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Some results on range labeling

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Abstract

Let G=(,) be a graph with finite and simple . Let (G) and (G) be the vertex set and edge set of G respectively. A Range labeling of a graph G is an injective function. $(G) \rightarrow \{1, 2. \ldots\}$ such that the edge labeling $\alpha* : (G) \rightarrow \{1, 2. \ldots.\}$ is defined by $\alpha[]$= maximum value ( ) - minimum Value (). A graph which admits such labeling is called a range graph. In this paper the range labeling is introduced and range labeling for a some trees as for star tree, spider tree and Banana tree are calculated.

Keywords: Graph; Labeling; Graceful labeling; Range labeling; Star trees; Spider trees; Banana trees.
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1 Introduction

Graph labeling is an assignment of set of integers to the set of vertices, edges or both based on certain conditions, In 1967 Rosa was first introduced the graph labeling. A graph labeling are useful family of mathematical models applied in many areas such as radar, missile guidance, radio frequency modulation and many more. A graph labeling is one topic in graph theory. So many kind of graph labeling among others: magic labeling, graceful labeling Afsana Ahmed munia [2014], o - Edge magic labeling, 1- Edge magic labeling, mean labeling, and etc. Every year and updated survey comes about various types of labeling by J. A. Gallian. From the survey, various types of labeling analyzed and introduced a new type of labeling called Range Labeling Moinin Al Aziz Md [2014]. In this paper are to proof that some trees namely Star tree, Spider tree, and Banana tree admit range labeling.

This article assumed for all graphs are finite and simple. The graph $G = (\Psi, \tau)$ where $\Psi(G)$ set of vertices and $\tau(G)$ set of edges. A labeling is a most one of the part of the graph theory, R.Uma [2012]. A labeling is the assignment of labels, traditionary defined by integers to edges or vertices or both of vertices and edges Moinin Al Aziz Md [2014].

The origin of labeling is Rosa by 1967. R. Jahir Hussain [2022] and Afsana Ahmed munia [2014] was First developed Range labeling in 2022. The article Range labeling apply for star trees, spider trees, Banana tress A. N.Mohamed [2013].

2 Preliminary

Definition 2.1. *R.Uma [2012] A graceful labeling of a graph G is a vertex labeling $\alpha : \Psi \to [0, m]$ such that α is injective and induced mapping*

$$\alpha(\tau) = |\alpha(\Psi_k) - \alpha(\Psi_{k+1})|, \textit{ for every } \Psi_k\Psi_{k+1} \in \tau(G).$$

Assigns different labels to different edges of G. The differences $|\alpha(\Psi_k) - \alpha(\Psi_{k+1})|$ is labeled weight of the edges $\Psi_k\Psi_{k+1}$. A graph G is called graceful.

Definition 2.2. *Let $G = (\Psi, \tau)$ be a graph with n vertices. A bijection on α : $\Psi \to \{1, 3, 6, 10, 15, ...\frac{n^2+n}{2}\}$ is called a range labeling if for each edge τ is distinct and τ is defined by $\alpha^*(\tau)$ = Maximum value $(\Psi_k, \Psi_{k+1})-$ Minimum value (Ψ_k, Ψ_{k+1}).*

Definition 2.3. *A tree for 1-internal vertex and k edges is called star S_1, K that appear to be complete by bipartite graph K_1, K.*

Definition 2.4. *A Spider tree with atmost one vertex of degree greater than* 2.

Definition 2.5. *An* (m, t)*-Banana tree is a graph attained by attaching 1-edge of all* m *copies of an* t*-star graph for a 1-root vertex is different from each stars.*

3 Main results

Theorem 3.1. *All star tree take a range labeling.*

Proof. Let $G = (\Psi, \tau)$, be a graph for Ψ_1 is an interval vertex, 10 edges.
Consider, $\alpha : \Psi \to \{1, 3, 6, 10, 15, ...\frac{n^2+n}{2}\}$
$\alpha^*(\tau) =$ Maximum value $(\Psi_k, \Psi_{k+1})-$ Minimum value (Ψ_k, Ψ_{k+1})
$\alpha^*(\tau_1) =$ Maximum value $(\Psi_1, \Psi_2)-$ Minimum value (Ψ_1, Ψ_2)
if Ψ_1 is a maximum value, Ψ_2 is a minimum value.
$\alpha^*(\tau_1) = (\Psi_1 - \Psi_2)$.
$= 1$ is an integer.
Suppose Ψ_2 is a maximum value, Ψ_1 is a minimum value.
$\alpha^*(\tau_1) = (\Psi_2 - \Psi_1)$.
$= 1$ is an integer.
$\alpha^*(\tau_2) =$ Maximum value $(\Psi_1, \Psi_3)-$ Minimum value (Ψ_1, Ψ_3)
if Ψ_1 is a maximum value, Ψ_3 is a minimum value.
$\alpha^*(\tau_2) = (\Psi_1 - \Psi_3)$.
$= 1$ is an integer.
Suppose Ψ_3 is a maximum value, Ψ_1 is a minimum value.
$\alpha^*(\tau_2) = (\Psi_3 - \Psi_1)$.
$= 1$ is an integer.
$\alpha^*(\tau_9) =$ Maximum value $(\Psi_1, \Psi_{10})-$ Minimum value (Ψ_1, Ψ_{10})

if Ψ_1 is a maximum value, Ψ_{10} is a minimum value.

$\alpha^*(\tau_9) = (\Psi_1 - \Psi_{10})$.
$= 1$ is an integer.
Suppose Ψ_{10} is a maximum value, Ψ_1 is a minimum value.
$\alpha^*(\tau_9) = (\Psi_{10} - \Psi_1)$.
$= 1$ is an integer.
$\alpha^*(\tau_{10}) =$ Maximum value $(\Psi_1, \Psi_{11})-$ Minimum value (Ψ_1, Ψ_{11})
if Ψ_1 is a maximum value, Ψ_{11} is a minimum value.
$\alpha^*(\tau_{10}) = (\Psi_1 - \Psi_{11})$.
$= 1$ is an integer.
Suppose Ψ_{11} is a maximum value, Ψ_1 is a minimum value.
$\alpha^*(\tau_{10}) = (\Psi_{11} - \Psi_1)$.

$= 1$ is an integer.
Hence, Every star tree is a received range labeling

Therefore, Any star tree is a Range graph.

Example 3.1.

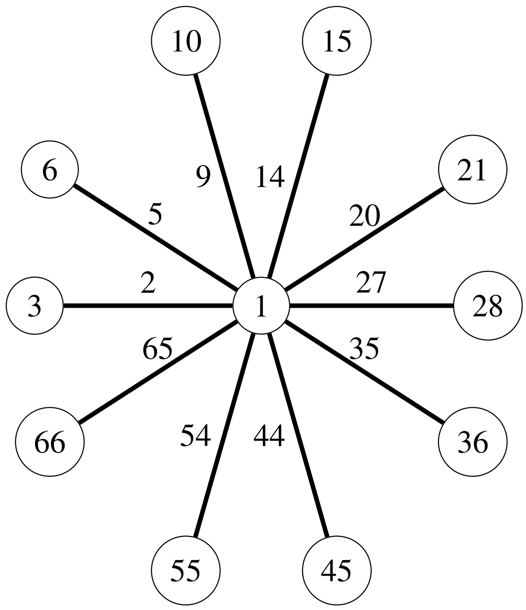

Fig.1. Range Labelling for star tree S_{10}.

Theorem 3.2. *Every Spider tree take a range labeling.*

Proof. Let $G = (\Psi, \tau)$, be a graph.
Let $\alpha : \Psi \to \{1, 3, 6, 10, 15, ...\frac{n^2+n}{2}\}$
A spider tree for atmost 1-node of degree greater than 2 and this node is said to be section node and is denoted by Ψ_0. A stage of a spider graph is a path from the section node to edge of the tree. The edge is denoted by Ψ_1, Ψ_2, Ψ_3.

$\alpha^*(\tau) =$ Maximum value $(\Psi_k, \Psi_{k+1})-$ Minimum value (Ψ_k, Ψ_{k+1})

$\alpha^*(\tau_1) =$ Maximum value $(\Psi_0, \Psi_1)-$ Minimum value (Ψ_0, Ψ_1)

if Ψ_0 is a maximum value, Ψ_1 is a minimum value.

$\alpha^*(\tau_1) = (\Psi_0 - \Psi_1)$.
$= 1$ is an integer.
Suppose Ψ_1 is a maximum value, Ψ_0 is a minimum value.

$\alpha^*(\tau_1) = (\Psi_1 - \Psi_0)$.
$= 1$ is an integer.

$\alpha^*(\tau_2) =$ Maximum value $(\Psi_0, \Psi_2)-$ Minimum value (Ψ_0, Ψ_2)

if Ψ_0 is a maximum value, Ψ_2 is a minimum value.
$\alpha^*(\tau_2) = (\Psi_0 - \Psi_2)$.
$= 1$ is an integer.
Suppose Ψ_2 is a maximum value, Ψ_0 is a minimum value.

$\alpha^*(\tau_2) = (\Psi_2 - \Psi_0)$.
$= 1$ is an integer.
$\alpha^*(\tau_3) =$ Maximum value $(\Psi_0, \Psi_3)-$ Minimum value (Ψ_0, Ψ_3)

if Ψ_0 is a maximum value, Ψ_3 is a minimum value.

$\alpha^*(\tau_3) = (\Psi_0 - \Psi_3)$.
$= 1$ is an integer.
Suppose Ψ_3 is a maximum value, Ψ_0 is a minimum value.

$\alpha^*(\tau_3) = (\Psi_3 - \Psi_0)$.
$= 1$ is an integer.
So, All spider tree accepted range labeling

Thus, Every spider tree is a range graph.

Example 3.2.

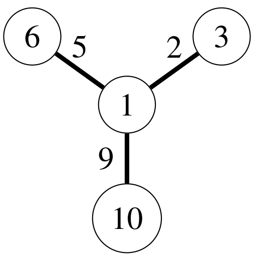

Fig.2. Range Labelling for Spider Tree.

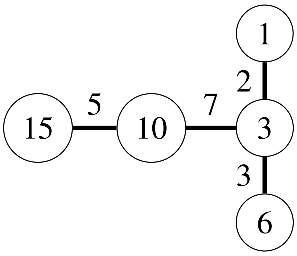

Fig.3. Range Labelling for Spider Tree.

Example 3.3.

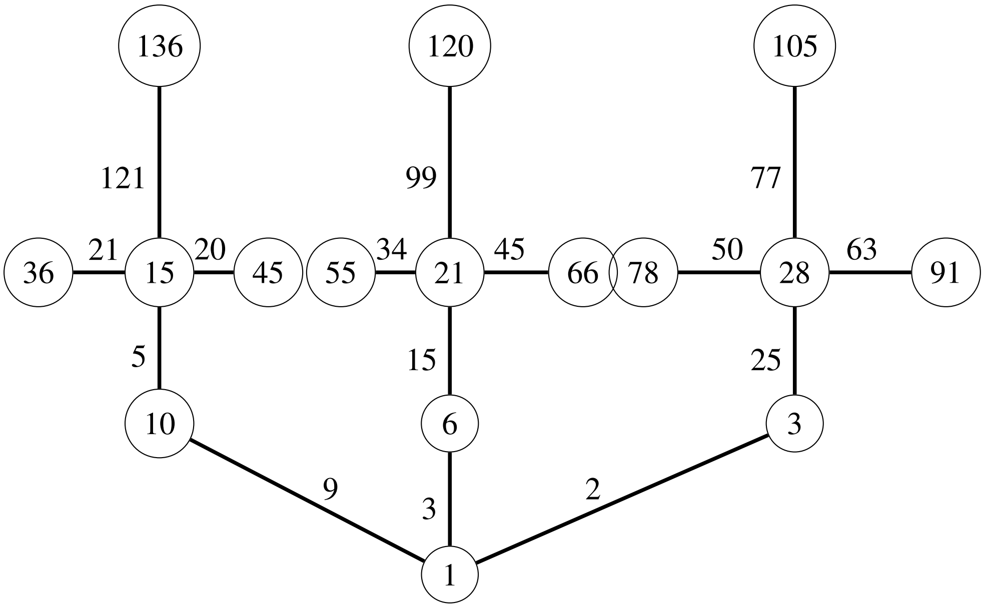

Fig.1. Range Labelling for Banana tree B(3,5).

Theorem 3.3. *All Banana tree take a range labeling.*

Proof. Let $G = (\Psi, \tau)$, be a graph.
Let $\alpha : \Psi \to \{1, 3, 6, 10, 15, ...\frac{n^2+n}{2}\}$

$B(3, 5)$ is a Banana tree by attaching 1-edge of every 3 copies (Ψ_1, Ψ_2, Ψ_3) of an $5-$star $(\Psi_1., \Psi_2., \Psi_3.)$ graph for 1-root vertex is different from every stars.
$\alpha^*(\tau) =$ Maximum value $(\Psi_k, \Psi_{k+1})-$ Minimum value (Ψ_k, Ψ_{k+1})
$\alpha^*(\tau_1) =$ Maximum value $(\Psi_0, \Psi_1)-$ Minimum value (Ψ_0, Ψ_1)
if Ψ_0 is a maximum value, Ψ_1 is a minimum value.
$\alpha^*(\tau_1) = (\Psi_0 - \Psi_1)$.
$= 1$ is an integer.
Suppose Ψ_1 is a maximum value, Ψ_0 is a minimum value.
$\alpha^*(\tau_1) = (\Psi_1 - \Psi_0)$.
$= 1$ is an integer.
$\alpha^*(\tau_2) =$ Maximum value $(\Psi_0, \Psi_2)-$ Minimum value (Ψ_0, Ψ_2)
if Ψ_2 is a maximum value, Ψ_0 is a minimum value.
$\alpha^*(\tau_2) = (\Psi_2 - \Psi_0)$.
$= 1$ is an integer.
Suppose Ψ_0 is a maximum value, Ψ_2 is a minimum value.
$\alpha^*(\tau_2) = (\Psi_0 - \Psi_2)$.
$= 1$ is an integer.
$\alpha^*(\tau_3) =$ Maximum value $(\Psi_0, \Psi_3)-$ Minimum value (Ψ_0, Ψ_3)

if Ψ_0 is a maximum value, Ψ_3 is a minimum value.
$\alpha^*(\tau_3) = (\Psi_0 - \Psi_3).$
$= 1$ is an integer.
Suppose Ψ_3 is a maximum value, Ψ_0 is a minimum value.
$\alpha^*(\tau_3) = (\Psi_3 - \Psi_0).$
$= 1$ is an integer.
$\alpha^*(\tau_4) =$ Maximum value $(\Psi_1, \Psi_{11})-$ Minimum value (Ψ_1, Ψ_{11})
if Ψ_1 is a maximum value, Ψ_{11} is a minimum value.
$\alpha^*(\tau_4) = (\Psi_1 - \Psi_{11}).$
$= 1$ is an integer.
Suppose Ψ_{11} is a maximum value, Ψ_1 is a minimum value.
$\alpha^*(\tau_4) = (\Psi_{11} - \Psi_1).$
$= 1$ is an integer.
$\alpha^*(\tau_7) =$ Maximum value $(\Psi_1, \Psi_{14})-$ Minimum value (Ψ_1, Ψ_{14})

if Ψ_1 is a maximum value, Ψ_{14} is a minimum value.
$\alpha^*(\tau_7) = (\Psi_1 - \Psi_{14}).$
$= 1$ is an integer.
Suppose Ψ_{14} is a maximum value, Ψ_1 is a minimum value.
$\alpha^*(\tau_7) = (\Psi_{14} - \Psi_1).$
$= 1$ is an integer.
$\alpha^*(\tau_8) =$ Maximum value $(\Psi_2, \Psi_{21})-$ Minimum value (Ψ_2, Ψ_{21})
if Ψ_2 is a maximum value, Ψ_{21} is a minimum value.
$\alpha^*(\tau_8) = (\Psi_2 - \Psi_{21}).$
$= 1$ is an integer.
Suppose Ψ_{21} is a maximum value, Ψ_2 is a minimum value.
$\alpha^*(\tau_8) = (\Psi_{21} - \Psi_2).$
$= 1$ is an integer.
$\alpha^*(\tau_{11}) =$ Maximum value $(\Psi_2, \Psi_{24})-$ Minimum value (Ψ_2, Ψ_{24})

if Ψ_2 is a maximum value, Ψ_{24} is a minimum value.
$\alpha^*(\tau_{11}) = (\Psi_2 - \Psi_{24}).$
$= 1$ is an integer.
Suppose Ψ_{24} is a maximum value, Ψ_2 is a minimum value.
$\alpha^*(\tau_{11}) = (\Psi_{24} - \Psi_2).$
$= 1$ is an integer.
$\alpha^*(\tau_{12}) =$ Maximum value $(\Psi_3, \Psi_{31})-$ Minimum value (Ψ_3, Ψ_{31})
if Ψ_3 is a maximum value, Ψ_{31} is a minimum value.
$\alpha^*(\tau_{12}) = (\Psi_3 - \Psi_{31}).$
$= 1$ is an integer.

Suppose Ψ_{31} is a maximum value, Ψ_3 is a minimum value.
$\alpha^*(\tau_{12}) = (\Psi_{31} - \Psi_3)$.
$= 1$ is an integer.
$\alpha^*(\tau_{15}) =$ Maximum value $(\Psi_3, \Psi_{34})-$ Minimum value (Ψ_3, Ψ_{34})

if Ψ_3 is a maximum value, Ψ_{34} is a minimum value.
$\alpha^*(\tau_{15}) = (\Psi_3 - \Psi_{34})$.
$= 1$ is an integer.
Suppose Ψ_{34} is a maximum value, Ψ_3 is a minimum value.
$\alpha^*(\tau_{15}) = (\Psi_{34} - \Psi_3)$.
$= 1$ is an integer.
Hence, Every Banana tree received range labeling.
Therefore, All Banana tree is a range labeling.

4 Conclusions

In this article discussed for some trees received range labeling, so this trees star tree, spider tree, banana tree is also a range graph. Further more Analysis for this labeling apply for some special graphs.

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