_a^D(M)=X$, $Cl_a^D(N)=\{s\}$, $Cl_a^D(M)\cap Cl_a^D(N)=\{s\}$ *and* $Cl_a^D(M\cap N)=\phi$

Lemma 3.3. *In* (X,τ)*,let* $M\subset X$*.Then*
(1) $Cl_a^D(X\backslash M)=X\backslash Int_a^D(M)$,
(2) $Int_a^D(X\backslash M)=X\backslash Cl_a^D(M)$.

Theorem 3.4. *In* (X,τ)*,let* $M,N,G\subset X$,
(1)$aInt(M)\subseteq Int_a^D(M)\subseteq M$, $Int_{g\delta}(M)\subseteq Int_a^D(M)$.
(2) $Int_a^D(M)$ *is a Da-open set.*
(3) If G is a Da-open set, and $G\subset M$*,then* $G\subset Int_a^D(M)$.
i.e.,$Int_a^D(M)$ *is the largest Da-open set contained in M.*
(4)M is Da-open set if and only if $Int_a^D(M)=M$.
(5) $Int_a^D(Int_a^D(M))=Int_a^D(M)$.
(6)$M\subseteq N$ *implies* $Int_a^D(M)\subseteq Int_a^D(N)$.
(7) $p\in Int_a^D(M)$ *if and only if there exists Da-open set N containing p such that* $N\subseteq M$.
(8) $Int_a^D(M\cap N)\subseteq Int_a^D(M)\cap Int_a^D(N)$.
(9) $Int_a^D(M)\cup Int_a^D(N)\subseteq Int_a^D(M\cup N)$.
Proof:*Similar to the proof of Theorem 3.3*

Remark 3.4. *(8)*$Int_a^D(M\cap N)\neq Int_a^D(M)\cap Int_a^D(N)$*, in general, as seen from Example 3.1,where* $M=\{p,q,s\}$, $N=\{q,r,s\}$ *and* $M\cap N=\{q,s\}$*.Then* $Int_a^D(M)=\{p,q,s\}$, $Int_a^D(N)=\{q,r,s\}$, $Int_a^D(M)\cap Int_a^D(N)=\{q,s\}$ *and* $Int_a^D(M\cap N)=\{q\}$.
(9) $Int_a^D(M)\cup Int_a^D(N)\neq Int_a^D(M\cup N)$*,in general, as seen from Example 3.1, where* $M=\{p,q,r\}$, $N=\{s\}$ *and* $M\cup N=X$*.Then* $Int_a^D(M)=\{p,q,r\}$, $Int_a^D(N)=\phi$, $Int_a^D(M)\cup Int_a^D(N)=\{p,q,r\}$ *and* $Int_a^D(M\cup N)=X$.

Lemma 3.4. *In* (X,τ)*,let* $M\subset X$*. Then*
(1)M is Da-open if and only if $M=M\cap Int_{g\delta}(Cl_\delta(Int_{g\delta}(M))$.
(2)M is Da-closed if and only if $M=M\cup Cl_{g\delta}(Int_\delta(Cl_{g\delta}(M))$.

Proof:*(1) Let M be an Da-open. Then,*
$M\subseteq Int_{g\delta}(Cl_\delta(Int_{g\delta}(M))$ *implies* $M\cap Int_{g\delta}(Cl_\delta(Int_{g\delta}(M))=M$.
Conversely,let $M=M\cap Int_{g\delta}(Cl_\delta(Int_{g\delta}(M))$ *implies* $M\subset Int_{g\delta}(Cl_\delta(Int_{g\delta}(M))$.
(2)It follows from (1)

Lemma 3.5. *In (X,τ),let $M \subset X$. Then*
(i)$M \cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M))$ is Da-open
(ii)$M\cup Cl_{g\delta}(Int_{\delta}(Cl_{g\delta}(M))$ is Da-closed.
Proof: *(i) $Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M \cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M)))))) = Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(A)\cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M)))))$ $= Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M)))$. This implies that $M \cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M))) = M \cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M \cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M)))))) \subseteq Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M \cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M))))))$. Therefore $M \cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M)))$ is Da-open.*
(ii) From (i) we have $X\backslash(M\cup Cl_{g\delta}(Int_{\delta}(Cl_{g\delta}(M))) = (X\backslash M) \cap Cl_{g\delta}(Int_{\delta}(Cl_{g\delta}(X\backslash M)))$ is Da-open so that $M \cup Cl_{g\delta}(Int_{\delta}(Cl_{g\delta}(M)))$ is Da-closed.

Lemma 3.6. *In (X,τ),let $M \subset X$. Then*
(i)$Int_a^D(M)=M \cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M))$.
(ii)$Cl_a^D(M)=M\cup Cl_{g\delta}(Int_{\delta}(Cl_{g\delta}(M))$.
Proof:*(i)Let $N=Int_a^D(M)$,then $N\subset M$.Since N is Da-open,$N\subset Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(N))$ $\subset Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M))$.Then $N\subset M\cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M))\subset M$.Therefore,by Lemma 3.5, it follows that $M\cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M))$ is a Da-open set contained in M. But $Int_a^D(M)$ is the largest Da-open set contained in M it follows that $M\cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M))\subset Int_a^D(M)=N$.Then $N=M\cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M))$. Therefore,$Int_a^D(M)=M \cap Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(M))$.*
(ii)It follows from (i)

4 Da-Continuous functions.

Definition 4.1. *A function $f:(X,\tau) \to (Y,\eta)$ is said be a Da-continuous if for each $p\in X$ and each $N\in O(Y,f(p))$, there exists $M \in DaO(X,p)$ such that $f(M)\subset N$.*

Theorem 4.1. *For a function $f:(X,\tau) \to (Y,\eta)$,the following are equivalent*
(1)f is Da-continuous;
(2)For each $N\in O(Y)$,$f^{-1}(V)\in DaO(X)$.
Proof:*(1)$\longrightarrow$(2)Let $N\in O(Y)$ and $p\in f^{-1}(N)$. Since $f(p) \in N$,then by(1),there exists $M_p \in DaO(X,p)$ such that $f(M_p) \subset N$.It follows that $f^{-1}(N)=\cup\{M_p\text{: } p\in f^{-1}(N)\}\in DaO(X)$, by Theorem 3.2 .*
(2)$\longrightarrow$(1) Let $p \in X$ and $N \in O(Y,f(p))$.Then,by (2), $f^{-1}(N)\in DaO(X,p)$. Take $M = f^{-1}(N)$, then $f(M) \subset N$.

Corolary 4.1. *A function $f:(X,\tau) \to (Y,\eta)$ is Da-continuous if and only if $f^{-1}(F)\in DaC(X)$ for each $F\in C(Y)$.*

Remark 4.1. *The following implications hold for a function* f*:*$(X,\tau) \rightarrow (Y,\eta)$*:*

$$\begin{tikzcd}
\textit{continuity} \arrow[dd] & \delta\textit{-continuity} \arrow[l] \arrow[r] \arrow[dd] & a\textit{-continuity} \arrow[d] \\
 & & Da\textit{-continuity} \\
g\textit{-continuity} \arrow[r] & g\delta\textit{-continuity} \arrow[ur] &
\end{tikzcd}$$

Example 4.1. *Consider* (X,τ) *as in Example 3.1 and* $\eta=\{X,\phi,\{p\},\{q\},\{p,q\},\{p,q,r\}\}$.
Define f*:*$(X,\sigma)\rightarrow(X,\eta)$ *by* $f(p)=s, f(q)=p, f(r)=q$ *and* $f(s)=r$*.Then* f *is Da-continuous but neither a-continuous nor* $g\delta$*-continuous since* $\{p,q,r\}$ *is open in* (X,η)*,*
$f^{-1}(\{p,q,r\}) = \{q,r,s\} \in DaO(X)$ *but* $\{q,r,s\} \notin aO(X)$ *and* $\{q,r,s\} \notin g\delta O(X)$.
The other Examples are shown in[3,5,21]

Theorem 4.2. *The following conditions are equivalent for a function*
f*:*$(X,\tau) \rightarrow (Y,\eta)$*:*
(1) f *is Da-continuous;*
(2) For each subset N of Y, $Cl_{g\delta}(Int_{\delta}(Cl_{g\delta}(f^{-1}(N))) \subset f^{-1}(Cl(N)$;
(3)For each subset N of Y, $f^{-1}(Int(N)) \subset Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(f^{-1}(N))$;
(4)For each subset N of Y,$Cl_a^D(f^{-1}(N)) \subset f^{-1}(Cl(N))$;
(5)For each subset M of X,$f(Cl_a^D(M)) \subset Cl(f(M))$;
(6)For each subset N of Y, $f^{-1}(Int(N)) \subset Int_a^D(f^{-1}(N))$.
Proof: *(1)→(2) Let* $N \subset Y$*.Then by (1),*$f^{-1}(Cl(N)) \in DaC(X)$ *implies*
$f^{-1}(Cl(N) \supset Cl_{g\delta}(Int_{\delta}(Cl_{g\delta}(f^{-1}(Cl(N))) \supset Cl_{g\delta}(Int_{\delta}(Cl_{g\delta}(f^{-1}(N)))$.
(2)→(3).Replace N by $Y\backslash N$ *in (2), we have*
$Cl_{g\delta}(Int_{\delta}(Cl_{g\delta}(f^{-1}(Y\backslash N))) \subset f^{-1}(Cl(Y\backslash N)$*, and therefore*
$f^{-1}(Int(N)) \subset Int_{g\delta}(Cl_{\delta}(Int_{g\delta}(f^{-1}(N))$ *for each subset N of Y.*
(3)→(1). Clear
(1)→(4). Let $N \subset Y$ *.Then by (1),* $f^{-1}(Cl(N)) \in DaC(X)$*. Thus*
$Cl_a^D(f^{-1}(N)) \subset Cl_a^D(f^{-1}(Cl(N)) = f^{-1}(Cl(N)$ *by Theorem 3.3(4).*
(4)→(1). Let $N \in C(Y)$*.Then by (4),*
$Cl_a^D(f^{-1}(N)) \subset f^{-1}(Cl(N) = f^{-1}(N)$ *implies* $Cl_a^D(f^{-1}(N)) = f^{-1}(N)$.
Then by Theorem 3.3(4), $f^{-1}(N) \in DaC(X)$.
(4)→(5).Let $M \subset X$*.Then* $f(M) \subset Y$*.By (4), we have*
$f^{-1}(Cl(f(M))) \supset Cl_a^D(f^{-1}(f(M))) \supset Cl_a^D(M)$.
Therefore, $f(Cl_a^D(M)) \subset f(f^{-1}(Cl(f(M))) \subset Cl(f(M)$.
(5)→(4).Let $N \subset Y$ *and* $M = f^{-1}(N) \subset X$*.Then by (5),*
$f(Cl_a^D(f^{-1}(N))) \subset Cl(f(f^{-1}(N)) \subset Cl(N)$ *implies* $Cl_a^D(f^{-1}(N)) \subset f^{-1}(Cl(N))$.
(4)→(6).Replace N by $Y \backslash N$ *in (4), we get*
$Cl_a^D(f^{-1}(Y\backslash N)) \subset f^{-1}(Cl(Y\backslash N))$ *implies* $Cl_a^D(X\backslash f^{-1}(N)) \subset f^{-1}(Y\backslash Int(N))$
Therefore,$f^{-1}(int(N)) \subset Int_a^D(f^{-1}(N))$ *for each subset N of Y.*

(6)→(1).Let G⊂Y be open.Then f^{-1}(G)=f^{-1}(Int(G)) ⊂ Int^D_a(f^{-1}(G) implies Int^D_a(f^{-1}(G)=f^{-1}(G).So by Theorem 3.4(4),f^{-1}(G)∈DaO(X).

Definition 4.2. *Two non-empty subsets A and B of a topological space (X,τ) are said to be Da-separated if there exist two Da-open sets G and H,such that A⊂G,B⊂H, A∩H=ϕ and B∩G=ϕ.*

Definition 4.3. *Two non-empty subsets A and B of a topological space (X,τ) are said to be strongly Da-separated if there exist two Da-open sets U and V,such that A⊂U,B⊂V and U∩V=ϕ.*

Definition 4.4. *A topological space (X,τ) is said to be*
(1) Da-T_2 if any two distinct points are strongly Da-separated in (X,τ)
(2) Da-T_1 if every pair of distinct points is Da-separated in (X,τ).

Remark 4.2. *The following implications are hold for a topological space (X,τ)*

$$\begin{array}{ccccc} a\text{-}T_2 & \longrightarrow & Da\text{-}T_2 & \longleftarrow & T_2 \\ \downarrow & & \downarrow & & \downarrow \\ a\text{-}T_1 & \longrightarrow & Da\text{-}T_1 & \longleftarrow & T_1 \end{array}$$

Theorem 4.3. *If an injective function f:(X,τ) → (Y,η) is Da-continuous and (Y,η) is T_1, then (X,τ) is Da-T_1.*
Proof: *Let (Y,σ) be T_1 and p,q∈X with p≠q. Then there exist open subsets G, H in Y such that f(p) ∈ G, f(q) ∉ G, f(p) ∉ H and f(q) ∈ H. Since f is Da-continuous, f^{-1}(G) and f^{-1}(H) ∈ DaO(X) such that p ∈ f^{-1}(G), q ∉ f^{-1}(G), p ∉ f^{-1}(H) and q ∈ f^{-1}(H). Hence,(X,σ) is Da-T_1 .*

Theorem 4.4. *If an injective function f: (X,τ) → (Y,η) is Da-continuous and (Y,η) is T_2, then (X,τ) is Da-T_2.*
Proof: *Similar to the proof of Theorem 4.3*

Recall that for a function f:(X,τ) → (Y,η), the subset
G_f={(x,f(x)):x ∈X} ⊂ X×Y is said to be graph of f.

Definition 4.5. *A graph G_f of a function f:(X,τ) → (Y,η) is said to be Da-closed if for each (p,q) ∉ G_f, there exist U∈DaO(X,p) and V∈O(Y,q) such that (U×V)∩ G_f = ϕ.*

As a consequence of Definition 4.5 and the fact that for any subsets C ⊂ X and D ⊂ Y, (C×D)∩ G_f=ϕ if and only if f(C)∩D = ϕ,we have the following result.

Lemma 4.1. *For a graph G_f of a function f:(X,τ) → (Y,η), the following properties are equivalent:*
(1)G_f is Da-closed in X×Y;
(2)For each (p,q) ∉G_f, there exist U∈DaO(X,p) and V∈O(Y,q) such that f(U)∩V = ϕ.

Theorem 4.5. *If f:$(X,\tau) \to (Y,\eta)$ is Da-continuous and (Y,η) is T_2 , then G_f is Da-closed in $X\times Y$.*
Proof: *Let $(p,q) \notin G_f$, $f(p)\neq q$. Since Y is T_2, there exist $V,W \in O(Y)$ such that $f(p)\in V$, $q\in W$ and $V\cap W=\phi$. Since f is Da-continuous, $f^{-1}(V)\in DaC(X,p)$.Set U $=f^{-1}(V)$, we have $f(U)\subset V$. Therefore, $f(U)\cap W=\phi$ and G_f is Da-closed in $X\times Y$*

Theorem 4.6. *Let f:$(X,\tau) \to (Y,\eta)$ have a Da-closed graph G_f. If f is injective, then (X,τ) is Da-T_1.*
Proof:*Let $x_1,x_2\in X$ with $x_1\neq x_2$.Then $f(x_1)\neq f(x_2)$ as f is injective So that $(x_1,f(x_2))$ $\notin G_f$.Thus there exist $U\in DaO(X,x_1)$ and $V\in O(Y,f(x_2))$ such that $f(U)\cap V = \phi$.Then $f(x_2)\notin f(U)$ implies $x_2\notin U$ and it follows that X is Da-T_1.*

Theorem 4.7. *Let f:$(X,\tau) \to (Y,\eta)$ have a Da-closed graph G_f. If f is surjective, then (Y,η) is T_1.*
Proof:*Let $y_1,y_2\in Y$ with $y_1\neq y_2$.Since f is surjective,$f(x)=y_2$ for some $x\in X$ and $(x,y_2)\notin G_f$.By Lemma 4.1,there exist $U\in DaO(X,x)$ and $V\in O(Y,y_1)$ such that $f(U)\cap V = \phi$.It follows that $y_2\notin V$.Hence Y is T_1.*

Theorem 4.8. *Let f:$(X,\tau) \to (Y,\eta)$ have a Da-closed graph G_f. If f is surjective, then (Y,η) is Da-T_1.*
Proof:*Similar to the proof of Theorem 4.7*

Corolary 4.2. *Let f:$(X,\tau) \to (Y,\eta)$ have a Da-closed graph G_f. If f is bijective, then both (X,τ) and (Y,η) are Da-T_1*
Proof:*Follows from Theorems 4.6 and 4.8*

Definition 4.6. *A graph G_f of a function f:$(X,\tau) \to (Y,\eta)$ is said to be (D,a)-closed if for each $(p,q) \notin G_f$, there exist $U\in DaO(X,p)$ and $V\in aO(Y,q)$ such that $(U\times aCl(V))\cap G_f = \phi$.*

Lemma 4.2. *For a graph G_f of a function f:$(X,\tau) \to (Y,\eta)$, the following properties are equivalent:*
(1)G_f is Da-closed in $X\times Y$;
(2)For each $(p,q) \notin G_f$, there exist $U\in DaO(X,p)$ and $V\in aO(Y,q)$ such that $f(U)\cap aCl(V)) = \phi$.

Theorem 4.9. *Let $M \subset X$.Then $x\in$ a-Cl(M) if and only if $G \cap M \neq \Phi$, for every a-open set G containing x.*
Proof:*Similar to the proof of Theorem 3.3(7)*

Theorem 4.10. *Let f:$(X,\tau) \to (Y,\eta)$ have a (D,a)-closed graph G_f . If f is surjective, then (Y,η) is a-T_2(resp,a-T_1).*
Proof:*Let $y_1,y_2\in Y$ with $y_1\neq y_2$.Since f surjective, $f(x_1)=y_1$ $x_1\in X$ and hence $(x_1,y_2)\notin G_f$. By Lemma 4.2,there exist $E\in DaO(X,x_1)$ and $F\in aO(Y,y_2)$ such that $f(E)\cap$ aCl(F) $= \phi$. Now, $x_1\in E$ implies $f(x_1)=y_1\in f(E)$ so that $y_1\notin aCl(F)$.By Theorem 4.9,there exists $D\in aO(Y,y_1)$ such that $D\cap F=\phi$.Hence Y is a-T_2.*

Theorem 4.11. *Let f:$(X,\tau) \rightarrow (Y,\eta)$ have a (D,a)-closed graph G_f . If f is surjective, then (Y,η) is Da-T_2(resp,Da-T_1).*
Proof:*Similar to the proof of Theorem 4.10*

Theorem 4.12. *Let f:$(X,\tau) \rightarrow (Y,\eta)$ have a (D,a)-closed graph G_f . If f is injective, then (X,τ) is Da-T_1.*
Proof:*Similar to the proof of Theorem 4.6*

Corolary 4.3. *Let f:$(X,\tau) \rightarrow (Y,\eta)$ have a (D,a)-closed graph G_f. If f is bijective, then both (X,τ) and (Y,η) are Da-T_1*
Proof:*Follows from Theorems 4.11 and 4.12*

5 Almost Da-Continuous functions.

Definition 5.1. *A function f:$(X,\tau) \rightarrow (Y,\eta)$ is said to be almost Da-continuous if for each point $p \in X$ and each open subset V of Y containing f(p), there exists $U \in$ DaO(X,p) such that f(U) $\subset$ int(Cl(V)).*

Theorem 5.1. *If f:$(X,\tau) \rightarrow (Y,\eta)$ is Da-continuous function , then f is an almost Da-continuous,but not conversely.*
Proof:*Obvious*

Example 5.1. *Consider (X,τ) and (X,η) as in 4.1. Define f:$(X,\tau) \rightarrow (X,\eta)$ by f(p)=p,f(q)=s,f(r)=q and f(s)=r Then f is almost Da-continuous but not Da-continuous since $\{p,q,r\}$ is open in (X,η), $f^{-1}(\{p,q,r\})=\{p,r,s\}\notin DaO(X,\tau)$*

Definition 5.2. *[Noiri and Popa, 1998] A space X is said to be semi-regular if for any open set U of X and each point $x \in U$ there exists a regular open set V of X such that $x \in V \subset U$.*

Theorem 5.2. *If f:$(X,\tau) \rightarrow (Y,\eta)$ is an almost Da-continuous function and Y is semi-regular, then f is Da-continuous.*
Proof: *Let $p \in X$ and let $V \in$ O(Y,f(p)). By the semi-regularity of Y , there exists G$\in$RO(Y,f(p)) such that $G \subset V$. Since f is almost Da-continuous, there exists $U \in$ DaO(X, x) such that f(U) $\subset$ Int(Cl(G)) = G $\subset$ V and hence f is Da-continuous.*

Lemma 5.1. *Let (X,τ) be a space and let A be a subset of X. The following statements are true:*
(1) $A \in PO(X)$ if and only if sCl(A) = Int(Cl(A)) [Janković, 1985].
(2) $A \in \beta O(X)$ if and only if Cl(A) is regular closed [Abd El-Monsef, 1983].

Theorem 5.3. *Let f:(X,τ) $\rightarrow$ (Y,η) be a function. Then the following conditions are equivalent:*
(1) f is almost Da-continuous;
(2) For every N$\in$RO(Y), f^{-1}(N)$\in$DaO(X);
(3) For every M$\in$RC(Y), f^{-1}(M)$\in$DaC(X);
(4) For each subset C of X, f(Cl_a^D(C)) $\subset$ Cl_δ(f(C));
(5)For each subset D of Y, Cl_a^D(f^{-1}(D)) $\subset$ f^{-1}(Cl_δ(D));
(6)For every G$\in\delta$C(Y), f^{-1}(G)$\in$DaC(X);
(7)For every H$\in\delta$O(Y), f^{-1}(H)$\in$DaO(X);
(8) For every N$\in$O(Y), f^{-1}(Int(Cl(N)$\in$DaO(X);
(9) For every M$\in$C(Y), f^{-1}(Cl(Int(M)$\in$DaC(X);
(10) For every N$\in\beta$O(Y), Cl_a^D(f^{-1}(N)) $\subset$ f^{-1}(Cl(N));
(11) For every M$\in\beta$C(Y), f^{-1}(Int(M)) $\subset$ Int_a^D(f^{-1}(M));
(12) For every M$\in$SC(Y), f^{-1}(Int(M)) $\subset$ Int_a^D(f^{-1}(M));
(13) For every N$\in$SO(Y), Cl_a^D(f^{-1}(N)) $\subset$ f^{-1}(Cl(N));
(14) For every M$\in$PO(Y), f^{-1}(M) $\subset$ Int_a^D(f^{-1}(Int(Cl(M));
(15) For each p$\in$ X and each N$\in$O(Y,f(p)), there exists M $\in$ DaO(X,p) such that f(M) $\subset$ sCl(N);
(16) For each p$\in$ X and each N$\in$RO(Y,f(p)), there exists M $\in$ DaO(X,p) such that f(M) $\subset$ N;
(17) For each p$\in$ X and each N$\in\delta$O(Y,f(p)), there exists M $\in$ DaO(X,p) such that f(M) $\subset$ N.
Proof: *(1)$\longrightarrow$(2) Similar to the proof of (1)$\longrightarrow$(2) of Theorem 4.1.*
(2)$\longrightarrow$(3) It follows from the fact that f^{-1}(Y$\setminus$F) = X $\setminus f^{-1}$(F).
(3)$\longrightarrow$(4) Suppose that D$\in$ δC(Y) such that f(C)$\subset$ D. Observe that D = Cl_δ(D) =$\bigcap\{$F:D$\subset$F and F$\in$RC(Y)$\}$ and so f^{-1}(D) = $\bigcap\{f^{-1}$(F):D$\subset$F and F$\in$RC(Y)$\}$. By (3) and Corollary 3.1,we have f^{-1}(D)$\in$DaC(X) and C$\subset$ f^{-1}(D). Hence Cl_a^D(C) $\subset f^{-1}$(D), and it follows that f(Cl_a^D(C)) $\subset$ D. Since this is true for any δ-closed set D containing f(C), we have f(Cl_a^D(C))$\subset$ Cl_δ(f(C)).
(4)$\longrightarrow$(5) Let D $\subset$ Y, then f^{-1}(D) $\subset$ X. By (4),
f(Cl_a^D(f^{-1}(D)))$\subset$ Cl_δ(f(f^{-1}(D)))$\subset$$Cl_\delta$(D). So that
Cl_a^D(f^{-1}(D)) $\subset$ f^{-1}(Cl_δ(D)).
(5)$\longrightarrow$(6) Let G$\in\delta$C(Y) Then by (5), Cl_a^D(f^{-1}(G)) $\subset$ f^{-1}(Cl_δ(G))=f^{-1}(G). In consequence, Cl_a^D(f^{-1}(G))=f^{-1}(G) and hence by Theorem 3.3(4), f^{-1}(G)$\in$DaC(X).
(6)$\longrightarrow$(7):Clear.
(7)$\longrightarrow$(1): Let p$\in$ X and let O$\in$O(Y,f(p)). Set D = Int(Cl(O)) and C =f^{-1}(D). Since D$\in$ δO(Y), then by (7), C = f^{-1}(D) $\in$ DaO(X). Now, f(p) $\in$ O= Int(O)$\subset$ Int(Cl(O)) = D it follows that p$\in f^{-1}$(D)=C and f(C)=f(f^{-1}(D)$\subset$D=Int(Cl(O).
(2)$\longleftrightarrow$(8): Let N $\in$O(Y). Since Int(Cl(N))$\in$RO(Y),by (2), f^{-1}(Int(Cl(N))$\in$DaO(X). The converse is similar.
(3)$\longleftrightarrow$(9)It is similar to (8)$\longleftrightarrow$(2).

(3)$\longrightarrow$(10): Let N$\in\beta$O(Y).Then by Lemma 5.1(2),Cl(N) $\in$ RC(Y).So by(3),f^{-1}(Cl(N)) $\in$DaC(X) .Since f^{-1}(N) $\subset$ f^{-1}(Cl(N)) and by Theorem 3.3(4),Cl_a^D(f^{-1}(N))$\subset$ f^{-1}(Cl(N)).
(10)$\longrightarrow$(11): and (12)$\longrightarrow$(13):Follows from Lemma 3.3
(11)$\longrightarrow$(12):It follows from the fact that SC(Y)$\subset\beta$C(Y)
(13)$\longrightarrow$(3):It follows from the fact that RC(Y)$\subset$SO(Y).
(2)$\longleftrightarrow$(14): Let N $\in$PO(Y). Since Int(Cl(N)) $\in$ RO(Y),then by (2), f^{-1}(Int(Cl(N))) $\in$ DaO(X) and hence f^{-1}(N) $\subset$ f^{-1}(int(Cl(N))) = Int_a^D(f^{-1}(int(Cl(N)))). Conversely,let N$\in$RO(Y). Since N $\in$ PO(Y), f^{-1}(N) $\subset$ Int_a^D(f^{-1}(int(Cl(N)))) =Int_a^D(f^{-1}(N)). In consequence, Int_a^D(f^{-1}(N))=f^{-1}(N) and by Theorem 3.4, f^{-1}(N) $\in$ DaO(X).
(1)$\longrightarrow$(15): Let p$\in$X and N$\in$O(Y,f(p)). By (1), there exists M$\in$ DaO(X,p) such that f(M) $\subset$ Int(Cl(N)).Since N$\in$PO(Y),by Lemma 5.1, f(M) $\subset$ sCl(N).
(15)$\longrightarrow$(16): Let p$\in$ X and N$\in$RO(Y,f(p)). Since N$\in$O(Y,f(p)) and by (15), there exists M$\in$DaO(X,p) such that f(M)$\subset$ sCl(N). Since N $\in$PO(Y), then by Lemma 5.1, f(M) $\subset$Int(Cl(N)) = N.
(16)$\longrightarrow$(17):Let p$\in$ X and V$\in\delta$O(Y,f(p)). Then, there exists G$\in$O(Y.f(p))such that G $\subset$ Int(Cl(G)) $\subset$ N. Since Int(Cl(G))$\in$RO(Y,f(p)), by (16), there exists M$\in$ DaO(X,p) such that f(M) $\subset$ Int(Cl(G))$\subset$ N.
(17)$\longrightarrow$(1). Let p$\in$ X and N$\in$O(Y,f(p)). Then Int(Cl(N))$\in$ δO(Y,f(p)). By (17), there exists M$\in$ DaO(X,p) such that f(M) $\subset$ Int(Cl(N)). Therefore,f is almost continuous

Theorem 5.4. *If f:(X,τ) $\rightarrow$ (Y,η) is an almost Da-continuous injective function and (Y,η) is r-T_1 , then (X,σ) is Da-T_1 .*
Proof: *It is similar to the proof of Theorem 4.3*

Theorem 5.5. *If f:(X,τ) $\rightarrow$ (Y,σ) is an almost Da-continuous injective function and (Y,σ) is r-T_2 , then (X,τ) is Da-T_2 .*
Proof: *It is similar to the proof of Theorem 4.4*

Lemma 5.2. *[Ayhan and Ozkoç, 2016] Let (X,τ) be a space and let A be a subset of X. Then:*
A $\in e^*$O(X) if and only if Cl_δ(A) is regular closed.

Theorem 5.6. *For a function f:(X,τ) $\rightarrow$ (Y,η),the following are equivalent:*
(a) f is almost Da-continuous;
(b) For every e^*-open set N in Y, f^{-1}(Cl_δ(N)) is Da-closed in X;
(c) For every δ-semiopen subset N of Y, f^{-1}(Cl_δ(N)) is Da-closed set in X;
(d) For every δ-preopen subset N of Y, f^{-1}(Int(Cl_δ(N))) is Da-open set in X;
(e) For every open subset N of Y, f^{-1}(Int(Cl_δ(N))) is Da-open set in X;
(f) For every closed subset N of Y, f^{-1}(Cl(Int_δ(A))) is Da-closed set in X .
Proof: *(a)$\rightarrow$(b):Let N$\in e^*$O(Y) Then by Lemma 5.2,Cl_δ(N)$\in$RC(Y).*

By (a), $f^{-1}(Cl_\delta(N)) \in DaC(X)$.
(b)→(c):Obvious since $\delta SO(Y) \subset e^*O(Y)$.
(c)→(d):Let $N \in \delta PO(Y)$*,then* $Int_\delta(Y \backslash N) \in \delta\text{-}SO(Y)$.*By (c),*
$f^{-1}(Cl_\delta(Int_\delta(Y \backslash N)) \in DaC(X)$ *which implies* $f^{-1}(Int(Cl_\delta(N)) \in DaO(X)$.
(d)→(e):Obvious since $O(Y) \subset \delta PO(Y)$.
(e)→(f):Clear
(f)→(a):Let $N \in RO(Y)$.*Then* $N = Int(Cl_\delta(N))$ *and hence* $Y \backslash N \in C(X)$. *By (f),*
$f^{-1}(Y \backslash N) = X \backslash f^{-1}(Int(Cl_\delta(N))) = f^{-1}(Cl(Int_\delta(Y \backslash N)) \in DaC(X)$.
Thus $f^{-1}(N) \in DaO(X)$.

Lemma 5.3. *[Ayhan and Ozkoç, 2016] Let* (X,τ) *be a space and let* $A \subset X$. *The following statements are true:*
(a) For each $A \in e^*O(X)$, $a\text{-}Cl(A) = Cl_\delta(A)$
(b)For each $A \in \delta SO(X)$, $\delta\text{-}pCl(A) = Cl_\delta(A)$.
(c)For each $A \in \delta PO(X)$,$\delta\text{-}sCl(A) = Int(Cl_\delta(A))$.

As a consequence of Theorem 5.6 and Lemma 5.3, we have the following result:

Theorem 5.7. *The following are equivalent for a function* $f:(X,\tau) \to (Y,\eta)$:
(a) f is almost Da-continuous;
(b) For every e^**-open subset G of Y,*$f^{-1}(a\text{-}Cl(G))$ *is Da-closed set in X;*
(c) For every δ*-semiopen subset G of Y,*$f^{-1}(\delta\text{-}pCl(G))$ *is Da-closed set in X;*
(d) For every δ*-preopen subset G of Y,*$f^{-1}(\delta\text{-}sCl(G)))$ *is Da-open set in X;*

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