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Performance evaluation in Public Administration: P-AHP and PROMETHEE a comparative analysis*

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Abstract

The performance evaluation is a crucial phase for each organization, and nowadays this is particularly relevant also for Public Administrations (PAs), where a high level of efficiency and effectiveness has a meaningful impact on citizens and companies. In this work we propose the adoption of Multi-Criteria Decision Making approaches, in particular PROMETHEE I-II and Parsimonious AHP, for the performance evaluation of PA's services. The novelty of our approach is mainly related to the application of these methods to the performance evaluation process carried out by PAs, showing the features that can allow their adoption in a real-life context. The methods have been tested on a real PA in Southern Italy, and the results show their effectiveness and significance as control and planning support tools for decision-makers.

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1 Introduction

The Public Administration (PA), having the task of carrying out the functions of management, direction and coordination of the State, has an increasingly crucial role in the lives of citizens. In particular, PAs are in charge to make decisions in continuously evolving territorial contexts characterized by a high degree of complexity [Reina et al., 2008]. It is clear that their actions have consequences for an entire community. These considerations lead us to reflect on the importance of introduction of decision-making support tools in the public sector. More and more studies are analyzing the decision-making problems faced by PAs, proposing the use of decision support tools to reach more reasoned and effective choices. The areas of study of PA are different. Recent studies have dealt with the management of public tenders. In particular, for the analysis of the most economically advantageous offer, many authors suggest the use of the AHP, see e.g. Marcarelli and Squillante [2020], Diabagaté et al. [2015], Salvatore et al. [2021]. Recent studies instead propose innovative methods such as the ELimination Et Choix Traduisant la REalité (ELECTRE) III and Parsimonious AHP (P-AHP) methods [Fattoruso and Marcarelli, 2022], Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) and the Multi-Attribute Utility Theory (MAUT) [Dotoli et al., 2020].

Another topic of interest in the PA is public transport. The study of Kiciński and Solecka [2018] proposes the use of the AHP and ELECTRE III method which allows reasoned decisions to be made on the implementation of the transport system by governments, taking into account a plurality of analysis criteria such as the respect for the environment. Of particular interest is also the topic of efficient management for the use of public funds: the study of Vavrek et al. [2017] proposes the use of the Weighted Sum Approach (WSA) method and of the Technique for Order Preference by Similarity methods to Ideal Solution (TOPSIS) method, proposing an analysis of the benefits and drawbacks of using these methods. Among the issues addressed within the PA is the the one related to the implementation of policies to support renewable energy by governments. For this analysis, the study by García et al. [2024] proposes the use of AHP and TOPSIS methods, while Kang et al. [2024] propose a model that combines the Analytic Hierarchy Process (AHP) and ELECTRE III, using the Stepwise Weighted Assessment Ratio Analysis (SWARA) method.

Among the topics that we believe are of great interest in the PA, in addition to those already mentioned, there is the performance evaluation that has a crucial role in PA as it allows the efficiency and effectiveness of public

services offered to citizens to be assessed, identifying any areas for improvement. This task also helps to guarantee responsibility and transparency in the work of public employees as well as in the decisions taken by the PA. Furthermore, performance evaluation can promote the professional development of public employees, stimulating them to improve their skills and achieve pre-established objectives. In the literature, the authors who have dealt with performance evaluation in the PA field have used various multi-criteria tools. Kuo and Liang [2012] and Goyal et al. [2022] analyse the public transport performance evaluation with Multi-Criteria Decision Making (MCDM) based on interval-valued fuzzy numbers. Lai et al. [2008] analyzes the performance of research and development projects using the Fuzzy Multiple Criteria Decision Making (FMCDM) approach. The work of Wu et al. [2012] and Fattoruso et al. [2022] discuss the high level instruction performances using respectively the AHP and VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method and the AHPSort II.

This paper proposes the use of Multi-criteria Methods analysis tools, which provide an exhaustive and in-depth evaluation of the decision process, allowing decision-makers to make informed and rational decisions in a complex context such as that of performance evaluation within the PA organization. In particular, a comparison is proposed between two multi-criteria methods such as the PROMETHEE I-II [Brans and De Smet, 2016] method and the Parsimoniuos AHP [Abastante et al., 2019] method. In addition to the novelty of using these methods in the public sector, the paper proposes to verify the effectiveness of multi-criteria methods for evaluating the performance of public administration offices. We apply the methods in a real case study provided by the Metropolitan City of Reggio Calabria, a PA located in Southern Italy, which provided us with a structured framework of the performance evaluation problem by defining the decision-making criteria and evaluations of each office. A comparative analysis has been performed by using PROMETHEE I-II and P-AHP methods, in order to assess the most suitable method for evaluating performance in the PA, highlighting its gaps and strengths. The methodology is described in Section 2; Section 3 contains the description of the case study and the results obtained; Sections 4 and 5 respectively discuss the results and conclude the paper.

2 Methodology

2.1 PROMETHEE I-II

The PROMETHEE method Brans et al. [1986] belongs to the family of outranking methods. This section will illustrate the methods PROMETHEE I which focuses on the detailed evaluation of alternatives with respect to individual criteria, allowing Decision Makers (DMs) to understand the im-

plications of their choices on each criterion and PROMETHEE II which aggregates the evaluations of the criteria to generate a global ranking of the alternatives based on the preferences of the decision makers, allowing to obtain a synthetic and ordered vision of the alternatives, facilitating the decision-making process and allowing a direct comparison between them [Ishizaka and Nemery, 2011] [Brans and De Smet, 2016]. Given a criterion $g_j \in G$, and for each pair of ordered alternatives $(a, b) \in A$, the decision maker expresses her/his preference by means of a degree of preference obtained using the preference function $P_j(a, b)$ which is in function of $d_j(a, b) = g_j(a) - g_j(b)$; therefore, $P_j(a, b)$ indicates whether an alternative a is preferred (or not) over b on the criterion g_j based on the difference between their evaluation $d_j(a, b)$. The construction of $P_j(a, b)$ may require the use of indifference threshold q_j and preference threshold p_j, such that

$$d_j(a,b) < q_j \implies P_j(a,b) = 0$$

$$d_j(a,b) > p_j \implies P_j(a,b) = 1$$

To evaluate to what extent the action a is preferred to b, the aggregate preference index $\pi(a, b)$ is then constructed. Given the pair of alternatives a and b, the aggregate preference index can be defined as follows:

$$\pi(a,b) = \sum_{j=1}^{k} P_j(a,b) \cdot w_j \tag{1}$$

where w_j represents the weight of each criterion g_j and k the number of criteria. The π value fluctuates between 0 and 1: if $\pi \approx 0$ the overall preference of one alternative over another is weak; if $\pi \approx 1$ the preference is strong. The positive and negative outranking flows are then calculated, which express the outranking (or not) of an alternative compared to the others:

$$\phi^+(a) = \frac{1}{n-1} \sum_{x \in A} \pi(a,x) \tag{2}$$

$$\phi^-(a) = \frac{1}{n-1} \sum_{x \in A} \pi(x,a) \tag{3}$$

where n represent the number of alternatives and x the generic alternative. Based on positive (2) and negative (3) flows, the partial classification according to PROMETHEE I is show in Table 1.

Using PROMETHEE II, ϕ^+ and ϕ^- flows are generally combined to obtain net flows defining a complete ranking. The net flow is defined as follows:

$$\phi(a) = \phi^+(a) - \phi^-(a) \tag{4}$$

ϕ^+	ϕ^-	Classification
$\phi^+(a) \geq \phi^+(b)$	$\phi^-(a) < \phi^-(b)$	$a \succ b$
$\phi^+(a) > \phi^+(b)$	$\phi^-(a) \leq \phi^-(b)$	$a \succ b$
$\phi^+(a) = \phi^+(b)$	$\phi^-(a) = \phi^-(b)$	$a \sim b$
		a is incomparable to b in other cases

Table 1: Partial classification according to PROMETHEE I

The definition of ϕ allows, unlike flows ϕ^+ and ϕ^-, to compare all the alternatives with each other and to obtain a complete ranking of the alternatives. A deeper discussion of net flow scores can be found in Brans and Mareschal et al. [2008].

2.2 P-AHP

The P-AHP [Abastante et al., 2019] is an extension of the classic AHP method, which overcomes the limitations of classical AHP [Abastante et al., 2018], allowing you to analyze problems with a high number of alternatives by reducing the number of pairwise comparisons for the Decision Maker. Given a decision matrix M in which there are the evaluations $m_j(a_i)$ of each alternative a_i with respect to each criterion g_j, the DM (with the support of an analyst) identifies a set of reference points r_{jp_j}. In particular, for each criterion g_j the DM identifies a specific number of reference points $(r_{j1}, \ldots, r_{jp_j})$ within the range of alternatives a_i taking into account the numerousness of the alternatives [Abastante et al., 2019].

Pairwise Comparison Matrices (PCM) are then constructed to allow the derivation of the criteria weights (w_j) and reference points weights $\upsilon_{r_{jp_j}}$; the respective weights are defined with the [Saaty, 1980] eigenvector method. At this point the consistency of the matrices is verified by analyzing the consistency index:

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

where λ_{max} is the maximum eigenvalue associated with the matrix and n represents the numerousness of the matrix [Saaty, 2008] [Abastante et al., 2019].

If the matrices present inconsistency problems then the evaluations will be reviewed by the DM [Vincke, 1981]. We proceed to derive the local priorities through the use of linear interpolation:

$$\psi(m_j(a_i)) = \upsilon_{r_{jp_j}} + \frac{\upsilon_{r_{jp_j+1}} - \upsilon_{r_{jp_j}}}{r_{jp_j+1} - r_{jp_j}}(d_j(a_i) - r_{jp_j}) \tag{5}$$

and finally the global priorities are defined as follows:

$$\Psi_i = \sum_{j=1}^{n} \psi(m_j(a_i))w_j \tag{6}$$

The definition of the global priority Ψ_i allows obtaining the final ranking of the alternatives, allowing ranking from most to least preferred.

3 Analysis of a real case

3.1 Description of PA

The Metropolitan City of Reggio Calabria, a PA located in southern Italy, provided us with a structural framework of the organization and functions of its services, allowing us to conduct an in-depth and analytical study on the evaluation of the performance of the services offered. The data provided describes in detail the skills and responsibilities of each Service, including the Offices within them and their related tasks. This documentation will form the basis for an accurate evaluation of the activities carried out by the administration and for the analysis of the evaluation criteria that could be applied to evaluate the performance of individual offices. The organizational structure of the Sector which we will call *Sector X*, for reasons of confidentiality, is shown in Figure 1.

The *Sector X* is led by a chief manager and supported by the Secretariat Office. It is divided into four Services:

- *Service A* is headed by a manager and is divided into two offices.
- *Service B* is headed by a manager holding an Organizational Position and is divided into four offices.
- *Service C* is headed by a manager and is divided into two offices.
- *Service D* is headed by a manager holding an Organizational Position and is divided into three offices.

Each Service has in charge specific functions, crucial to the operation and development of the public administration and each office is headed by a manager. Below are the main responsibilities assigned to each service:

- the u_{a1} office provides support to the Sole Procedural Manager (RUP) during tenders, assisting in the various phases of the procedure and in the preparation of the related documents;
- the u_{a2} office manages the Executive Management Plan (PEG), financing and finance;

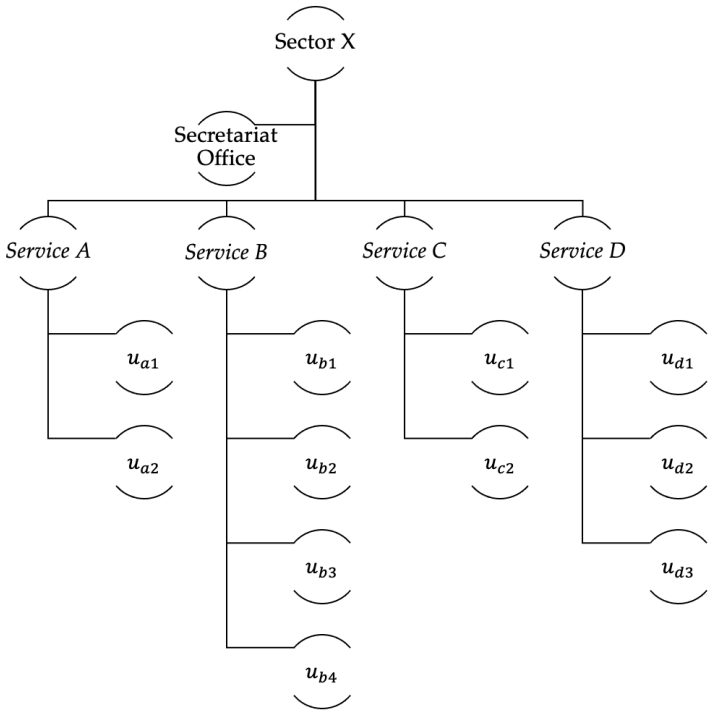

Figure 1: Organizational structure of the PA

- the u_{b1} office oversees the institutional, technical and administrative process of drafting and approving the Strategic Plan of the Metropolitan City (art. 44, paragraph 1, letter a, of Law 56/2014);
- the u_{b2} office oversees the institutional, technical and administrative process of approving variants of the Provincial Territorial Coordination Plan (PTCP) and the subsequent Territorial Plan of the Metropolitan City (PTCM) and its variants, as well as drawing up the annual list and the three-year program of public works (LL.PP.), and related updates, monitors the implementation status of the LL.PP. and evaluates the coherence of municipal planning instruments with the PTCP and subsequently with the PTCM; furthermore, it provides technical assistance to municipalities in the drafting of urban planning instruments and in verification activities.
- the u_{b3} office manages the issuing of landscape authorizations and verifies landscape compatibility in the areas subject to environmental restrictions, it also deals with the obligations relating to authorization procedures and the issuing of landscape opinions in compliance with the laws on building amnesty;
- the u_{b4} office coordinates and takes care of expropriation procedures,

carrying out administrative and, where necessary, technical functions referred to in the Consolidated Law. regarding expropriations for public utility (Presidential Decree no. 327/2001);

- the u_{c1} office manages urban reforestation projects and management plans for protected natural areas;
- the u_{c2} office deals with the drafting and management of Integrated Urban Plans and projects for the enhancement of the architectural and landscape heritage;
- the u_{d1} office deals with the preliminary investigation for the declaration of admissibility of the interventions, guaranteeing the regularity of the documentation and coordination between the Sole Procedural Managers (RUP). Furthermore, it instructs and controls the acts for programming and reprogramming of the Pact, preparing manuals and control tools to guarantee homogeneity in implementation;
- the u_{d2} office deals with controls, archiving of documentation and expense certification procedures;
- the u_{d3} office verifies the accounting regularity for the payment of the Beneficiaries' expenses, providing technical and accounting support to the ORs for the processing and transmission of data relating to the disbursement requests.

We would like to point out that the specific skills were assigned through an internal service order, which also defined the percentage of assignment of some employees to different offices. The DM who supported us in the construction of the methods and in the analysis of the PA's performance is a top figure within the PA. With the DM we decided that for a matter of privacy of the PA the services will be renamed $\{u_1, u_2, \ldots, u_{11}\}$ and will be ordered randomly so that the corresponding service cannot be traced. The DM defined a set of criteria $G = \{g_1, g_2, \ldots, g_j\}$ described below:

1. Location and physical accessibility (g_1): evaluates the physical location and accessibility of the Services and Offices;
2. Multi-channel accessibility (g_2): evaluates the availability of communication and assistance channels for citizens;
3. Timeliness (g_3): evaluates the readiness and efficiency in managing practices and procedures.

The DM has therefore provided us with the evaluation of the individual offices, expressed with a 10-point scale, widely validated in the literature [Dawes, 2008], [Loken et al., 1987] and [Kumar, 2018]. The evaluation are shown in the Table 2.

	g_1	g_2	g_3
u_1	4	6	7
u_2	4	6	8
u_3	6	7	5
u_4	5	6	6
u_5	7	7	8
u_6	5	8	7
u_7	6	8	8
u_8	5	7	6
u_9	6	7	7
u_{10}	5	8	8
u_{11}	5	8	7

Table 2: Evaluations of the offices with respect to each criterion g_j

3.2 Performance evaluation with PROMETHEE I-II

The method described in the Section 2.1 was applied. For both methods application, the PCM was constructed to define the weight of the criteria; the preferences of the decision makers were expressed through the use of Saaty's semantic scale [Saaty, 2001]. The criterion weights are obtained by the right eigenvector method [Saaty, 2003]. The Table 3 shows the PCM for criteria.

	g_1	g_2	g_3	w_j
g_1	1	$\frac{1}{6}$	$\frac{1}{8}$	0.066
g_2	6	1	2	0.551
g_3	8	$\frac{1}{2}$	1	0.382
				Inc. 0.10

Table 3: PCM of criteria

At this point we have calculated the positive, negative and net flows for each office considered. The results are shown in Table 4.

The Partial Rankings obtained by applying PROMETHEE I are shown in Figure 2, where two bars are depicted: one on the left which represents the ranking of the shares according to the positive preference flow ϕ^+, and one on the right which shows the ranking according to the negative preference flow ϕ^-. A line is drawn for each action between its ϕ^+ score on the left bar and its ϕ^- score on the right bar: when one line is above another it means that the action is preferred to the other, when two lines cross each other means that the actions are incomparable in PROMETHEE I.

The complete rankings obtained with PROMETHEE II are shown in Fig-

ϕ	ϕ^-	ϕ^+
0,6934	0,7000	0,0066
0,6405	0,6670	0,0264
0,3346	0,5140	0,1794
0,3346	0,5140	0,1794
0,2786	0,4992	0,2206
-0,0538	0,3264	0,3802
-0,2330	0,2677	0,5007
-0,3360	0,2169	0,5530
-0,3979	0,2117	0,6096
-0,5389	0,1147	0,6537
-0,7221	0,0515	0,7739

Table 4: Net, Positive and negative flows according to PROMETHEE I-II

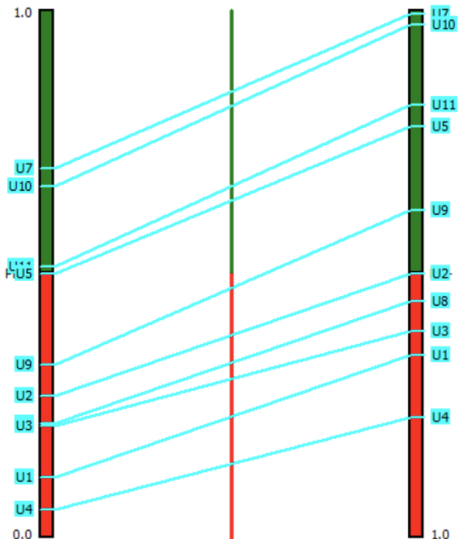

Figure 2: Partial Ranking with PROMETHEE I

ure 3, which shows the complete rankings by displaying the ϕ scores on a vertical bar: the upper half of the scale (in green) corresponds to the positive scores and the lower half (in red) to the negative scores. From the results obtained we can see that the best performing office is u_7, followed by the offices u_{10}, u_{11} and u_6 in equal merit and followed by u_5. Among the least performing offices we find u_4, u_1 and u_3 in the last three positions.

With the use of the VisualPROMETHEE[1] software we have also defined the efficient frontier which is a graphical representation of the best possi-

[1]https://visual-promethee.software.informer.com/1.4/

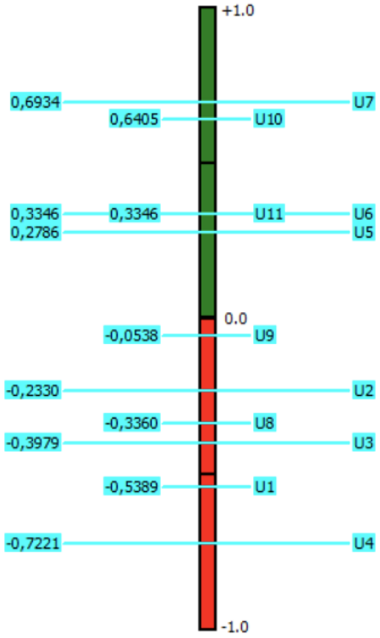

Figure 3: Complete Ranking with PROMETHEE II

ble performance in an Input/Output optimization problem. It represents the maximum boundary of performance that can be achieved, given a given set of actions and evaluation criteria, helping decision makers to identify the optimal options, or those that come closest to them, in their decision. In this context, Input/Output efficiency refers to the ability of an Office to obtain desirable results (output) compared to the resources used (input). We show in Figure 4 the efficient frontier drawn in red through the points representing the actions in the two-dimensional graph of Inputs and Outputs. The actions that are on the efficient frontier are considered efficient as they maximize the output for a given level of input, therefore representing optimal solutions or at least comparable to each other.

In our analysis only u_7 is located directly on the efficient frontier: this indicates that it is maximizing output for the level of input used, making it an efficient and highly competitive option compared to other offices. On the other hand, the other Offices are all far from the border, which suggests that they could improve their performance in terms of output compared to the resources used. The u_{10} is located very close to u_7 and just below the efficient frontier: this position indicates that it has relatively good performance.

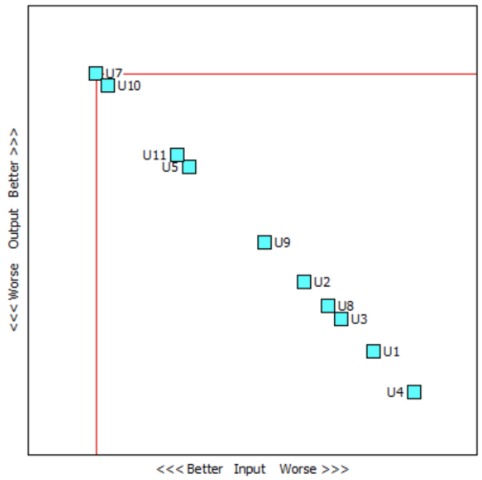

Figure 4: Efficient frontier

3.3 Performance evaluation with P-AHP

In order to be able to apply the P-AHP method, the reference points r_{jp_j} have been defined for each criterion g_j; in particular 6 were identified as the reference points which for greater ease of reading were renamed $\{RP_1, \ldots, RP_6\}$; the values are shown in Table 5.

	g_1	g_2	g_3
RP_1	1	1	1
RP_2	3	3	3
RP_3	4	4	4
RP_4	6	6	6
RP_5	7	7	7
RP_6	10	10	10

Table 5: Reference points r_{jp_j} for each criterion g_j

The PCMs were then constructed to obtain the weights of the criteria (Table 3) and reference points. All PCMs provided by the DM have an acceptable CI. The weights of the reference points are reported in Table 6. The local priorities $\psi(m_j(u_i))$ for each office respect each criterion g_j were then calculated using the formula (5) as well as the global priorities Ψ_i using the formula (6). The results are reported in the Table 7.

As we can see, the best performing offices are u_7, u_{10}, u_{11} and u_6 in equal merit and followed by u_5. Among the least performing offices we find u_4, u_1 and u_3 in the last three positions.

	$\upsilon_{r_{jp_j}}(g_1)$	$\upsilon_{r_{jp_j}}(g_2)$	$\upsilon_{r_{jp_j}}(g_3)$
RP_1	0.032	0.028	0.043
RP_2	0.056	0.053	0.064
RP_3	0.099	0.104	0.101
RP_4	0.170	0.163	0.160
RP_5	0.251	0.267	0.250
RP_6	0.391	0.385	0,382

Table 6: Weights of the reference points $\upsilon_{r_{jp_j}(g_j)}$

	$\psi(g_1)$	$\psi(g_2)$	$\psi(g_3)$	Ψ_i
u_1	0.099	0.163	0.250	0.192
u_2	0.099	0.163	0.294	0.209
u_3	0.170	0.267	0.131	0.208
u_4	0.135	0.163	0.160	0.160
u_5	0.089	0.267	0.294	0.265
u_6	0.135	0.306	0.250	0.273
u_7	0.170	0.306	0.294	0.292
u_8	0.135	0.267	0.160	0.217
u_9	0.170	0.267	0.250	0.254
u_{10}	0.135	0.306	0.294	0.290
u_{11}	0.135	0.306	0.250	0.273

Table 7: Local and Global priority for each office u

3.4 Sensitivity analysis

In order to validate the results obtained and the stability of the two methods applied, we conducted a sensitivity analysis by decreasing each criterion considered in the analysis by 20% with the consequent increase in all the other criteria; e.g.: g_1 decreases by 20% while g_2 and g_3 increase by 20%. The offices rankings are shown in the Tables 8 and 9.

decrease g_1	decrease g_2	decrease g_3
u_7	u_7	u_7
u_{10}	u_{10}	u_{10}
u_6	u_6	u_5
u_{11}	u_{11}	u_6
u_5	u_5	u_{11}
u_9	u_9	u_9
u_8	u_8	u_2
u_3	u_3	u_8
u_2	u_2	u_1
u_1	u_1	u_3
u_4	u_4	u_4

Table 8: Sensitivity analysis of P-AHP

decrease g_1	decrease g_2	decrease g_3
u_7	u_7	u_7
u_{10}	u_{10}	u_{10}
u_6	u_6	u_6
u_{11}	u_{11}	u_{11}
u_5	u_5	u_5
u_9	u_9	u_9
u_8	u_2	u_2
u_3	u_8	u_8
u_2	u_3	u_3
u_1	u_1	u_1
u_4	u_4	u_4

Table 9: Sensitivity analysis of PROMETHEE I-II

As we can see from the rankings obtained, for the P-AHP method just small changes are present when g_3 decreases w.r.t. the decrease of the other criteria. For example, u_5 is ranked over u_6 and u_{11}, differently from the ranking in Table 7. Also for the PROMETHEE I-II method there are small

ranking shifts for the offices u_2, u_8 and u_3; in any case, the top positions remain stable in both the cases.

4 Discussion

As we have seen from the results obtained from the application of the PROMETHEE I-II (see Section 3.2) method and the P-AHP (see Section 3.3) method, both the approaches report the same ordering with the exception only of the ranking of the offices u_2 and u_8, which have undergone a ranking reversal. We point out that although the two methods applied use completely different methodological approaches, one an outranking method (PROMETHEE I-II) and the other a hierarchical method (P-AHP), both the approaches allowed us to obtain a performance of the offices from the most to the least performing, a clear benefit from a managerial insight standpoint. Obtaining these results demonstrates that both methods are valid for addressing performance evaluation problems. The choice of method by Public Administrations, at this point, is limited to the consideration of the characteristics of the methods themselves and the pros or cons of using one method rather than another. In particular, we remind that both methods allow to use qualitative and quantitative criteria and both define a valid decision support system even though they use different tools (see Section 2). Among the cons of the PROMETHEE I-II method we remember that rank reversal problems may occur, that transitivity may be violated and finally incomparability is admitted. The P-AHP method does not present rank reversal problems but among the disadvantages we remember the compensation between good and bad scores and that the transitivity may be violated. In Table 10, we report the principal pros and cons of the methods.

5 Conclusion

This paper provided the use of multi-criteria methods in the evaluation of Public Administration services. The results that emerged can guide public sector operators in optimizing the services offered by promoting transparency, efficiency and effectiveness in the provision of public services. With the results obtained using the PROMETHEE I-II and P-AHP methods, we defined a detailed framework of the performance of the offices within the Public Administration under examination. The analysis of the PROMETHEE I-II flows made it possible to determine the partial and complete rankings of the alternatives, highlighting the preferences of the decision maker. The visualization of the efficient frontier identified the offices considered efficient and those that could improve their performance. The P-AHP method allowed the use of reference points to reduce the number

	PROMETHEE I-II	P-AHP
Pros	a. Large number of alternatives and criteria b. Non-compensation between good and bad scores	a. Large number of alternatives and criteria b. Preventing the rank reversal problem
Cons	a. Loss of some information in defining the final sort b. Incomparability is allowed between alternatives c. Transitivity may be violated	a. Compensation between good and bad scores b. Transitivity may be violated

Table 10: Pros and Cons of PROMETHEE I-II and P-AHP,

of pairwise comparisons for the DM. The use of linear interpolation also made it possible to compare the representative points with the office assessments provided by the PA. Both methods provided the same ranking of offices except for a reverse ordering for offices u_2 and u_8. This result demonstrates that both methods are effective for analyzing the performance of a Public Administration. The application of the PROMETHEE I-II and P-AHP methods therefore provided an exhaustive and in-depth evaluation of the alternatives considered, allowing decision makers to make informed and rational decisions in a complex context such as that of evaluating the performance of offices within the Public Administration.

Future studies could concern the use of further multi-criteria methods and their integration with optimization methods, like for example knapsack-like or capital budgeting formulations, for the definition of corrective actions to improve the performance of PA offices.

Declarations

Conflict of interest. The authors declare that they have no conflict of interest.

References

Francesca Abastante, Salvatore Corrente, Salvatore Greco, Alessio Ishizaka, and Isabella M Lami. Choice architecture for architecture choices: Evaluating social housing initiatives putting together a parsimonious ahp methodology and the choquet integral. *Land Use Policy*, 78:748–762, 2018.

Francesca Abastante, Salvatore Corrente, Salvatore Greco, Alessio Ishizaka, and Isabella M Lami. A new parsimonious ahp methodology: Assigning priorities to many objects by comparing pairwise few reference objects. *Expert Systems with Applications*, 127:109–120, 2019.

Jean-Pierre Brans and Yves De Smet. Promethee methods. *Multiple criteria decision analysis: state of the art surveys*, pages 187–219, 2016.

Jean-Pierre Brans, Ph Vincke, and Bertrand Mareschal. How to select and how to rank projects: The promethee method. *European journal of operational research*, 24(2):228–238, 1986.

JP Brans. Mareschal (2005). promethee methods. *Multiple Criteria Decision Analysis: State of the Art Surveys*, pages 163–195.

John Dawes. Do data characteristics change according to the number of scale points used? an experiment using 5-point, 7-point and 10-point scales. *International journal of market research*, 50(1):61–104, 2008.

Amadou Diabagaté, Abdellah Azmani, and Mohamed El Harzli. The choice of the best proposal in tendering with ahp method: case of procurement of it master plan's realization. *International Journal of Information Technology and Computer Science (IJITCS)*, 7(12):1–11, 2015.

Mariagrazia Dotoli, Nicola Epicoco, and Marco Falagario. Multi-criteria decision making techniques for the management of public procurement tenders: A case study. *Applied Soft Computing*, 88:106064, 2020.

Gerarda Fattoruso and Gabriella Marcarelli. A multi-criteria approach for public tenders. electre iii and parsimonious ahp: a comparative study. *Soft Computing*, 26(21):11771–11781, 2022.

Gerarda Fattoruso, Paola Mancini, and Gabriella Marcarelli. The ahpsort ii to evaluate the high level instruction performances. *Ratio Mathematica*, 42:283, 2022.

Isabel C Gil García, Ana Fernández-Guillamón, M Socorro García-Cascales, and Ángel Molina-García. Multi-factorial methodology for

wind power plant repowering optimization: A spanish case study. *Energy Reports*, 11:179–196, 2024.

Swati Goyal, Shivi Agarwal, Narinderjit Singh Sawaran Singh, Trilok Mathur, and Nirbhay Mathur. Analysis of hybrid mcdm methods for the performance assessment and ranking public transport sector: a case study. *Sustainability*, 14(22):15110, 2022.

Alessio Ishizaka and Philippe Nemery. Selecting the best statistical distribution with promethee and gaia. *Computers & Industrial Engineering*, 61 (4):958–969, 2011.

Daekook Kang, Krishnan Suvitha, Samayan Narayanamoorthy, Michael Sandra, and Dragan Pamucar. Evaluation of wave energy converters based on integrated electre approach. *Expert Systems with Applications*, 242: 122793, 2024.

Marcin Kiciński and Katarzyna Solecka. Application of mcda/mcdm methods for an integrated urban public transportation system–case study, city of cracow. *Archives of Transport*, 46(2):71–84, 2018.

Ranjit Kumar. Research methodology: A step-by-step guide for beginners. *Research methodology*, pages 1–528, 2018.

Ming-Shin Kuo and Gin-Shuh Liang. A soft computing method of performance evaluation with mcdm based on interval-valued fuzzy numbers. *Applied Soft Computing*, 12(1):476–485, 2012.

Wen-Hsiang Lai, Pao-Long Chang, and Ying-Chyi Chou. Fuzzy mcdm approach to r&d project evaluation in taiwan's public sectors. In *PICMET'08-2008 Portland International Conference on Management of Engineering & Technology*, pages 1523–1532. IEEE, 2008.

Barbara Loken, Phyllis Pirie, Karen A Virnig, Ronald L Hinkle, and Charles T Salmon. The use of 0-10 scales in telephone surveys. *Journal of the Market Research Society*, 29(3):353–362, 1987.

Gabriella Marcarelli and Massimo Squillante. A group-ahp-based approach for selecting the best public tender. *Soft Computing*, 24(18):13717–13724, 2020.

Bertrand Mareschal, Yves De Smet, and Philippe Nemery. Rank reversal in the promethee ii method: some new results. In *2008 IEEE International Conference on Industrial Engineering and Engineering Management*, pages 959–963. IEEE, 2008.

Rocco Reina et al. *Gestione del personale e cambiamento organizzativo nell'amministrazione pubblica*. Rubbettino Editore, 2008.

Thomas L Saaty. The analytic hierarchy process: Planning, priority setting, resources allocation. mcgraw-hill, new york. 1980.

Thomas L Saaty. Deriving the ahp 1-9 scale from first principles. *Proceedings 6th ISAHP. Berna, Suiza*, pages 397–402, 2001.

Thomas L Saaty. Decision-making with the ahp: Why is the principal eigenvector necessary. *European journal of operational research*, 145(1):85–91, 2003.

Thomas L Saaty. Decision making with the analytic hierarchy process. *International journal of services sciences*, 1(1):83–98, 2008.

Fiorella Pia Salvatore, Simone Fanelli, Gianluca Lanza, and Michele Milone. Public food procurement for italian schools: results from analytical and content analyses. *British Food Journal*, 123(8):2936–2951, 2021.

Roman Vavrek, Rastislav Kotulic, Peter Adamisin, Elena Sira, and Ivana Kravcakova Vozarova. Effectiveness of use of mcdm methods in the terms of local self-government. In *Advances in Applied Economic Research: Proceedings of the 2016 International Conference on Applied Economics (ICOAE)*, pages 279–288. Springer, 2017.

Philippe Vincke. Preference modelling: a survey and an experiment. In *Operational research'81: Proceedings of the ninth IFORS International Conference on Operational Research*. North Holland, 1981.

Hung-Yi Wu, Jui-Kuei Chen, I-Shuo Chen, and Hsin-Hui Zhuo. Ranking universities based on performance evaluation by a hybrid mcdm model. *Measurement*, 45(5):856–880, 2012.