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Transversal core of intuitionistic fuzzy k-partite hypergraphs

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Abstract

In graph theory, a transversal is a set of vertices incident to every edge in a graph but in Intuitionistic Fuzzy k-Partite Hypergraph(IFk-PHG), the transversal is a hyperedge which cuts every hyperedges. In this article, Intuitionistic Fuzzy Transversal(IFT), minimal IFT, locally minimal IFT, IFTC(Intuitionistic Fuzzy Transversal Core) of IFk-PHG has been defined. It has been proved that every IFk-PHG has a nonempty IFT. Also few of the properties relating to the transversal of IFk-PHG were discussed.

Keywords: IFT; minimal IFT; locally minimal IFT; IFTC of IFk-PHG.
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1 Introduction

Euler was the first author who found graph theory in 1736. The Graph theoretical approach is widely used to solve numerous issues in different areas like computer science, optimization, algebra and number theory. As an application part, the concept of graph has been extended to hypergraph, an edge with more than one or two vertices. An idea of graph and hypergraph was popularized by Berge [1976] in 1976. Fuzzy graph and fuzzy hypergraph concepts are developed by the authors in J.N.Mordeson and Nair [2000]. In K.T.Atanassov [1999], the author wrote ideas of Intuitionistic Fuzzy Sets(IFS). According to K.T.Atanassov [2002], K.T.Atanassov [2012], the researcher putforth ideas of intuitionistic fuzzy relations and cartesian products are defined.

In Myithili and Keerthika [2020a], the authors proposed the notion of k-partite hyperedges in IFHGs(Intuitionistic Fuzzy Hypergraphs). Certain operations like Union, Intersection, Ringsum, Cartesian Product were discussed in Myithili and Keerthika [2020b] . It has numerous application problems in decision-making. In Myithili and Parvathi [2015], Myithili and Parvathi [2016], Myithili et al. [2014] transversals and its properties on intuitionistic fuzzy directed hypergraphs were discussed.

The authors in Goetschel [1995], Goetschel [1998], Goetschel et al. [1996] initiated the concepts like fuzzy transversal and fuzzy coloring in fuzzy hypergraph. In this article an attempt has been made to analyze the transversal and its related properties in IFk-PHGs.

2 Symbolic representation

MNMV-membership and non-membership values
FSV-Finite set of vertices
FIFS-family of intuitionistic fuzzy subsets
IFH-intuitionistic fuzzy hyperedge
ONV-Open Neighborhood of the vertex
CNV-Closed Neighborhood of the vertex
$\aleph = (\vee, \Xi, \psi)$ - Intuitionistic fuzzy(IF) k-partite hypergraph with edge set Ξ, vertex set $\vee$ and k-partite hyperedge ψ
$h(\aleph)$ - Height of IFk-PHG
$F_k(\aleph)$ - Fundamental sequence (FS) of IFk-PHG
$c(\aleph)$ - Core set(CS) of IFk-PHG
$I_k(\aleph)$ - Induced fundamental sequence(IFS) of IFk-PHG

$\aleph^{(a_i,b_i)}$ - (a_i, b_i)-level of IFk-PHG
(a_i, b_i) - Edge membership(EM) and non-membership values(ENMV)
$\mathscr{TR}(\aleph)$ - minimal intuitionistic fuzzy transversal(MIFT) of IFk-PHG

3 Preliminaries

Definition 3.1. *Myithili and Keerthika [2020a] The IFk-PHG $\aleph$ is an ordered triple $\aleph = (\vee, \Xi, \psi)$ where,*
- $\vee = \{g_1, g_2, g_3, \cdots, g_n\}$ *is a FSV,*
- $\Xi = \{\Xi_1, \Xi_2, \Xi_3, \cdots, \Xi_m\}$ *is a FIFS of* $\vee$*,*
- $\Xi_j = \{(g_i, \omega_j(g_i), \nu_j(g_i)) : \omega_j(g_i), \nu_j(g_i) \geq 0,\ \omega_j(g_i) + \nu_j(g_i) \leq 1\}$, $1 \leq j \leq m$*,*
- $\Xi_j \neq \emptyset$, $1 \leq j \leq m$*,*
- $\bigcup_j supp(\Xi_j) = \vee$, $1 \leq j \leq m$.

For all $g_i \in \Xi\ \exists\ k$ - disjoint sets ψ_i, $i = 1, 2, \cdots, k$ and no two vertices in the same set are adjacent such that $\Xi_k = \bigcap_{i=1}^{k} \psi_i = \emptyset$

Definition 3.2. *Myithili and Keerthika [2020a] Let an IFk-PHG be $\aleph = (\vee, \Xi, \psi)$. The height of IF$k$-PHG is defined by $h(\aleph) = \{max(min(\omega_{k_{ij}})), max(max(\nu_{k_{ij}}))\}$ for all $1 \leq i \leq m$ and $1 \leq j \leq n$. Also $\omega_{k_{ij}}$ and $\nu_{k_{ij}}$ are MNMV of the k-partite hyperedge ψ_{ij}.*

Definition 3.3. *Myithili and Keerthika [2020a] Let $\aleph$ be an IFk-PHG. Suppose ψ_j,ψ_k $\in\psi$ and $0 < \delta,\varepsilon \leq 1$. The (δ, ε)-level is defined by $(\psi_j, \psi_k)^{(\delta,\varepsilon)} = \{g_i \in \vee | min(\omega^{\delta}_{k_{ij}}(g_i)) \geq \delta, max(\nu^{\varepsilon}_{k_{ij}}(g_i)) \leq \varepsilon\}$.*

Definition 3.4. *Myithili and Keerthika [2020a] Let $\aleph$ be IFk-PHG, $\aleph^{a_i,b_i} = \left\langle \vee^{a_i,b_i}, \Xi^{a_i,b_i} \right\rangle$ be the (a_i, b_i)-level of $\aleph$. The sequence of real numbers $\{a_1, a_2, \cdots, a_k; b_1, b_2, \cdots, b_k\}$ $\ni 0 \leq a_i \leq h_\omega(\aleph)$ and $0 \leq b_i \leq h_\nu(\aleph)$, satisfies:*

(i) If $a_1 < \delta \leq 1$ & $0 \leq \varepsilon < b_1$ then $\psi^{\delta,\varepsilon} = \emptyset$,
(ii) If $a_{i+1} \leq \delta \leq a_i$; $b_i \leq \varepsilon \leq b_{i+1}$ then $\psi^{\delta,\varepsilon} = \psi^{a_i,b_i}$,
(iii) $\psi^{a_i,b_i} \sqsubset \psi^{a_{i+1},b_{i+1}}$ is fundamental sequence of IFk-PHG and it is denoted as $F_k(\aleph)$.

Definition 3.5. *Myithili and Keerthika [2020a] Let $c(\aleph) = \{\aleph^{a_1,b_1}, \aleph^{a_2,b_2}, \cdots, \aleph^{a_k,b_k}\}$ be core set of $\aleph$. The analogous set of (a_i, b_i)-level hypergraphs is $\aleph^{a_1,b_1} \subset \aleph^{a_2,b_2} \subset \cdots \subset \aleph^{a_k,b_k}$ is said to be $\aleph$-IFS and it is denoted by $I_k(\aleph)$. The (a_k, b_k)-level is known as support level of $\aleph$. $\aleph^{a_k,b_k}$ is known as the support of $\aleph$.*

Definition 3.6. *Myithili and Keerthika [2020a] Let $\aleph = (\vee, \Xi, \psi)$ & $\aleph' = (\vee', \Xi', \psi')$ are IFk-PHGs, $\aleph$ is known as partial IFk-PHG of $\aleph'$, if*

$$\vee' = \begin{cases} min\,(supp\,(\omega_{k_{ij}}))\,|\,\omega_{k_{ij}} \in \psi' \\ max\,(supp\,(\nu_{k_{ij}}))\,|\,\nu_{k_{ij}} \in \psi' \end{cases}$$

the partial IFk-PHG generated by ψ' and is represented as $\aleph \subseteq \aleph'$. Also, if $\aleph \subseteq \aleph'$ and $\aleph \neq \aleph'$ exists then $\aleph \subset \aleph'$.

Definition 3.7. *Myithili and Keerthika [2020a] Let $\aleph$ be the IFk-PHG, $c(\aleph) = \{\aleph^{a_1,b_1}, \aleph^{a_2,b_2}, \cdots, \aleph^{a_k,b_k}\}$. $\aleph$ is called as ordered if $c(\aleph)$ is ordered (i.e) $\aleph^{a_1,b_1} \subset \aleph^{a_2,b_2} \subset \cdots \subset \aleph^{a_k,b_k}$. The IF$k$-PHG is known as simply ordered if $\{\aleph^{a_i,b_i} | i = 1, 2, \cdots, k\}$ is simply ordered, (i.e) if it is ordered and if $\psi \in \aleph^{a_{i+1},b_{i+1}} \backslash \aleph^{a_i,b_i}$ then $\psi \nsubseteq \aleph^{a_i,b_i}$.*

4 Main results

Definition 4.1. *Consider an IFk-PHG $\aleph$. An IFT $\mathscr{T}$ of IFk-PHG is an IF subset of $\vee$ with $\mathscr{T}^{(\psi_j,\psi_k)} \cap \mathscr{A}^{(\psi_j,\psi_k)} \neq \emptyset$ for each $\mathscr{A} \in \psi$ where $\psi_j = min(\omega_{k_{ij}})$ and $\psi_k = max(\nu_{k_{ij}})$ $\forall$ $1 \leq i \leq m$, $1 \leq j \leq n$. Also $\omega_{k_{ij}}$ and $\nu_{k_{ij}}$ is the MNMV of k^{th} partition of j^{th} edge in i^{th} vertex.*

Definition 4.2. *A minimal IFT $\mathscr{T}$ for IFk-PHG be a transversal of $\aleph$, which satisfies the condition that if $\mathscr{T}_1 \subset \mathscr{T}$, then $\mathscr{T}_1$ is not IFT of $\aleph$.*
Note: *The set of minimal IFT of IFk-PHG is denoted as $\mathscr{T}\mathscr{R}(\aleph)$. Always $\mathscr{T}\mathscr{R}(\aleph) \neq \emptyset$.*

Example 4.1. *An IFH (intuitionistic fuzzy hypergraph) with $\vee = \{g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8\}$, $\Xi = \{\Xi_1, \Xi_2, \Xi_3, \Xi_4\}$ has been considered.*

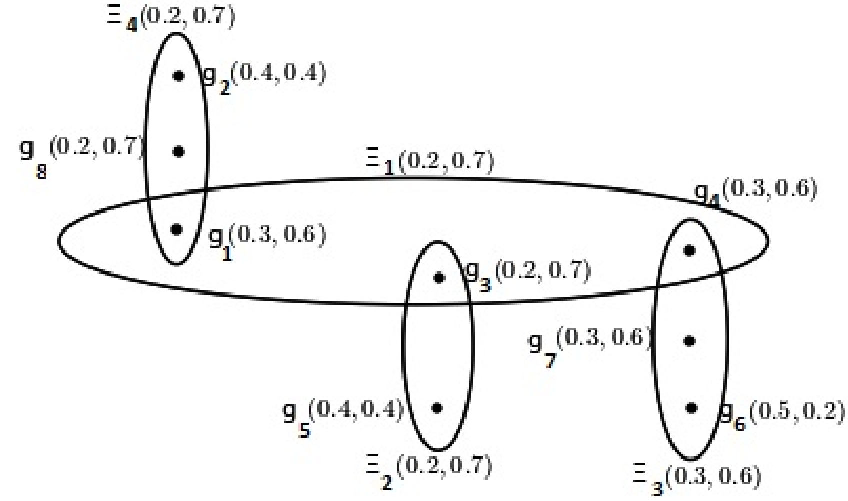

Figure 1: Intuitionistic Fuzzy Hypergraph

Using the above figure we can construct an IFk-PHG $\aleph$, with $\psi = \{\psi_1, \psi_2, \psi_3\}$ disjoint hyperedges which are represented below as incidence matrix

$$\begin{array}{c} \\ \mathsf{g}_1 \\ \mathsf{g}_2 \\ \mathsf{g}_3 \\ \mathsf{g}_4 \\ \mathsf{g}_5 \\ \mathsf{g}_6 \\ \mathsf{g}_7 \\ \mathsf{g}_8 \end{array} \begin{array}{c} \begin{array}{ccc} \psi_1 & \psi_2 & \psi_3 \end{array} \\ \begin{pmatrix} \langle 0.3, 0.6\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0.4, 0.4\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0.2, 0.7\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0.3, 0.6\rangle \\ \langle 0.4, 0.4\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0.5, 0.2\rangle & \langle 0, 1\rangle \\ \langle 0.3, 0.6\rangle & \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle & \langle 0.2, 0.7\rangle \end{pmatrix} \end{array}$$

The minimal IFT of IFk-PHG is attained as follows,

$$\begin{array}{c} \\ \mathsf{g}_1 \\ \mathsf{g}_2 \\ \mathsf{g}_3 \\ \mathsf{g}_4 \\ \mathsf{g}_5 \\ \mathsf{g}_6 \\ \mathsf{g}_7 \\ \mathsf{g}_8 \end{array} \begin{array}{c} \begin{array}{cc} \mathscr{T}_1 & \mathscr{T}_2 \end{array} \\ \begin{pmatrix} \langle 0.3, 0.6\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0.2, 0.7\rangle & \langle 0.2, 0.7\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0, 1\rangle \\ \langle 0, 1\rangle & \langle 0.3, 0.6\rangle \\ \langle 0.2, 0.7\rangle & \langle 0.2, 0.7\rangle \end{pmatrix} \end{array}$$

The corresponding graph is shown below.

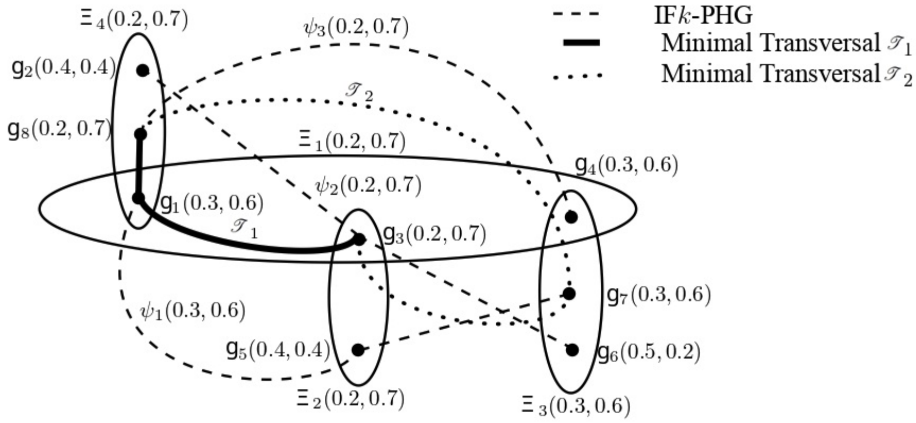

Figure 2: $\aleph$ and minimal IFT of $\aleph$

Definition 4.3. *If $\mathscr{T}$ is an IFS with $\mathscr{T}^{(a_i,b_i)}$ as minimal IFT (MIFT) of $\aleph^{(a_i,b_i)}$ for each $(a_i, b_i) \in (0, 1)$ then $\mathscr{T}$ is called as locally minimal IFT (LMIFT) of IFk-PHG. The set containing LMIFT of IFk-PHG is written as $\mathscr{T}\mathscr{R}^*(\aleph)$*

Theorem 4.1. *If $\mathscr{T}$ is an IFT of $\aleph$ then $h(\mathscr{T}) \geq h(\psi_j)$ for $\psi_j \in \psi$. Also, if $\mathscr{T}$ is the minimal IFT of IFk-PHG, then*

$h(\mathscr{T}) = \{max(min(\omega_{k_{ij}})), max(max(\nu_{k_{ij}})) \mid \omega_{k_{ij}}, \nu_{k_{ij}} \in \psi\} = h(\aleph)$.

Theorem 4.2. *Every IFk-PHG has a nonempty IFT.*
Note: *Every IFT of IFk-PHG contains a MIFT.*

Theorem 4.3. *If $\mathscr{T}' \in \mathscr{T}\mathscr{R}(\aleph)$ and for every $g \in \vee$, $\mathscr{T}'(g) \in F_k(\aleph)$, then $F_k(\mathscr{T}\mathscr{R}(\aleph)) \subseteq F_k(\aleph)$.*

Theorem 4.4. *$\mathscr{T}\mathscr{R}(\aleph)$ is sectionally elementary.*

Proof. Let $F_k(\mathscr{T}\mathscr{R}(\mathscr{H})) = a_1, a_2, \cdots, a_k;\ b_1, b_2, \cdots, b_k$. Assume that $\mathscr{T}' \in \mathscr{T}\mathscr{R}(\mathscr{H})$ and some $\delta, \varepsilon \in (a_i, b_i)$ such that $\mathscr{T}^{(a_i,b_i)} \subset \mathscr{T}^{(\delta,\varepsilon)}$. Since $\mathscr{T}\mathscr{R}(\aleph^{a_i,b_i}) = \mathscr{T}\mathscr{R}(\aleph^{\delta,\varepsilon})$, $\exists$ some $\mathscr{A} \in \mathscr{T}\mathscr{R}(\mathscr{H}) \ni \mathscr{A}^{a_i,b_i} = \mathscr{T}^{\delta,\varepsilon}$. Then $\mathscr{T}^{\delta,\varepsilon} \subset \mathscr{A}^{a_i,b_i}$ implies the IFS $\vee(g_i)$ defined by

$$\vee(g_i) = \begin{cases} (\delta, \varepsilon) & if\ \ x \in \mathscr{A}^{a_i,b_i} \setminus \mathscr{T}^{a_i,b_i} \\ \mathscr{A}(g_i) & Otherwise \end{cases}$$

is an IFT of IFk-PHG. Here $\vee < \mathscr{A}$, implies the contradiction of minimality (CM) of $\mathscr{A}$.

Theorem 4.5. *For every $\mathscr{A} \in \mathscr{T}\mathscr{R}(\aleph)$, $\mathscr{A}^{a_1,b_1}$ is a minimal IFT of $\aleph^{a_1,b_1}$.*

Proof. For any IFk-PHG $\aleph = (\vee, \Xi, \psi)$, consider a minimal IFT $\mathscr{T}$ of $\aleph^{a_1,b_1}$ such that $\mathscr{T} \subset \mathscr{A}^{a_1,b_1}$.
Define the IFS $\vee(g_i)$ where

$$\vee(g_i) = \begin{cases} (a_2, b_2) & if\ \ x \in \mathscr{A}^{a_1,b_1} \setminus \mathscr{T} \\ \mathscr{A}(g_i) & Otherwise \end{cases}$$

By the above theorem, $\vee$ is an IFT of IFk-PHG, CM of $\mathscr{A}$.

Definition 4.4. *Let $\aleph$ be IFk-PHG. The Intuitionistic Fuzzy Transversal Core $(IFTC)$ of $\aleph$ is $\aleph' = (\vee', \Xi', \psi')$ with the following condition that*
(i) min $\mathscr{T}\mathscr{R}(\aleph)$ = min $\mathscr{T}\mathscr{R}(\aleph')$,
(ii) $\aleph'$ = $\cup$ min $\mathscr{T}\mathscr{R}(\aleph)$,
(iii) $\psi \setminus \psi'$ is exactly the set containing vertices of $\aleph$ which does not belong to $\mathscr{T}\mathscr{R}(\aleph)$,where ψ' is the remaining hyperedge set, after deleting hyperedges that are correctly contained in another hyperedge.
The remarks of the statement is,
(i) For any IFk-PHG without spike hyperedges, $\exists$ transversal core which are always unique.
(ii) The definition also holds good for IFk-PHGs with spike (a hyperedge with single vertex) hyperedges.

Definition 4.5. *In IFk-PHG, the ONV g_i is the set containing adjacent vertices of g_i except itself in a k-partite hyperedge and is denoted as $N_k(g_i)$.*

Example 4.2. *Consider an IFk-PHG with $\vee = \{g_1, g_2, g_3, g_4, g_5, g_6, g_7\}$,*
$\Xi = \{\Xi_1, \Xi_2, \Xi_3\}$ where,
$\Xi_1 = \{g_1 \langle 0.5, 0.2\rangle, g_2 \langle 0.3, 0.4\rangle, g_3 \langle 0.6, 0.3\rangle\}$,
$\Xi_2 = \{g_2 \langle 0.3, 0.4\rangle, g_4 \langle 0.2, 0.5\rangle, g_5 \langle 0.3, 0.4\rangle\}$,
$\Xi_3 = \{g_3 \langle 0.6, 0.3\rangle, g_6 \langle 0.4, 0.3\rangle, g_7 \langle 0.1, 0.7\rangle\}$ with
$\psi_1 = \{g_1 \langle 0.5, 0.2\rangle, g_4 \langle 0.2, 0.5\rangle, g_7 \langle 0.1, 0.7\rangle\}$, $\psi_2 = \{g_2 \langle 0.3, 0.4\rangle, g_6 \langle 0.4, 0.3\rangle\}$,
$\psi_3 = \{g_3 \langle 0.6, 0.3\rangle, g_5 \langle 0.3, 0.4\rangle\}$
Here g_1 and g_7 are the ONV g_4 in ψ_1.

Definition 4.6. *In IFk-PHG, the CNV g_i is the set containing adjacent vertices of g_i including the vertex in a k-partite hyperedge and is denoted as $N_k[g_i]$.*

Example 4.3. *From the above example it is clear that the Closed Neighborhood of the vertex g_3 is g_3 and g_5 in ψ_3.*

Theorem 4.6. *In $\aleph$, the following claims are related*
(i) $\mathscr{T}$ is an IFT of IFk-PHG,
(ii) $\mathscr{T}^{a_i,b_i} \cap \mathscr{A}^{a_i,b_i} \neq \emptyset$, for all IFH $\mathscr{A} \in \psi$ and every (a_i, b_i) with $0 < a_i \leq h_\omega(\aleph)$, $0 < b_i \leq h_\nu(\aleph)$,
(iii) $\mathscr{T}^{a_i,b_i}$ is an IFT of $\aleph^{a_i,b_i}$, for each (a_i, b_i) with $0 < a_i \leq \delta$, $0 < b_i \leq \varepsilon$.

Proof. From the definition, "A minimal IFT $\mathscr{T}$ for IFk-PHG is a transversal of $\aleph$, which satisfies the property that if $\mathscr{T}_1 \subset \mathscr{T}$, then $\mathscr{T}_1$ is not an IFT of $\aleph$" the result is immediate.

Theorem 4.7. *For a simple IFk-PHG, $\mathscr{T}\mathscr{R}(\mathscr{T}\mathscr{R}(\aleph)) = \aleph$.*

Theorem 4.8. *For any IFk-PHG, $\mathscr{T}\mathscr{R}(\mathscr{T}\mathscr{R}(\aleph)) \subseteq \aleph$.*

Proof. From definition 4.4, $\exists$ a $\aleph'$(partial hypergraph) of a simple IFk-PHG $\ni \mathscr{T}\mathscr{R}(\aleph') = \mathscr{T}\mathscr{R}(\aleph)$. From Theorem 4.7, $\mathscr{T}\mathscr{R}(\mathscr{T}\mathscr{R}(\aleph)) = \mathscr{T}\mathscr{R}(\mathscr{T}\mathscr{R}(\aleph'))$ implies $\aleph' \subseteq \aleph$.

Theorem 4.9. *Let $\aleph$ be an IFk-PHG and suppose that $\mathscr{T} \in \mathscr{T}\mathscr{R}(\aleph)$. If $\aleph' \subseteq supp(\mathscr{T}) \subseteq \aleph$, then $\exists$ a hyperedge of IFk-PHG $\mathscr{A}$, $(a_i, b_i) \in \mathscr{A}$ represents the MNMV of $\mathscr{A} \ni$*
(i) $(a_i, b_i) = h(\mathscr{A}) = h(\mathscr{T}^{a_i,b_i}) > 0$,
(ii) $\mathscr{T}_{h(\mathscr{A})} \cap \mathscr{A}_{h(\mathscr{A})} = \aleph$.

Proof. Let $0 < h(\mathscr{T}^{a_i,b_i}) \leq 1$ and ψ' be the set of all IF k-partite hyperedges where $h(\tau^{a_i,b_i}) \geq h(\mathscr{T}^{a_i,b_i})$ for each $\tau \in \psi'$.

Since $\mathscr{T}^{a_i,b_i}$ is an IFT of $\aleph^{a_i,b_i}$ and $\aleph^{'} \subseteq \mathscr{T}^{a_i,b_i}$ is nonempty. Further, for each $\tau \in \psi^{'}$, $h(\tau) \geq h(\tau^{a_i,b_i}) \geq h(\mathscr{T}^{a_i,b_i})$ is true. Also, assume that $\mathscr{T}^{a_i,b_i}$ is the MIFT, then for all $\tau \in \psi^{'}$, $h(\tau) > h(\mathscr{T}^{a_i,b_i})$ and $\exists\, \aleph_\tau \neq \aleph$ with $\aleph_\tau \in \tau_{h(\tau)} \cap \mathscr{T}_{h(\tau)}$. Define an IF$k$-PHG $\aleph_1 \ni$

$$\aleph_1(U) = \begin{cases} \mathscr{T}(U)\ whenever\ U \neq \aleph^{'}, \\ min\ (h(\mathscr{A})/h(\mathscr{A}) < h(\mathscr{T}^{a_i,b_i})), max\ (h(\mathscr{A})/h(\mathscr{A}) < h(\mathscr{T}^{a_i,b_i})) \\ \qquad whenever\ U = \aleph^{'} \end{cases}$$

Hence $\aleph_1$ is an IFT of IFk-PHG and $h(\aleph_1^{a_i,b_i}) < h(\mathscr{T}^{a_i,b_i})$, It does not meet the basic requirement of $\mathscr{T}$. For each $\tau \in \psi^{'}$ satisfies the first part of the theorem 4.9 and has $\aleph_\tau$ which is not in $\aleph$ with $\aleph_\tau \in \tau_{h(\tau)} \cap \mathscr{T}_{h(\tau)}$. The procedure is repeated, and the argument of (i) provides a contradiction and bringing close to the proof.

Theorem 4.10. *Let $\aleph$ be an IFk-PHG. Then, $\exists\, \mathscr{T} \in \mathscr{T}\mathscr{R}(\aleph)$ with $\aleph^{'} \subseteq supp(\mathscr{T}) \subseteq \aleph$, if and only if for $\mathscr{A} \in \psi$ it meets the following requirements:*
(i) $(a_i, b_i) = h(\mathscr{A})$,
(ii) The level cut (a_j, b_j) of $h(\mathscr{A}^{'})$ is not a subhypergraph of the level cut (a_i, b_i) of $h(\mathscr{A})$, for all $\mathscr{A}^{'} \in \psi$ with $h(\mathscr{A}^{'}) > h(\mathscr{A})$,
(iii) The level cut (a_i, b_i) of $h(\mathscr{A})$ does not contain any other hyperedge of $\aleph_{h(\mathscr{A})}$, where (a_i, b_i) denotes MNMV of $\mathscr{A}$.

Proof. Necessary Part:
(i) Let $\mathscr{T} \in \mathscr{T}\mathscr{R}(\aleph)$ and $0 < h(\mathscr{T}^{a_i,b_i}) \leq 1$. Condition (i) is followed from Theorem 4.9.
(ii) Suppose that for each $\mathscr{A}$ satisfying (i) $\exists\, \mathscr{A}^{'} \in \psi \ni h(\mathscr{A}^{'}) > h(\mathscr{A})$ and $\mathscr{A}^{'}{}_{h(\mathscr{A}^{'})} \subseteq \mathscr{A}_{h(\mathscr{A})}$, then $\exists\, U \neq \aleph^{'}$, with $U \in \mathscr{A}^{'}{}_{h(\mathscr{A}^{'})} \cap \mathscr{T}_{h(\mathscr{A}^{'})} \subseteq \mathscr{A}_{h(\mathscr{A})} \cap \mathscr{T}_{h(\mathscr{A})}$ which differs from the concept of Theorem 4.9.
(iii) Assume for each $\mathscr{A}$ satisfying (i) and (ii) then $\exists\, \mathscr{A}^{'} \in \psi$ so that $\emptyset \neq \mathscr{A}^{'}{}_{h(\mathscr{A})} \subset \mathscr{A}_{h(\mathscr{A})}$. Since $\mathscr{A}^{'}{}_{h(\mathscr{A})} \neq \emptyset$ and by (ii), we have $h(\mathscr{A}^{'}) = h(\mathscr{A}) = (a_i, b_i)$.
If $(a_j, b_j) = h(\mathscr{A}')$ and $\mathscr{A}'' \in \psi$ such that $\emptyset \neq \mathscr{A}''_{h(\mathscr{A})} \subset \mathscr{A}'_{h(\mathscr{A})} \subset \mathscr{A}_{h(\mathscr{A})}$. The process is continued and the chain ends finitely, without loss of abstraction assume $(a_i, b_i) < h(\mathscr{A})$. But, $\exists\, U \neq \aleph^{'} \ni U \in \mathscr{A}^{'}{}_{h(\mathscr{A})} \cap \mathscr{T}_{h(\mathscr{A})} \subseteq \mathscr{A}_{h(\mathscr{A})} \cap \mathscr{T}_{h(\mathscr{A})}$, which contradicts Theorem 4.9.
Sufficient Part:
Let $\mathscr{A} \in \psi$ satisfy the condition (i), (ii) and (iii). By condition (i), the process is trivial. By condition (ii) and (iii) $\exists\, U \in \mathscr{A}^{'}{}_{h(\mathscr{A}^{'})} \setminus \mathscr{A}_{h(\mathscr{A})}$ for every $\mathscr{A}^{'} \in \psi \ni \mathscr{A}^{'} \neq \mathscr{A}$ and $h(\mathscr{A}^{'}) \geq h(\mathscr{A})$. Let $\vee_{\mathscr{A}}$ be the set of all vertices of $\aleph \ni \vee_{\mathscr{A}} \cap \mathscr{A}_{h(\mathscr{A})} = \emptyset$.

The initial sequence of transversals are constructed. So $\tau_s \subseteq \vee$ for all s,

$1 \leq s < i$ and $\tau_i \subseteq \vee_{\mathscr{A}} \cup \vee_i$. Hence, $\vee_i \in \tau_i$ for each i. The process is terminated till it reaches a minimal IFT with $(a_i, b_i) = h(\mathscr{A}) = h(\mathscr{T}^{a_i,b_i})$.

Theorem 4.11. *Let $\aleph$ be an IFk-PHG with $F_k(\aleph) = \{a_1, a_2, \cdots, a_k; b_1, b_2, \cdots, b_k\}$ so that $0 \leq a_i \leq h_\omega(\aleph)$, $0 \leq b_i \leq h_\nu(\aleph)$. Also, $\aleph^{a_i,b_i} \subseteq \mathscr{A}'$, be the elementary IF$k$-PHG if and only if $h(\mathscr{A}') = (a_i, b_i)$ and $supp(\mathscr{A}')$ is a hyperedge of $\aleph^{a_i,b_i}$. Then $\mathscr{TR}(\mathscr{TR}(\aleph))$ is the partial IFk-PHG of $\aleph^{a_i,b_i}$.*

Proof. From Theorem 4.5 and by the construction of minimal IFT, the (a_i, b_i)-level hypergraph of $\mathscr{TR}(\aleph)$ is $\mathscr{TR}(\aleph^{a_i,b_i})$ which means that $(\mathscr{TR}(\aleph))^{a_i,b_i} = \mathscr{TR}(\aleph^{a_i,b_i})$. Let τ belongs to $\mathscr{TR}(\mathscr{TR}(\aleph))$. From Theorem 4.9, $h(\tau(\vee_i)) > 0$, this implies that $\exists\ \mathscr{T} \in \mathscr{TR}(\aleph)$ with $h(\tau(\vee_i)) = h(\mathscr{T})$. From Theorem 4.1, $h(\mathscr{T}) = (max(min(\omega_{k_{ij}})), max(max(\nu_{k_{ij}}))) = h(\aleph)$ for all minimal IFT $\mathscr{T}$. Hence τ is elementary with $h(a_i, b_i)$. Since $supp(\tau) = \tau^{a_i,b_i}$, Theorem 4.5 suggest that $supp(\tau)$ is the minimal IFT of $(\mathscr{TR}(\aleph))^{a_i,b_i}$. It is obvious that $supp(\tau)$ is a hyperedge of $\aleph^{a_i,b_i}$. Hence τ is a hyperedge of $\aleph^{a_i,b_i}$.

Theorem 4.12. *Let $\aleph$ be an IFk-PHG with $\aleph^{a_i,b_i}$ is a simple. Then $\mathscr{TR}(\mathscr{TR}(\mathscr{H})) = \aleph^{a_i,b_i}$.*

Proof. By the above theorem, $\mathscr{TR}(\mathscr{TR}(\aleph)) \subseteq \aleph^{a_i,b_i}$. Let τ be an elementary with $h(\mathscr{T}) = (a_i, b_i)$ and $supp(\tau) \in \aleph^{a_i,b_i}$. By Theorem 4.11, $supp(\tau)$ is a minimal IFT of $(\mathscr{TR}(\aleph))^{a_i,b_i}$. Since each minimal IFT of $\mathscr{TR}(\aleph)$ is elementary by definition of minimal IFT the process ends at (a_i, b_i)-level and $\tau \in \mathscr{TR}(\mathscr{TR}(\aleph))$.
Hence $\aleph^{a_i,b_i} \subseteq \mathscr{TR}(\mathscr{TR}(\aleph))$ which implies $\aleph^{a_i,b_i} = \mathscr{TR}(\mathscr{TR}(\aleph))$.

5 Conclusion

In this article, some interesting concepts like IFT, minimal IFT, locally minimal IFT and IFTC of IFk-PHGs were discussed. It is important to note that IFTC exists for both spike and non-spike intuitionistic fuzzy k-partite hyperedges. In future, the authors planned to work on Robotics with multi-task concept as an application part of IFk-PHG.

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