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Anti-homomorphism in Q-fuzzy subgroups and normal subgroups

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Abstract

The fuzzy set has been applied in wide area by many researchers. We define the concept of anti-homomorphism in Q-fuzzy subgroups and Q-fuzzy normal subgroups and establish some result in this research article and develop some theory of anti- homomorphism in Q-fuzzy subgroups, normal subgroups and also extend results on Q-fuzzy abelian subgroup and Q- fuzzy normal subgroup. Many researchers have explored the fuzzy set extensively. We propose the notion of anti-homomorphism in Q is fuzzy subgroups and normal subgroups. It is establish some findings in this study article and build the theory of anti-homomorphism in Q-fuzzy subgroups, normal subgroups. It is also extend results on Q-fuzzy abelian subgroup and Q-fuzzy normal subgroup.

Keywords: Fuzzy, subgroup, Q-fuzzy, fuzzy abelian, fuzzy normal subgroup, anti-homomorphism,.
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1 Introduction

Zadeh L.A. Zadeh [1965] introduced the fuzzy set concept. Numerous scholars have used the fuzzy set in several different contexts. Fuzzy subgroups are first discussed by Rosenfeld Rosenfeld [1971]. Biswas.R Biswas [1990] was introduced, the anti-fuzzy subgroups. The novel structure of Q-fuzzy subgroups was introduced by Solairaju.A and Nagarajan.R Solairaju and Nagarajan [2009]. Fuzzy subgroups and fuzzy homomorphisms were defined by Choudhury, F.P., Chakraborty, A. B, and Khare Choudhury et al. [1988] Sheik Anti-homomorphism in fuzzy subgroups was defined by Abdullah A. and Jeyaraman K. Sheik Abdullah and Jeyaraman [2010]. In this study, we demonstrate various results and define the notion of anti-homomorphism in Q-fuzzy subgroups and fuzzy normal subgroups.

2 Preliminaries

Definition 2.1. *Zadeh [1965] A function of fuzzy subset* $\delta \neq S$ *is* $\delta : S \to [0, 1]$.

Definition 2.2. *Rosenfeld [1971] A fuzzy subset* δ *of a group* $J = J$ *(fuzzy subgroup) if it is satisfying the following conditions,*

(i) $\delta\,(\varrho) \geq \min\{\delta\,(\varrho)\,,\ \delta\,(\gamma)\}$

(ii) $\delta\,(\varrho^{-1}) =\ \delta(\varrho)$, $\forall\ \varrho,\ \gamma\ \in J$.

Definition 2.3. *Solairaju and Nagarajan [2009] A Q-fuzzy set* $\delta = J$ *if* $\forall\ \varrho$, $gammaf\ \in J$, *and* $\kappa\ \in Q$

(i) $\delta\,(\varrho\gamma,\ \kappa) \geq \min\{\delta\,(\varrho,\ \kappa)\,,\ \delta\,(\gamma,\ \kappa)\}$

(ii) $\delta\,(\varrho^{-1},\ \kappa) =\ \delta\,(\varrho,\ \kappa)$

Definition 2.4. *Zadeh [1965]* $\nu \subseteq S$ *(fuzzy subset of a set) . For* $\beta \in [0, 1]$, *the level subset of* δ *is defined by*

$$\delta_\beta = \{e \in S :\ \delta_\nu(\varrho) \geq \beta$$

Definition 2.5. *Solairaju and Nagarajan [2009]* $\nu \subseteq S$. *For* $\beta \in [0, 1]$, *the set* $\delta_\beta = \{e \in S,\ \kappa \in Q :\ \delta_\nu(\varrho,\ \kappa) \geq \beta\}$ *is called a* $Q \subseteq \delta$.

Definition 2.6. *Palaniappan and Muthuraj [2004] Consider* $\delta < J$. *The fuzzy subgroup* δ *is said to be fuzzy normal subgroup if* $\delta\,(\varrho\gamma) = \delta\,(fe)$, $\forall\ \varrho,\ \gamma\ \in J$.

Definition 2.7. *Palaniappan and Muthuraj [2004] A fuzzy subgroup* δ *of a group* J *is a Q-fuzzy normal subgroup if* $\delta\,(\varrho\gamma,\ \kappa) = \delta\,(\gamma\varrho,\ \kappa)$, $\forall\ varrho\ \gamma\ \in J$, *and* $\kappa\ \in Q$.

Definition 2.8. *Choudhury et al. [1988] Let* $(J_1,\ \bullet)$ *and* $J_2,\ \bullet)$ *be the function* $g : J_1 \to J_2$ *is called a group homomorphism if* $g\,(\varrho\gamma) = g\,(\varrho)\,.g(\gamma)$*,* $\forall\ \varrho,\ \gamma\ \in J_1$.

Definition 2.9. *Sheik Abdullah and Jeyaraman [2010] Let* $(J_1,\ \bullet)$ *and* $J_2,\ \bullet)$ *be the function* $g : J_1 \to J_2$ *is called a group anti homomorphism if* $g\,(\varrho\gamma) = g\,(\gamma)\,.g\,(\varrho)$*,* $\forall\ \varrho,\ \gamma\ \in J_1$.

Definition 2.10. *Sheik Abdullah and Jeyaraman [2010] Let* $g : J_1 \to J_2$ *is called anti automorphism if* $g\,(\varrho\gamma) = g\,(f)\,.g(\varrho)\ \forall\ \varrho,\ \gamma\ \in J_1$.

Definition 2.11. *Sheik Abdullah and Jeyaraman [2010] The function* δ *is a fuzzy characteristic subgroup of a group* J *if* $\delta\,(h\,(\varrho)) = \delta(\varrho)$.

3 Some results On Q -fuzzy subgroups in anti- homomorphism

Theorem 3.1. *Let* $g : J \to J^*$ *be an anti-homomorphism, if* δ^* *is a Q<*J^**. Then* $g^{-1}(\delta^*)$ *is a Q < J.*

Proof. Let $e,\ \gamma\ \in J$. Then

$$\begin{aligned} g^{-1}\,(\delta^*)\,(\varrho\gamma,\ \kappa) &= \delta^*(h\,(\varrho\gamma,\ \kappa)) \\ &= \delta^*\{g\,(\gamma,\ \kappa)\,.g\,(\varrho,\ \kappa)\} \\ &\geq \min\{\delta^*\,(h\,(\gamma,\ \kappa))\,,\ \delta^*(h\,(\varrho,\ \kappa))\} \\ &= \min\{(h^{-1}\delta^*)(\gamma,\ \kappa),\ (h^{-1}\delta^*)(\varrho,\ \kappa)\} \end{aligned} \tag{1}$$

and

$$\begin{aligned} g^{-1}\,(\delta^*)\left(\varrho^{-1},\ \kappa\right) &= \delta^*(h\left(\varrho^{-1},\ \kappa\right)) \\ &= \delta^*(h\,(\varrho,\ \kappa)) \\ &= g^{-1}\,(\delta^*)\,(\varrho,\ \kappa) \end{aligned} \tag{2}$$

From (1) and (2), $g^{-1}(\delta^*)$ is a Q < J. □

Theorem 3.2. *If* δ *is a Q <*J *and* $g : S \to S^*$ *is an anti-homomorphism, then* $g^{-1}(\delta)$ *is a Q-fuzzy normal subgroup of* S^**.*

Proof. For every $\varrho,\ \gamma\ \in S$. We get,

$$\begin{aligned} g^{-1}\,(\delta)\,(\varrho,\ \kappa) &= \delta(h(\varrho,\ \kappa) \\ &= \delta\{g\,(\gamma,\ \kappa)\,,\ g\,(\varrho,\ \kappa)\} \\ &= \delta(h(\gamma\varrho,\ \kappa) \\ &= g^{-1}\,(\delta)\,(\gamma\varrho,\ \kappa) \end{aligned}$$

Hence $g^{-1}(\delta)$ is a Q $<J$. □

Theorem 3.3. *A fuzzy characteristic subgroup of a $Q < Q$. It is a fuzzy normal subgroup.*

Proof. Given g is an anti automorphism of S. For all $\varrho,\ \gamma\ \in S$, and $\kappa\ \in Q$. Then

$$g\left(\varrho\gamma\right) = g\left(\gamma\right).g(\varrho),\ \text{for every}\ e,\ \gamma\ \in S$$

Now,

$$\begin{aligned}\delta\left(\varrho\gamma,\ \kappa\right) &= \delta(h(\varrho\gamma,\ \kappa) \\ &= \delta\{g\left(f,\ \kappa\right), g\left(\varrho,\ \kappa\right)\}\end{aligned}$$

Since δ is a characteristics Q $<S$.
Then $\delta\left(\varrho\gamma,\ \kappa\right) = \delta\{g\left(\gamma,\ \kappa\right), g\left(\varrho,\ \kappa\right)\}$
Since g is anti automorphism of S,

$$\begin{aligned}\delta\left(\varrho\gamma,\ \kappa\right) &= \delta(h(fe,\ \kappa)) \\ &= \delta(\gamma\varrho,\ \kappa),\ for\ all\ \varrho,\ \gamma\ \in\ S,\ and\,\kappa\ \in\ Q\end{aligned}$$

δ is a characteristics Q $<S$.
Hence δ is a Q $<S$. □

Definition 3.1. *The Q-fuzzy subgroup δ of a group S is called a Q -fuzzy abelian subgroup of S if $H = \{\varrho \in S : \delta\left(\varrho,\ \kappa\right) = \delta\left(i,\ \kappa\right)\}$, $\forall\ \varrho,\ \gamma\ \in S$, and $\kappa\ \in Q$.*

Theorem 3.4. *The commutative property satisfies all anti-homomorphism pre-images of a Q-fuzzy commutative subgroup.*

Proof. If δ is a Q $<S$.
To prove δ is a Q=[S, S]
Let us consider ν is a Q = [S^*, S^*] (Q fuzzy commutative subgroup of S^*).
Since δ is a Q = [S^*, S^*] and ν is Q = [S^*, S^*]
Then $V = \{\gamma \in S^*,\ \kappa\ \in Q:\ \nu\left(\gamma,\ \kappa\right) = \nu\left(i^*, \kappa\right)\}$ is a Q = [S^*, S^*], where i^* is the identify element of S^*.
Let $T = \{\varrho\ \in\ S,\ \kappa\ \in Q : \delta\left(\varrho,\ \kappa\right) = \delta\left(i,\ \kappa\right)\}$ where i is the identify element of S.
Take $\varrho,\ \gamma\ \in\ T$ this implies $\varrho\gamma\ \in T \subseteq S$. Then

$$\begin{aligned}\delta\left(\varrho\gamma,\ \kappa\right) &= \delta(i,\ \kappa) \\ \nu\left(h\left(\varrho\gamma,\ \kappa\right)\right) &= \nu(h\left(i, \kappa\right)) \\ &= \nu\left(i^*, \kappa\right) \\ \nu\left\{g\left(\gamma,\ \kappa\right).g\left(\varrho,\ \kappa\right)\right\} &= \nu\left(i^*, \kappa\right)\end{aligned}$$

Since $g\,(\gamma,\ \kappa)\,.g\,(\varrho,\ \kappa) \in V$ and V is abelian,

$$\begin{aligned}
g\,(f,\ \kappa)\,.g\,(\varrho,\ \kappa) &= g\,(\varrho,\ \kappa)\,.g\,(\gamma,\ \kappa)\\
\nu(h\,(\gamma,\ \kappa)\,.g\,(\varrho,\ \kappa)) &= \nu(h\,(\varrho,\ \kappa)\,.g\,(\gamma,\ \kappa))\\
\nu\,(\gamma\,(\gamma\varrho,\ \kappa)) &= \nu(\gamma\,(\varrho\gamma,\ \kappa)),\ since\ g\ is\ anti-homomorphism.\\
\delta\,(\varrho\gamma,\ \kappa) &= \delta\,(\gamma\varrho,\ \kappa)\\
\delta\,(i,\ \kappa) &= \delta\,(\gamma\varrho,\ \kappa)
\end{aligned}$$

i.e $(\gamma\varrho,\ \kappa) = \delta\,(i,\ \kappa)$, this implies $fe\ \in T$ and $\kappa\ \in Q$.
For all $\varrho,\ \gamma\ \in T$, $\varrho\gamma\ \in T$ and $\gamma\varrho \in T$.
This implies $\varrho\gamma = \gamma\varrho$
T satisfies commutative. Therefore δ satisfies commutative property. □

Theorem 3.5. *Anti-homomorphism image of a Q -fuzzy commutative subgroup is also satisfies commutative level.*

Proof. Let ν be a Q -fuzzy subgroup of S^*.
To prove: ν is a Q -fuzzy commutative subgroup of S^*.
Let g be an anti-homomorphism from S to S^*.
Since δ is a Q=[S, S].
Then $T = \{\varrho\ inS,\ \kappa\ \in Q : \delta\,(\varrho,\ \kappa) = \delta\,(i,\ \kappa)\}$ is an commutative Q -fuzzy subgroup of S^* where i is the identity element of S.
Let ν be the Q - fuzzy subgroup of S^*.
Let $V = \{x \in S^*,\ \kappa\ \in Q : \nu\,(\varrho,\ \kappa) = \delta\,(i^*,\ \kappa)\}$ where $i^*$is the identity element S^*.
Let $e,\ \gamma\ in\ V\ \subseteq S^*$

$$\begin{aligned}
\nu\,(\varrho\gamma,\ \kappa) &= \nu(i^*,\ \kappa)\\
Sup\,\delta\,(r,\ \kappa) &= Sup\,\delta\,(r,\ \kappa)\\
r \in g^{-1}(\varrho\gamma) &\quad,\quad r \in g^{-1}(i^*)\\
\delta\,(\varrho\gamma,\ \kappa) &= \delta(i,\ \kappa)
\end{aligned}$$

Then $\varrho\gamma\ \in T$ and T is an commutative Q - fuzzy subgroup.

$$\begin{aligned}
(\varrho\gamma, \kappa) &= (\gamma\varrho,\ \kappa)\\
\delta\,(\varrho\gamma, \kappa) &= \delta(\gamma\varrho,\ \kappa)\\
Sup\,\delta\,(r,\ \kappa) &= Sup\,\delta\,(r,\ \kappa)\\
r \in g^{-1}(\varrho\gamma) &\quad,\quad r \in g^{-1}(\gamma\varrho)\\
\nu\,(\varrho\gamma, \kappa) &= \nu(\gamma\varrho,\ \kappa)\\
\nu\,(i^*, \kappa) &= \nu(\gamma\varrho,\ \kappa)
\end{aligned}$$

That is $\nu\,(\varrho\gamma, \kappa) = \nu\,(i^{*}, \kappa)$, this implies $\gamma\varrho\ \in V$ and $\kappa\ \in Q$.
For all $\varrho,\ \gamma\ \ \in V$, $\varrho\gamma\ \in V$
This implies $\gamma\varrho\ \in V$ and $\varrho\gamma = \gamma\varrho$.
Then $V = [S^{*}$, $S^{*}]$ (commutative subgroup of S^{*}).
Therefore $\nu = [S^{*}$, $S^{*}]$. Where Q fuzzy commutative subgroup of S^{*}. $\square$

4 Conclusions

Many results can be found from the research article. But, in this paper we found few concepts of anti-homomorphism in Q-fuzzy subgroups. Further this paper used to developing the concept of Q-fuzzy abelian subgroup. There are so many concepts can be availed by future research work.

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