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An Overview of Topological and Fuzzy Topological Hypergroupoids

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It is my honour to dedicate this paper to Professor Thomas Vougiouklis lifetime work.

Abstract

On a hypergroup, one can define a topology such that the hyperoperation is pseudocontinuous or continuous. This concepts can be extend to the fuzzy case and a connection between the classical and fuzzy (pseudo)continuous hyperoperations can be given. This paper, that is his an overview of results received by S. Hoskova-Mayerova with coauthors I. Cristea, M. Tahere and B. Davaz, gives examples of topological hypergroupoids and show that there is no relation (in general) between pseudotopological and strongly pseudotopological hypergroupoids. In particular, it shows a topological hypergroupoid that does not depend on the pseudocontinuity nor on strongly pseudocontinuity of the hyperoperation.
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1 Introduction

As was mentioned e.g. in [32], in various branches of mathematics we encounter important examples of topologico-algebraical structures like topological groupoids, groups, rings, fields etc. Therefore, there was a natural interest to generalize the concept of topological groupoid to topological hypergroupoid. First results of this type can be found e.g. in [6, 32].

Hypergroups are generalizations of groups. Group is a set with a binary operation on it satisfying a number of conditions. If this binary operation is taken to be multivalued, then we arrive at a hypergroup. The motivation for generalization of the notion of group resulted naturally from various problems in non-commutative algebra, another motivation for such an investigation came from geometry.

Hypergroups theory, born in 1934 with Marty‘s paper [39] presented in the 8th Congress of Scandinavian Mathematicians where he had given this renowned definition. "*Marty, managed to do the greatest generalisation anybody would ever do, acting as a pure and clever researcher. He left space for future generalisations "between" his axioms and other hypergroups, as the regular hypergroups, join spaces etc. The reproduction axiom in the theory of groups is also presented as solutions of two equations, consequently, Marty got round that hitch, too.* " [53]. He was followed in 1938 by Dresher with Ore [23] as well as by Griffiths [27] and in 1940 by Eaton [24] is now studied from the theoretical point of view and for its applications to many subjects of pure and applied mathematics (see [9, 15, 16, 57]) like algebra, geometry, convexity, topology, cryptography and code theory, graphs and hypergraphs, lattice theory, Boolean algebras, logic, probability theory, binary relations, theory of fuzzy and rough sets [12, 20], automata theory, economy, etc. [10, 11, 15].

Hypergroupoids, [17] quasi-hypergroups, semihypergroups [41, 42], hypergroups [1, 2], hyperrings [40, 52], hyperfields, [60] hyper vector spaces, hyperlattices, up to all kinds of fuzzy hyperstructures [49], have been studied theoretically as well as from the perspective of particular applications, see e.g. [5, 18, 21, 30, 33, 56]. In 1990, Th. Vougiouklis introduced the class of Hv-structures which satisfy the weak axioms where the non-empty intersection replaces the equality [55].

Moreover, topological and algebraic structures in fuzzy sets are strategically located at the juncture of fuzzy sets, topology, [26] algebra [7], lattices, etc. They has these unique features: major studies in uniformities and convergence structures, fundamental examples in lattice-valued topology, modifications and extensions of sobriety, categorical aspects of lattice-valued subsets, logic and foundations of mathematics, t-norms and associated algebraic and ordered structures. In the last decade a number of interesting applications to social sciences appear, e.g. [3, 43, 44, 59, 61, 62].

In [6], Ameri presented the concept of topological (transposition) hypergroups. He introduced the concept of a (pseudo, strong pseudo) topological hypergroup and gave some related basic results. R. Ameri studied the relationships between pseudo, pseudo topological polygroups and topological polygroups. In [28], Heidari et al. studied the notion of topological hypergroups as a generalization of topological groups. They showed - by considering the quotient topology induced by the fundamental relation on a hypergroup - that if every open subset of a topological hypergroup is complete part, then it's fundamental group is a topological group. Moreover, in [29], Heidari et al. defined the notion of topological polygroups and they investigated the topological isomorphism theorems it. Later on, Salehi Shadkami et al. [47, 48] established various relations between its complete parts and open sets and they used these facts to obtain some new results in topological polygroups. For example, they investigated some properties of cp-resolvable topological polygroups. In [32], the author of this note introduced the concept of topological hypergroupoid and found necessary and sufficient conditions for having a τ_U-topological hypergroupoid, a τ_L-topological hypergroupoid and a $\tau_\aleph$-topological hypergroupoid by using the concepts of pseudocontinuity, strong pseudocontinuity and both respectively.

When in 1965 Zadeh [63] introduced the fuzzy sets, than the reconsideration of the concept of classical mathematics began. Since then the connections between fuzzy sets and hyperstructures was studied. Using the structure of a fuzzy topological space and that of a fuzzy group (introduced by Rosenfeld [46]), Foster [26] defined the concept of *fuzzy topological group*. Later, Ma and Yu [38] changed Foster's definition in order to make sure that an ordinary topological group is a special case of a fuzzy topological group. An interesting book concerning fuzzy topology was published in 1997 by Liu [36].

Inspired by the definition of the *topological groupoid* I. Cristea and S. Hoskova-Mayerova in [19] extended these notions on a *fuzzy topological space*.

This paper is an overview of results received by S. Hoskova-Mayerova in [32] as well as the results with coauthors I. Cristea [19], M. Tahere, B. Davaz [50]. Paper is organized as follows: Firstly, we review some basic definitions and results on hypergroups and topology and fuzzy topological spaces. Section 3 recall the results concerning topological hypergroupoids. In Section 4 we recall some basic results on the fuzzy topological spaces that we use in the following Section 6. In Section 5 we recall the definition of fuzzy (pseudo)continuous hyperoperations, we explain relations between fuzzy continuous and continuous hyperoperations, between fuzzy continuous and fuzzy pseudocontinuous hyperoperations, respectively. Moreover, we give the condition when a product hypergroupoid is a fuzzy pseudotopological hypergroupoid. Finally, in Section 6 we recall some results - published in [19] concerning fuzzy topological hypergroupoids.

2 Basic Definitions

In this section, we present some definitions related to hyperstructures and topology that are used throughout the paper. They can be found in e.g. [4, 19, 31, 22].

Definition 2.1. *Let H be a non-empty set. Then, a mapping $\circ : H \times H \to \mathcal{P}^*(H)$ is called a binary hyperoperation on H, where $\mathcal{P}^*(H)$ is the family of all non-empty subsets of H. The couple $(H, \circ)$ is called a hypergroupoid.*

If A and B are two non-empty subsets of H and $x \in H$, then we define:

$$A \circ B = \bigcup_{\substack{a \in A \\ b \in B}} a \circ b, \; x \circ A = \{x\} \circ A \text{ and } A \circ x = A \circ \{x\}.$$

A hypergroupoid $(H, \circ)$ is called a:

- *semihypergroup if for every $x, y, z \in H$, we have $x \circ (y \circ z) = (x \circ y) \circ z$;*
- *quasihypergroup if for every $x \in H$, $x \circ = H = H \circ x$ (This condition is called the reproduction axiom);*
- *hypergroup if it is a semihypergroup and a quasihypergroup.*

Definition 2.2. *Let (X, τ) be a topological space. Then*

1. *(X, τ) is a T_0-space if for all $x \neq y \in X$, there exists $U \in \tau$ such that $x \in U$ and y is not in U or $y \in U$ and x is not in U.*
2. *(X, τ) is a T_1-space if for all $x \neq y \in X$, there exist $U, V \in \tau$ such that $x \in U$ and y is not in U and $y \in V$ and x is not in V.*
3. *(X, τ) is a T_2-space if for all $x \neq y \in X$, there exist $U, V \in \tau$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*

So, every T_2-topological space is a T_1-topological space and every T_1-topological space is a T_0-topological space.

Definition 2.3. *Let $(H_1, \circ_1)$, $(H_2, \circ_2)$ be two hypergroupoids and define the topologies τ, τ' on H_1, H_2 respectively. A mapping f from H_1 to H_2 is said to be good topological homomorphism if for all $x, y \in H_1$,*

1. *$f(a \circ_1 b) = f(a) \circ_2 f(b)$;*
2. *f is continuous;*

3. f is open.

A good topological homomorphism is a topological isomorphism if f is one to one and onto and we say that H_1 is topologically isomorphic to H_2.

Let $(H, \circ)$ be a hypergroupoid and A, B be non empty subsets of H. By $A \approx B$ we mean that $A \cap B \neq \emptyset$.

3 Topological Hypergroupoids

Š. Hošková-Mayerová in [32] introduced some new definitions inspired by the definition of topological groupoid. Her results are summarized in this section.

Definition 3.1. [32] *Let $(H, \cdot)$ be a hypergroupoid and (H, τ) be a topological space. The hyperoperation " $\cdot$ " is called:*

1. *pseudocontinuous (p-continuous) if for every $O \in \tau$, the set $O_\star = \{(x, y) \in H^2 : x \cdot y \subseteq O\}$ is open in $H \times H$.*
2. *strongly pseudocontinuous (sp-continuous) if for every $O \in \tau$, the set $O^\star = \{(x, y) \in H^2 : x \cdot y \approx O\}$ is open in $H \times H$.*

A simple way to prove that a hyperoperation " $\cdot$ " is p- continuous (sp- continuous) is to take any open set O in τ and $(x, y) \in H^2$ such that $x \cdot y \subseteq O$ ($x \cdot y \approx O$) and prove that there exist $U, V \in \tau$ such that $u.v \subseteq O$ ($u.v \approx O$) for all $(u, v) \in U \times V$.

Definition 3.2. [32] *Let $(H, \cdot)$ be a hypergroupoid, (H, τ) be a topological space and $\tau_\star$ be a topology on $\mathcal{P}^*(H)$.*

The triple $(H, \cdot, \tau)$ is called a pseudotopological or strongly pseudotopological hypergroupoid if the hyperoperation " $\cdot$ " is p-continuous or sp-continuous respectively.

The quadruple $(H, \cdot, \tau, \tau_\star)$ is called $\tau_\star$-topological hypergroupoid if the hyperoperation " $\cdot$ " is $\tau_\star$-continuous.

Let (H, τ) be a topological space, $V, U_1, \ldots, U_k \in \tau$. We define S_V, I_V and $\aleph(U_1, \ldots, U_k)$ as follows:

- $S_V = \{U \in \mathcal{P}^*(H) : U \subseteq V\} = \mathcal{P}^*(V)$.
- $I_V = \{U \in \mathcal{P}^*(H) : U \approx V\}$.
- $\aleph(U_1, \ldots, U_k) = \{B \in \mathcal{P}^*(H) : B \subseteq \bigcup_{i=1}^k U_i \text{ and } B \approx U_i \text{ for } i = 1, \ldots, k\}$.

$S_\emptyset = I_\emptyset = \emptyset$. For all $V \neq \emptyset$, we have

$$S_V = \mathcal{P}^*(V) \text{ and } I_V \supseteq \{H, \mathcal{P}^*(V)\}.$$

Lemma 3.1. *Let (H, τ) be a topological space.*

Then $\{S_V\}_{V\in\tau}$ forms a base for a topology (τ_U) on $\mathcal{P}^*(H)$. Moreover, τ_U is called the upper topology.

Then $\{I_V\}_{V\in\tau}$ forms a subbase for a topology (τ_L) on $\mathcal{P}^*(H)$. Moreover, τ_L is called the lower topology.

Let (H, τ) be a topological space. Then $\{\aleph(U_1, \ldots, U_k)\}_{U_i\in\tau}$ forms a base for a topology ($\tau_\aleph$) on $\mathcal{P}^*(H)$. Moreover, $\tau_\aleph$ is called the Vietoris topology [51].

Following results was proved by S. Hoskova-Mayerova in [32].

Theorem 3.1. *Let $(H, \cdot)$ be a hypergroupoid and (H, τ) be a topological space.*

Then the triple $(H, \cdot, \tau)$ is a pseudotopological hypergroupoid if and only if the quadruple $(H, \cdot, \tau, \tau_U)$ is a τ_U-topological hypergroupoid.

Then the triple $(H, \cdot, \tau)$ is a strongly pseudotopological hypergroupoid if and only if the quadruple $(H, \cdot, \tau, \tau_L)$ is a τ_L-topological hypergroupoid.

Then the triple $(H, \cdot, \tau)$ is a pseudotopological hypergroupoid and strongly pseudotopological hypergroupoid if and only if the quadruple $(H, \cdot, \tau, \tau_\aleph)$ is a $\tau_\aleph$-topological hypergroupoid.

4 Fuzzy Topological Spaces

In this section we recall some basic results on the fuzzy topological spaces that we use in the following.

Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A : X \longrightarrow [0, 1]$. We denote by $FS(X)$ the set of all fuzzy sets on X.

In this paper we use the definition of a fuzzy topological space given by Chang [8].

Definition 4.1. [8] *A fuzzy topology on a set X is a collection $\mathcal{T}$ of fuzzy sets in X satisfying*

(i) *$\underline{0} \in \mathcal{T}$ and $\underline{1} \in \mathcal{T}$ (where $\underline{0}, \underline{1} : X \longrightarrow [0, 1]$, $\underline{0}(x) = 0, \underline{1}(x) = 1$, for any $x \in X$).*

(ii) *If $A_1, A_2 \in \mathcal{T}$, then $A_1 \cap A_2 \in \mathcal{T}$.*

(iii) *If $A_i \in \mathcal{T}$ for any $i \in I$, then $\bigcup_{i\in I} A_i \in \mathcal{T}$,* *where* $\mu_{A_1 \cap A_2}(x) = \mu_{A_1}(x) \wedge \mu_{A_2}(x)$ *and* $\mu_{\bigcup_{i \in I} A_i}(x) = \bigvee_{i \in I} \mu_{A_i}(x)$.

The pair $(X, \mathcal{T})$ *is called a fuzzy topological space.*

In the definition of a fuzzy topology of Lowen [37], the condition (i) is substituted by

(i') for all $c \in [0,1]$, $k_c \in \mathcal{T}$, where $\mu_{k_c}(x) = c$, for any $x \in X$.

Example 4.1. *Now we present some examples of fuzzy topologies on a set* X. *For more details see [19].*

(i) *The family* $\mathcal{T} = \{\underline{0}, \underline{1}\}$ *is called the indiscrete fuzzy topology on* X.

(ii) *The family of all fuzzy sets in* X *is called the discrete fuzzy topology on* X.

(iii) *If* τ *is a topology on* X, *then the collection* $\mathcal{T} = \{A_O \mid O \in \tau\}$ *of fuzzy sets* X, *where* μ_{A_O} *is the characteristic function of the open set O, is a fuzzy topology on* X.

(iv) *The collection of all constant fuzzy sets in* X *is a fuzzy topology on* X, *where a constant fuzzy set* A *in* x *has the membership function* μ_A *defined as follows :* $\mu_a \colon \longrightarrow [0,1]$, $\mu_A(x) = k$, *with* k *a fix constant in* $[0,1]$.

Definition 4.2. [8] *Given two topological spaces* $(X, \mathcal{T})$ *and* $(Y, \mathcal{U})$, *a function* $f : X \longrightarrow Y$ *is fuzzy continuous if, for any fuzzy set* $A \in \mathcal{U}$, *the inverse image* $f^{-1}[A]$ *belongs to* $\mathcal{T}$, *where* $\mu_{f^{-1}[A]}(x) = \mu_A(f(x))$, *for any* $x \in X$.

Proposition 4.1. [8] *A composition of fuzzy continuous functions is fuzzy continuous function.*

Definition 4.3. [36] *A base for a fuzzy topological space* $(X, \mathcal{T})$ *is a subcollection* $\mathcal{B}$ *of* $\mathcal{T}$ *such that each member* A *of* $\mathcal{T}$ *can be written as the union of members of* $\mathcal{B}$.

A natural question is: 'How to judge whether some fuzzy subsets just form a base of some fuzzy topological space?' We have the following rule:

Proposition 4.2. [36] *A family* $\mathcal{B}$ *of fuzzy sets in* X *is a base for a fuzzy topology* $\mathcal{T}$ *on* X *if and only if it satisfies the following conditions:*

(i) *For any* $A_1, A_2 \in \mathcal{B}$, *we have* $A_1 \cap A_2 \in \mathcal{B}$.

(ii) $\bigcup_{A \in \mathcal{B}} A = \underline{1}$.

If $(X_1, \mathcal{T}_1)$ and $(X_2, \mathcal{T}_2)$ are fuzzy topological spaces, we can speak about the product fuzzy topological space $(X_1 \times X_2, \mathcal{T}_1 \times \mathcal{T}_2)$, where the product fuzzy topology is given by a base like in the following result, which can be generalized to a family of fuzzy topological spaces.

Proposition 4.3. [36] *Let $(X_1, \mathcal{T}_1)$ and $(X_2, \mathcal{T}_2)$ be fuzzy topological spaces. The product fuzzy topology $\mathcal{T}$ on the product space $X = X_1 \times X_2$ has as a base the set of product fuzzy sets of the form $A_1 \times A_2$, where $A_i \in \mathcal{T}_i$, $i = 1, 2$, and $\mu_{A_1 \times A_2}(x_1, x_2) = \mu_{A_1}(x_1) \wedge \mu_{A_2}(x_2)$.*

Proposition 4.4. [26] *Let $\{(X_i, \mathcal{T}_i)\}_{i \in I}, \{(Y_i, \mathcal{U}_i)\}_{i \in I}$ be two families of fuzzy topological spaces and $(X, \mathcal{T}), (Y, \mathcal{U})$ the respective product fuzzy topological spaces. For each $i \in I$, let $f_i : (X_i, \mathcal{T}_i) \longrightarrow (Y_i, \mathcal{U}_i)$. Then the product mapping $f = \times f_i : (x_i) \longrightarrow (f_i(x_i))$ of $(X, \mathcal{T})$ into $(Y, \mathcal{U})$ is fuzzy continuous if f_i is fuzzy continuous, for each $i \in I$.*

5 Some Results on Relation between Topological Spaces on a Set and Topological Spaces on its Powerset

In this section, we use the results presented in [32] to study topological hypergroupoids. First, we show that there is no relation (in general) between pseudotopological and strongly pseudotopological hypergroupoids.

Proposition 5.1. [50] *Let $H = \{a, b\}$, $\tau = \{\emptyset, \{a\}, H\}$ and define a hyperoperation "$\circ_1$" on H as follows:*

$\circ_1$	a	b
a	b	H
b	H	H

Then $(H, \circ_1, \tau)$ is a pseudotopological hypergroupoid.

Thus, the quadruple $(H, \circ_1, \tau, \tau_U)$ is a τ_U- topological hypergroupoid.

Moreover, $(H, \circ_1, \tau)$ is not strongly pseudotopological hypergroupoid.

Proposition 5.2. [50] *Let $H = \{a, b\}$, $\tau = \{\emptyset, \{a\}, H\}$ and define a hyperoperation "$\circ_2$" on H as follows:*

$\circ_2$	a	b
a	H	a
b	a	b

Then $(H, \circ_2, \tau)$ is a strongly pseudotopological hypergroupoid.

Now we have: The quadruple $(H, \circ_2, \tau, \tau_L)$ is a τ_L- topological hypergroupoid. $(H, \circ_2, \tau)$ is not pseudotopological hypergroupoid. Not every strongly pseudotopological hypergroupoid is a pseudotopological hypergroupoid.

Let $(H, \circ)$ be a hypergroupoid and τ a topology on H. Then $(H, \circ, \tau)$ may be neither a pseudotopological hypergroupoid nor a strongly pseudotopological hypergroupoid. We illustrate this fact by the following example.

Example 5.1. [50] *Let $H = \{a, b\}$, $\tau = \{\emptyset, \{a\}, H\}$ and define a hyperoperation " $\circ_3$ " on H as follows:*

$\circ_3$	a	b
a	b	a
b	a	b

It is easy to check, by taking $O = \{a\}$ and $a \circ_3 b \in O$, that $(H, \circ_3, \tau)$ is neither a pseudotopological hypergroupoid nor a strongly pseudotopological hypergroupoid.

Proposition 5.3. *Let $(H, \circ, \tau, \tau_U)$ be a topological hypergroupoid. Then (H, τ) is the trivial topology if and only if $(\mathcal{P}^*(H), \tau_U)$ is the trivial topology.*

For the proof see [50].

Corolary 5.1. *Let $(H, \circ, \tau, \tau_\aleph)$ be a topological hypergroupoid. Then (H, τ) is the trivial topology if and only if $(\mathcal{P}^*(H), \tau_\aleph)$ is the trivial topology.*

Proposition 5.4. *Let $(H, \circ, \tau, \tau_L)$ be a topological hypergroupoid. Then (H, τ) is the trivial topology if and only if $(\mathcal{P}^*(H), \tau_L)$ is the trivial topology.*

The proof is similar to that of Proposition 5.3.

Proposition 5.5. *Let $(H, \circ, \tau, \tau_U)$ be a topological hypergroupoid, $|H| \geq 2$ and (H, τ) be the powerset topology.*

Then $(\mathcal{P}^*(H), \tau_U)$ is not the powerset topology on $\mathcal{P}^*(H)$.

Then $(\mathcal{P}^*(H), \tau_L)$ is not the powerset topology on $\mathcal{P}^*(H)$.

Proposition 5.6. *Let $(H_1, \circ_1, \tau)$ and $(H_2, \circ_2, \tau')$ be two topologically isomorphic hypergroupoids.*

If $(H_1, \circ_1, \tau, \tau_U)$ is a τ_U- topological hypergroupoid then $(H_2, \circ_2, \tau', \tau'_U)$ is a τ_U- topological hypergroupoid.

If $(H_1, \circ_1, \tau, \tau_L)$ is a τ_L- topological hypergroupoid then $(H_2, \circ_2, \tau', \tau'_L)$ is a τ_L- topological hypergroupoid.

Corolary 5.2. *Let $(H_1, \circ_1, \tau)$ and $(H_2, \circ_2, \tau')$ be two topologically isomorphic hypergroupoids. If $(H_1, \circ_1, \tau, \tau_\aleph)$ is a $\tau_\aleph$- topological hypergroupoid then $(H_2, \circ_2, \tau', \tau'_\aleph)$ is a $\tau_\aleph$- topological hypergroupoid.*

We present now some $\tau_\aleph$- topological hypergroupoids.

Proposition 5.7. *Let $(H, \circ)$ be the total hypergroup (i.e., $x \circ y = H$ for all $(x, y) \in H^2$) and τ be any topology on H. Then $(H, \circ, \tau)$ is both: pseudotopological hypergroupoid and strongly pseudotopological hypergroupoid.*

Corolary 5.3. *Let $(H, \circ)$ be the total hypergroup and τ be any topology on H. Then the quadruple $(H, \circ, \tau, \tau_\aleph)$ is a $\tau_\aleph$- topological hypergroupoid.*

Proposition 5.8. *Let $H = \mathbb{R}$, $(H, \circ)$ be the hypergroupoid defined by:*

$$x \circ y = \begin{cases} \{a \in \mathbb{R} : x \leq a \leq y\}, & \text{if } x \leq y; \\ \{a \in \mathbb{R} : y \leq a \leq x\}, & \text{if } y \leq x. \end{cases}$$

and τ be the topology on H defined by:

$$\tau = \{]-\infty, a[: a \in \mathbb{R} \cup \{\pm\infty\}\}.$$

Then $(H, \circ, \tau, \tau_\aleph)$ is a $\tau_\aleph$- topological hypergroupoid.

Proposition 5.9. *Let $H = \mathbb{R}$, $(H, \circ)$ be the hypergroupoid defined by:*

$$x \circ y = \begin{cases} \{a \in \mathbb{R} : x \leq a \leq y\}, & \text{if } x \leq y; \\ \{a \in \mathbb{R} : y \leq a \leq x\}, & \text{if } y \leq x. \end{cases}$$

and τ be the topology on H defined by:

$$\tau = \{]a, \infty[: a \in \mathbb{R} \cup \{\pm\infty\}\}.$$

Then $(H, \circ, \tau', \tau'_\aleph)$ is a $\tau'_\aleph$- topological hypergroupoid.

Proposition 5.10. *Let $(H, \star)$ be the hypergroupoid defined by $x \star y = \{x, y\}$ and τ be any topology on H. Then $(H, \star, \tau, \tau_\aleph)$ is a $\tau_\aleph$- topological hypergroupoid.*

Example 5.2. [50] *Let $H = \{a, b\}$, $\tau = \{\emptyset, \{a\}, H\}$ and define a hyperoperation "$\circ_4$" on H as follows:*

$\circ_4$	a	b
a	a	H
b	H	b

Then, by Proposition 5.10, $(H, \circ_4, \tau, \tau_\aleph)$ is a $\tau_\aleph$- topological hypergroupoid. Moreover, $\tau_\aleph = \{\emptyset, \{\{a\}\}, \mathcal{P}^*(H)\}$.

Proposition 5.11. *Let $(H, \cdot)$ be any hypergroupoid and τ be the power set topology on H. Then $(H, \cdot, \tau, \tau_{\aleph})$ is a $\tau_{\aleph}$- topological hypergroupoid.*

Proposition 5.12. *Let $(H, \cdot)$ be any hypergroupoid and τ be the trivial topology on H. Then $(H, \cdot, \tau, \tau_{\aleph})$ is a $\tau_{\aleph}$- topological hypergroupoid.*

Next, we present a topological hypergroupoid that does not depend on the pseudocontinuity nor on strongly pseudocontinuity of the hyperoperation.

Proposition 5.13. *Let $(H, \cdot)$ be any hypergroupoid, τ be any topology on H and $\tau_\star$ be the trivial topology on $\mathcal{P}^*(H)$. Then $(H, \cdot, \tau, \tau_\star)$ is a topological hypergroupoid.*

Next, we present some results on T_0, T_1, T_2- topological spaces.

Proposition 5.14. *Let $(H, \cdot, \tau, \tau_U)$ be a τ_U-topological hypergroupoid. If $(\mathcal{P}^*(H), \tau_U)$ is a T_0- topological space then (H, τ) is a T_0- topological space.*

The converse of Proposition 5.14 is not always true. An illustrating example is presented in [50].

Proposition 5.15. *Let $|H| \geq 2$ and $(H, \cdot, \tau, \tau_U)$ be a τ_U-topological hypergroupoid. Then $(\mathcal{P}^*(H), \tau_U)$ is neither a T_1- topological space nor a T_2- topological space.*

Proposition 5.16. *Let $|H| \geq 2$ and $(H, \cdot, \tau, \tau_L)$ be a τ_L-topological hypergroupoid. Then $(\mathcal{P}^*(H), \tau_L)$ is neither a T_1- topological space nor a T_2- topological space.*

Proposition 5.17. *Let $(H, \cdot, \tau, \tau_L)$ be a τ_L-topological hypergroupoid. If $(\mathcal{P}^*(H), \tau_L)$ is a T_0- topological space then (H, τ) is a T_0- topological space.*

Proposition 5.18. *Let $(H, \cdot, \tau, \tau_L)$ be a τ_L-topological hypergroupoid. If (H, τ) is a T_0- topological space then $(\mathcal{P}^*(H), \tau_L)$ may not be a T_0- topological space.*

It can be proved that: Let $(H, \cdot, \tau, \tau_{\aleph})$ be a $\tau_{\aleph}$-topological hypergroupoid. If $(\mathcal{P}^*(H), \tau_{\aleph})$ is a T_0- topological space then (H, τ) is a T_0- topological space.

Let $(H, \cdot, \tau, \tau_{\aleph})$ be a $\tau_{\aleph}$-topological hypergroupoid. Then $(\mathcal{P}^*(H), \tau_{\aleph})$ is neither a T_2- topological space nor a T_1- topological space.

6 Fuzzy Topological Hypergroupoids

In this section we recall some results - published in [19] concerning fuzzy topological hypergroupoids.

Definition 6.1. *Let $(H, \circ)$ be a hypergroupoid, $\mathcal{T}$ and $\mathcal{U}$ be fuzzy topologies on H and $\mathcal{P}^*(H)$, respectively. The hyperoperation "$\circ$" is called $\mathcal{U}$-fuzzy continuous if the map $\circ : H \times H \longrightarrow \mathcal{P}^*(H)$ is fuzzy continuous with respect to the fuzzy topologies $\mathcal{T} \times \mathcal{T}$ and $\mathcal{U}$.*

For any topology τ on a set X, we denote by $\mathcal{T}_c$ the fuzzy topology formed with the characteristic functions of the open sets of τ. In the following result we give a relation between the continuity and fuzzy continuity of a hyperoperation.

Proposition 6.1. *Let $(H, \circ)$ be a hypergroupoid, τ and τ^* be topologies on H and $\mathcal{P}^*(H)$, respectively. Let $\mathcal{T}_c$ and $\mathcal{U}_c$ be the fuzzy topologies on H and $\mathcal{P}^*(H)$, respectively, generated by τ and τ^*, respectively. The hyperoperation "$\circ$" is τ^*-continuous if and only if it is $\mathcal{U}_c$-fuzzy continuous.*

Let $(H, \mathcal{T})$ be a fuzzy topological space. Then the family $\mathcal{B} = \{\tilde{A} \in FS$ $(\mathcal{P}^*(H)) \mid A \in \mathcal{T}\}$, where $\mu_{\tilde{A}}(X) = \bigwedge\limits_{x \in X} \mu_A(x)$, is a base for a fuzzy topology $\mathcal{T}^*$ on $\mathcal{P}^*(H)$.

Definition 6.2. *Let $(H, \circ)$ be a hypergroupoid endowed with a fuzzy topology $\mathcal{T}$. The hyperoperation "$\circ$" is called fuzzy pseudocontinuous (or briefly fuzzy p-continuous) if, for any $A \in \mathcal{T}$, the fuzzy set A_* in $H \times H$ belongs to $\mathcal{T} \times \mathcal{T}$, where $\mu_{A_*}(x, y) = \bigwedge\limits_{u \in x \circ y} \mu_A(u)$.*

The triple $(H, \circ, \mathcal{T})$ is called a fuzzy pseudotopological hypergroupoid if the hyperoperation "$\circ$" is fuzzy p-continuous.

Now we can characterize a fuzzy pseudotopological hypergroupoid $(H, \circ, \mathcal{T})$ using the $\mathcal{T}^*$-fuzzy continuity of the hyperoperation "$\circ$", where the fuzzy topology $\mathcal{T}^*$ is that one given in Proposition 6.1.

Let $(H, \circ)$ be a hypergroupoid and $\mathcal{T}$ be a fuzzy topology on H. Then the triple $(H, \circ, \mathcal{T})$ is a fuzzy pseudotopological hypergroupoid if and only if the hyperoperation "$\circ$" is $\mathcal{T}^*$-fuzzy continuous.

Proposition 6.2. *Let $(H_1, \mathcal{T}_1)$ and $(H_2, \mathcal{T}_2)$ be two fuzzy topological spaces. We denote $H = H_1 \times H_2$ and $\mathcal{T} = \mathcal{T}_1 \times \mathcal{T}_2$. Then the mapping*

$$\alpha : (H, \mathcal{T}) \times (H, \mathcal{T}) \longrightarrow (H_1 \times H_1, \mathcal{T}_1 \times \mathcal{T}_1) \times (H_2 \times H_2, \mathcal{T}_2 \times \mathcal{T}_2),$$

defined by $\alpha((x_1, x_2), (y_1, y_2)) = ((x_1, y_1), (x_2, y_2))$ is fuzzy continuous.

Let $(H_1, \circ_1)$ and $(H_2, \circ_2)$ be two hypergroupoids. The product hypergroupoid $(H_1 \times H_2, \otimes)$ has the hyperoperation defined by $(x_1, x_2) \otimes (y_1, y_2) = (x_1 \circ_1 y_1, x_2 \circ_2 y_2)$, for any $(x_1, x_2), (y_1, y_2) \in H_1 \times H_2$.

So, we get:

If $(H_1, \circ_1, \mathcal{T}_1)$ and $(H_2, \circ_2, \mathcal{T}_2)$ are fuzzy pseudotopological hypergroupoids, then the product hypergroupoid $(H_1 \times H_2, \otimes, \mathcal{T}_1 \times \mathcal{T}_2)$ is a fuzzy pseudotopological hypergroupoid.

7 Conclusions

On a hypergroup, a topology such that the hyperoperation is pseudocontinuous can be defined. This paper highlighted the topological hypergroupoids by proving some of theirs properties. It illustrated the results achieved on topological hypergroupoids in [32] by examples and remarks. Moreover, it was shown that there is no relation (in general) between pseudotopological and strongly pseudotopological hypergroupoids. In particular, we presented a topological hypergroupoid that does not depend on the pseudocontinuity nor on strongly pseudocontinuity of the hyperoperation.

For future work, the existence of topological hypergroupoids on $\mathcal{P}^*(H)$ that are neither τ_U nor τ_L nor $\tau_\aleph$ can be investigated or the existence of n-ary topological hypergroupoids can be studied.

This work could be also continued in order to introduce the notion of fuzzy topological hypergroup as a generalization of a fuzzy topological group in the sense of Foster [26] or in the sense of Ma and Yu [38].

The author would like to express a wish for this beautiful discipline of mathematics to be continue and to be developed. Since there are already numbers of excellent mathematicians around the world who are concerned with this issue, lets believe their interest will not go away, and that the School of Professor P. Corsini and Professor T. Vougiouklis will find many followers.

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