Ratio Mathematica Volume 47, 2023

On Neutrosophic filter of BL-algebras

A. Ibrahim*
S. Karunya Helen Gunaseeli†

Abstract

In this paper, we introduce the notion of neutrosophic filter in BL-algebras and investigate several key characteristics with illustrations. Additionally, we obtain few conditions, order properties of neutrosophic filter of BL-algebras. Moreover, we prove that intersection of two neutrosophic filters of a BL-algebra is also a neutrosophic filter.

Keywords: BL-algebra; filter; neutrosophic set; neutrosophic filter.
2020 AMS subject classification: 03G25,03E72,03F55,06F35‡.

1. Introduction

In 1965, L. A. Zadeh [2, 3, 14] was the first to introduce the notion of fuzzy sets to describe vagueness mathematically. He rectified those problems by designating every feasible individual in the universe a number resembling its grade of membership in the fuzzy set. In lattice implication algebras, Xu and Qin[12] first proferred the concept of filters. An important aspect of researching various logical algebras is filter theory. In the argument of the completeness of various logical algebra, filters are crucial. Several academicians have investigated the filter theory of different logical algebras. In 1998, the ideology of neutrosophy was introduced by Smarandache [1,9]. This became a new idea of philosophy to characterize the neutralites. The main idea of neutrosophy is

*Assistant Professor, P.G. and Research Department of Mathematics, H.H. The Rajah's College, Pudukkottai, Affiliated to Bharathidasan University, Trichirappalli, India; dribra@hhrc.ac.in, dribrahimaadhil@gmail.com.
†Research Scholar, P.G. and Research Department of Mathematics, H.H. The Rajah's College, Pudukkottai, Affiliated to Bharathidasan University, Trichirappalli, India; karjes821@gmail.com.
‡Received on August 11, 2022. Accepted on May 17, 2023. Published on June 30, 2023. DOI:10.23755/rm.v47i0.816. ISSN: 1592-7415. eISSN: 2282-8214.

that behind every concept there also exists an indeterminant degree in addition to truth and falsity. Neutrosophy, a discipline of philosophy that has just recently been recognised as a science, examines the genesis, character, and range of neutralities as well as how they interact with various ideational spectra. Hajek[6] introduced the concept of BL-algebras (Basic Logic), is a type of logical algebra.

In this paper, the concept of neutrosophic filters of BL-algebras is discussed. In section 2, few fundamental definitions and results are explained. In section 3, we introduce the notion of neutrosophic filters of BL-algebras along with some of its related features.

2. Preliminaries

In this section, we recall few fundamental definitions and their characteristics that are useful for developing the primary findings.

Definition 2.1[6,11] A BL-algebra is an algebra $(\mathcal{B}, \vee, \wedge, \circ, \rightarrow, 0, 1)$ of type $(2, 2, 2, 2, 0, 0)$ such that the following are satisfied for all $j_1, k_1, l_1 \in \mathcal{B}$,

(i) $(\mathcal{B}, \vee, \wedge, 0, 1)$ is a bounded lattice,
(ii) $(\mathcal{B}, \circ, 1)$ is a commutative monoid,
(iii) $' \circ '$ and $' \rightarrow '$form an adjoint pair, that is, $l_1 \leq j_1 \rightarrow k_1$if and only if $j_1 \circ l_1 \leq k_1$for all $j_1, k_1, l_1 \in \mathcal{B}$,
(iv) $j_1 \wedge k_1 = j_1 \circ (j_1 \rightarrow k_1)$,
(v) $(j_1 \rightarrow k_1) \vee (k_1 \rightarrow j_1) = 1$.

Proposition 2.2[7,13] The following axioms are satisfied in a BL- algebra $\mathcal{B}$ for all $j_1, k_1, l_1 \in \mathcal{B}$,

(i) $k_1 \rightarrow (j_1 \rightarrow l_1) = j_1 \rightarrow (k_1 \rightarrow l_1) = (j_1 \circ k_1) \rightarrow l_1$,
(ii) $1 \rightarrow j_1 = j_1$,
(iii) $j_1 \leq k_1$ if and only if $j_1 \rightarrow k_1 = 1$,
(iv) $j_1 \vee k_1 = ((j_1 \rightarrow k_1) \rightarrow k_1) \wedge ((k_1 \rightarrow j_1) \rightarrow j_1)$,
(v) $j_1 \leq k_1$ implies $k_1 \rightarrow l_1 \leq j_1 \rightarrow l_1$,
(vi) $j_1 \leq k_1$ implies $l_1 \rightarrow j_1 \leq l_1 \rightarrow k_1$,
(vii) $j_1 \rightarrow k_1 \leq (l_1 \rightarrow j_1) \rightarrow (l_1 \rightarrow k_1)$,
(viii) $j_1 \rightarrow k_1 \leq (k_1 \rightarrow l_1) \rightarrow (k_1 \rightarrow l_1)$,
(ix) $j_1 \leq (j_1 \rightarrow k_1) \rightarrow k_1$,
(x) $j_1 \circ (j_1 \rightarrow k_1) = j_1 \wedge k_1$,
(xi) $j_1 \circ k_1 \leq j_1 \wedge k_1$
(xii) $j_1 \rightarrow k_1 \leq (j_1 \circ l_1) \rightarrow (k_1 \circ l_1)$,
(xiii) $j_1 \circ (k_1 \rightarrow l_1) \leq k_1 \rightarrow (j_1 \circ l_1)$,
(xiv) $(j_1 \rightarrow k_1) \circ (k_1 \rightarrow l_1) \leq j_1 \rightarrow l_1$,
(xv) $(j_1 \circ j_1{}^*) = 0$.

Note. In the above sequence, $\mathcal{B}$ is used to intend the BL- algebras and the operations

$'\vee'$, $'\wedge'$, $'\circ'$ have preference on the way to the operations $'\rightarrow'$.

Note. In a BL- algebra $\mathcal{B}$, $'*'$ is a complement defined as ${j_1}^* = j_1 \rightarrow 0$ for all $j_1 \in \mathcal{B}$.

Definition 2.3[15] A non-empty subset F of a BL- algebra $\mathcal{B}$ is a filter of $\mathcal{B}$ if the following axioms hold for all $j_1, k_1 \in \mathcal{B}$,

(i) If $j_1, k_1 \in F$, then $j_1 \circ k_1 \in F$,
(ii) If $j_1 \in F$ and $j_1 \leq k_1$, then $k_1 \in F$.

Proposition 2.4[15] A nonempty subset F of a BL- algebra $\mathcal{B}$ is a filter of $\mathcal{B}$ if and only if the following are satisfied for all $j_1, k_1 \in \mathcal{B}$,

(i) $1 \in F$,
(ii) $j_1, j_1 \rightarrow k_1 \in F$ implies $k_1 \in F$.

A filter F of a BL-algebra $\mathcal{B}$ is proper if $F \neq \mathcal{B}$.

Definition 2.5[8,9] Let X be a set. A neutrosophic subset R of X is a triple (T_R, I_R, F_R) where $T_R: X \rightarrow [0,1]$ is truth membership function, $I_R: X \rightarrow [0,1]$ is indeterminacy function and $F_R: X \rightarrow [0,1]$ is false membership function and $0 \leq T_R(j_1) + I_R(j_1) + F_R(j_1) \leq 3$ for all $j_1 \in X$. Hence, for each $j_1 \in X$, $T_R(j_1)$, $I_R(j_1)$ and $F_R(j_1)$ are all standard real numbers in $[0,1]$.

Note. The values of $T_R(j_1), I_R(j_1)$ and $F_R(j_1)$ have no limitations and we have the obvious condition $0 \leq T_R(j_1) + I_R(j_1) + F_R(j_1) \leq 3$.

Definition 2.6[4, 9] Let R and S be two neutrosophic sets on X. Define $R \leq S$ if and only if

$T_R(j_1) \leq T_S(j_1)$, $I_R(j_1) \geq I_S(j_1)$, $F_R(j_1) \geq F_S(j_1)$ for all $j_1 \in X$.

Definition 2.7[5,9] Let R and S be two neutrosophic sets on X. Define $R \wedge S = (T_R \wedge T_S, I_R \vee I_S, F_R \vee F_S)$; $R \vee S = (T_R \vee T_S, I_R \wedge I_S, F_R \wedge F_S)$

Where $'\wedge'$ is the minimum and $'\vee'$ is the maximum between real numbers.

Definition 2.8[10] Let R be a neutrosophic set in X and $\alpha, \beta, \gamma \in [0,1]$ with $0 \leq \alpha + \beta + \gamma \leq 3$ and (α, β, γ) – level set of R denoted by $R^{(\alpha,\beta,\gamma)}$ is defined as

$R^{(\alpha,\beta,\gamma)} = \{j_1 \in X / T_R(j_1) \geq \alpha, I_R(j_1) \leq \beta$ and $F_R(j_1) \leq \gamma\}$.

3. Properties of neutrosophic filter

In this section, we introduce the definition of neutrosophic filter of BL-algebra and obtain some relevant properties with illustrations.

Definition 3.1 A neutrosophic set R of algebra $\mathcal{B}$ is called a neutrosophic filter if it satisfies the subsequent conditions:

(i) $T_R(j_1) \leq T_R(1)$, $I_R(j_1) \geq I_R(1)$ and $F_R(j_1) \geq F_R(1)$,

(ii) $\min\{T_R(j_1 \to k_1), T_R(j_1)\} \leq T_R(k_1)$, $\min\{I_R(j_1 \to k_1), I_R(j_1)\} \geq I_R(k_1)$ and $\min\{F_R(j_1 \to k_1), F_R(j_1)\} \geq F_R(k_1)\}$ for all $j_1, k_1 \in \mathcal{B}$.

Example 3.2 Let $\mathcal{B} = \{0, u, v, 1\}$. The binary operations $'\circ'$ and $'\to'$ are given by the subsequent tables (3.1) and (3.2).

$\circ$	0	u	v	1
0	0	0	0	0
u	0	0	u	v
v	0	u	v	v
1	0	u	v	1

Table3.1: $'\circ'$ Operation

$\to$	0	u	v	1
0	1	1	1	1
u	u	1	1	1
v	0	u	1	1
1	0	u	v	1

Table 3.2: $'\to'$ Operation

Then, $(\mathcal{B}, \vee, \wedge, \circ, \to, 0, 1)$ is a BL- algebra.

Define a neutrosophic set R of $\mathcal{B}$ as follows:

$R = \{(1, [0.9,0.2,0.1]), (u, [0.5, 0.3, 0]), (v, [0.5,0.3,0]), (0, [0.9,0.2,0.1])\}$.

It is evident that R is a neutrosophic filter of $\mathcal{B}$ and assure the conditions (i) and (ii) of the definition 3.1.

Example 3.3 Let $\mathcal{B} = \{0, a, b, 1\}$. The binary operations are given by the tables (3.3) and (3.4). Let S of $\mathcal{B}$ be a neutrosophic set as follows:

$S = \{(1, [0.5,0.3,0.2]), (a, [0.9,0.2,0.1]), (b, [0.9,0.2,0.1]), (0, [0.9,0.2,0.1])\}$

Here, S is not a neutrosophic filter of $\mathcal{B}$.

Since $T_S(1) = 0.5 \ngeq 0.9 = \min\{T_S(a \circ 1), T_S(a)\}$,

$T_S(1) = 0.5 \ngeq 0.9 = \min\{T_S(b \circ 1), T_S(b)\}$.

$\circ$	0	a	b	1
0	0	0	0	b
a	0	0	a	b
b	0	a	b	b
1	0	a	b	1

Table 3.3: $'\circ'$ Operation

$\rightarrow$	0	a	b	1
0	1	1	1	1
a	a	1	1	1
b	0	a	1	1
1	0	a	b	1

Table 3.4:$'\longrightarrow'$Operation

Proposition 3.4 Let R be a neutrosophic filter in a BL- algebra $\mathcal{B}$.If $j_1 \leq k_1$then

$T_R(j_1) \leq T_R(k_1), I_R(j_1) \geq I_R(k_1), F_R(j_1) \geq F_R(k_1)$for all $j_1, k_1 \in \mathcal{B}$.

Proof: Let R be a neutrosophic filter of a BL-algebra$\mathcal{B}$. If $j_1 \leq k_1$, then $j_1 \longrightarrow k_1 = 1$for all $j_1, k_1 \in \mathcal{B}$ [From(iii) of the proposition 2.2]

Then, from (i) and(ii) of the definition 3.1,

$T_R(j_1) = \min\{T_R(1), T_R(j_1)\}$

$= \min\{T_R(j_1 \rightarrow k_1), T_R(j_1)\}$

$\leq T_R(k_1)$,

$I_R(j_1) = \min\{I_R(1), I_R(j_1)\}$

$= \min\{I_R(j_1 \rightarrow k_1), I_R(j_1)\}$

$\geq I_R(k_1)$,

$F_R(j_1) = \min\{F_R(1), F_R(j_1)\}$

$= \min\{F_R(j_1 \rightarrow k_1), F_R(j_1)\}$

$\geq F_R(k_1)$ for all $j_1, k_1 \in \mathcal{B}$. ■

Proposition 3.5 Let R be a neutrosophic filter of a BL-algebra $\mathcal{B}$. If $j_1 \leq k_1$then $T_R(j_1)$is order preserving and $I_R(j_1), F_R(j_1)$are order reversing.

Proof: Let R be a neutrosophic filter of a BL-algebra $\mathcal{B}$.

To prove: If $j_1 \leq k_1$then $T_R(j_1) \leq T_R(k_1)$, $I_R(j_1) \geq I_R(k_1)$, $F_R(j_1) \geq F_R(k_1)$ for all$j_1, k_1 \in \mathcal{B}$.

Then, from the proposition 3.4, the proof is straight forward. Thus $T_R(j_1)$is order preserving and $I_R(j_1), F_R(j_1)$are order reversing. ■

Proposition 3.6 Let R be a neutrosophic set of a BL–algebra $\mathcal{B}$. R is a neutrosophic filter of $\mathcal{B}$ if and only if $j_1 \longrightarrow (k_1 \longrightarrow l_1) = 1$ implies $T_R(l_1) \geq \min\{ T_R(j_1), T_R(k_1) \}$, $I_R(l_1) \leq \min\{ I_R(j_1), I_R(k_1)\}$ and $F_R(l_1) \leq \min\{ F_R(j_1), F_R(k_1)\}$ for all $j_1, k_1, l_1 \in \mathcal{B}$.

Proof: Let R be a neutrosophic filter of a BL- algebra $\mathcal{B}$.

Then, from (ii) of the definition 3.1, we have

$T_R(l_1) \geq \min\{T_R(l_1 \longrightarrow k_1), T_R(k_1)\}$ for all $j_1, k_1, l_1 \in \mathcal{B}$.

Now, $T_R(l_1 \longrightarrow k_1) \geq \min\{ T_R(j_1 \rightarrow (k_1 \rightarrow l_1)), T_R(j_1) \}$.

If $j_1 \rightarrow (k_1 \rightarrow l_1) = 1$, then we have $T_R(l_1 \longrightarrow k_1) \geq \min\{T_R(1), T_R(j_1)\}$

$= T_R(j_1)$.

So, $T_R(l_1) \geq \min\{T_R(j_1), T_R(k_1)\}$.

Similarly, $I_R(l_1) \leq \min\{ I_R(j_1), I_R(k_1)\}$ and

$F_R(l_1) \leq \min\{F_R(j_1), F_R(k_1)\}$.

Conversely, Let $j_1 \longrightarrow (j_1 \longrightarrow 1) = 1$ for all $j_1 \in \mathcal{B}$.
Then, $T_R(1) \geq \min\{T_R(j_1), T_R(j_1)\} = T_R(j_1)$.
On the other hand, from $(j_1 \rightarrow k_1) \rightarrow (j_1 \rightarrow k_1) = 1$ implies

$T_R(k_1) \geq \min\{T_R(j_1 \rightarrow k_1), T_R(j_1)\}$, $I_R(k_1) \leq \min\{ I_R(j_1 \rightarrow k_1), I_R(j_1)\}$ and $F_R(k_1) \leq \min\{ F_R(j_1 \rightarrow k_1), F_R(j_1)\}$.

Then, from the definition 3.1, R is a neutrosophic filter of $\mathcal{B}$. ■

Corollary 3.7 Let R be a neutrosophic set of BL- algebra $\mathcal{B}$. R is a neutrosophic filter of $\mathcal{B}$ if and only if $j_1 \circ k_1 \leq l_1$ or $k_1 \circ l_1 \leq l_1$ implies $T_R(l_1) \geq \min\{ T_R(j_1), T_R(k_1) \}$, $I_R(l_1) \leq \min\{I_R(j_1), I_R(k_1)\}$ and $F_R(l_1) \leq \min\{ F_R(j_1), F_R(k_1)\}$ for all $j_1, k_1, l_1 \in \mathcal{B}$.

Proof: From (i) of the proposition 2.2 and the proposition 3.6 the proof is obvious. ■

Proposition 3.8 Let R be a neutrosophic set of a BL-algebra $\mathcal{B}$. R is a neutrosophic filter of $\mathcal{B}$ if and only if

(i) If $j_1 \leq k_1$ then $T_R(j_1) \leq T_R(k_1)$, $I_R(j_1) \geq I_R(k_1)$ and $F_R(j_1) \geq F_R(k_1)$,

(ii) $T_R(j_1 \circ k_1) \geq \min\{T_R(j_1), T_R(k_1)\}$, $I_R(j_1 \circ k_1) \leq \min\{I_R(j_1), I_R(k_1)\}$ and $F_R(j_1 \circ k_1) \leq \min\{ F_R(j_1), F_R(k_1)\}$ for all $j_1, k_1 \in \mathcal{B}$.

Proof: Let R be a neutrosophic filter of a BL-algebra $\mathcal{B}$.

Then, from the proposition 3.4, we have $T_R(j_1) \leq T_R(k_1)$, $I_R(j_1) \geq I_R(k_1)$ and $F_R(j_1) \geq F_R(k_1)$ when $j_1 \leq k_1$.

Since $j_1 \circ k_1 \leq j_1 \circ k_1$ and from the corollary 3.7, we have $T_R(j_1 \circ k_1) \geq \min\{ T_R(j_1), T_R(k_1) \}$, $I_R(j_1 \circ k_1) \leq \min\{ I_R(j_1), I_R(k_1)\}$ and

$F_R(j_1 \circ k_1) \leq \min\{ F_R(j_1), F_R(k_1)\}$.

Conversely, Let R be a neutrosophic set and satisfies (i) and (ii).

If $j_1 \circ k_1 \leq l_1$ then from (i) and (ii) we get, $T_R(l_1) \geq \min\{T_R(j_1), T_R(k_1)\}$,

$I_R(l_1) \leq \min\{ I_R(j_1), I_R(k_1)\}$ and $F_R(l_1) \leq \min\{ F_R(j_1), F_R(k_1)\}$for all $j_1, k_1, l_1 \in \mathcal{B}$.

Then, from the corollary 3.7, we have R is a neutrosophic filter. ■

Proposition 3.9 Let R be a neutrosophic set of a BL-algebra $\mathcal{B}$. If R is a neutrosophic filter of $\mathcal{B}$, then it satisfies the following for all $j_1, k_1, l_1 \in \mathcal{B}$.

(i) If $T_R(j_1 \longrightarrow k_1) = T_R(1)$, then $T_R(j_1) \leq T(k_1)$,$I_R(j_1 \longrightarrow k_1) = I_R(1)$, then $I_R(j_1) \geq I_R(k_1)$,$F_R(j_1 \longrightarrow k_1) = F_R(1)$, then $F_R(j_1) \geq F_R(k_1)$.

(ii) $T_R(j_1 \longrightarrow k_1) \leq T_R(j_1 \circ l_1 \longrightarrow k_1 \circ l_1)$, $I_R(j_1 \longrightarrow k_1) \geq I_R(j_1 \circ l_1 \longrightarrow k_1 \circ l_1)$ and$F_R(j_1 \longrightarrow k_1) \geq F_R(j_1 \circ l_1 \longrightarrow k_1 \circ l_1)$.

(iii) $T_R(j_1 \longrightarrow k_1) \leq T_R((k_1 \to l_1) \to (j_1 \to l_1))$,$I_R(j_1 \longrightarrow k_1) \geq I_R((k_1 \to l_1) \to (j_1 \to l_1))$and $F_R(j_1 \longrightarrow k_1) \geq F_R((k_1 \to l_1) \to (j_1 \to l_1))$.

(iv) $T_R(j_1 \longrightarrow k_1) \leq T_R((l_1 \to j_1) \to (l_1 \to k_1))$,$I_R(j_1 \longrightarrow k_1) \geq I_R((l_1 \to j_1) \to (l_1 \to k_1))$ and$F_R(j_1 \longrightarrow k_1) \geq F_R((l_1 \to j_1) \to (l_1 \to k_1))$.

Proof: (i) Let R be a neutrosophic filter of a BL-algebra $\mathcal{B}$.

Then from the definition 3.1, and since $T_R(j_1 \longrightarrow k_1) = T_R(1)$,

we have $T_R(k_1) \geq \min\{ T_R(j_1), T_R(j_1 \longrightarrow k_1) \}$

$= \min\{T_R(j_1), T_R(1)\} = T_R(j_1)$.

Thus, $T_R(j_1) \leq T_R(k_1)$. Similarly we get, $I_R(j_1) \geq I_R(k_1)$, $F_R(j_1) \geq F_R(k_1)$.

From the proposition 2.2 and (i) of the proposition 3.8, we can prove (ii), (iii) and (iv) easily. ■

Proposition 3.10 Let R be a neutrosophic set of a BL- algebra $\mathcal{B}$. $R$is a neutrosophic filter of $\mathcal{B}$ if and only if

(i) $T_R(j_1) \leq T_R(1)$, $I_R(j_1) \geq I_R(1)$ and $F_R(j_1) \geq F_R(1)$,

(ii) $T_R((j_1 \to (k_1 \to l_1)) \to l_1) \geq \min\{ T_R(j_1), T_R(k_1)\}$,

$I_R((j_1 \to (k_1 \to l_1)) \to l_1) \leq \min\{I_R(j_1), I_R(k_1)\}$

and $F_R((j_1 \to (k_1 \to l_1)) \to l_1) \leq \min\{ F_R(j_1), F_R(k_1)\}$for all $j_1, k_1, l_1 \in \mathcal{B}$.

Proof: Let R be a neutrosophic filter of BL-algebra $\mathcal{B}$.

From the definition 3.1 (i) is straight forward.

Since,$T_R((j_1 \to (k_1 \to l_1)) \to l_1) \geq \min\{T_R((j_1 \to (k_1 \to l_1)) \to l_1), T_R(k_1)\}$ (3.1)

Now, we have, $(j_1 \to (k_1 \to l_1)) \to (k_1 \to l_1) = j_1 \vee (k_1 \to l_1) \geq j_1$.

$T_R((j_1 \to (k_1 \to l_1)) \to (k_1 \to l_1)) \geq T_R(j_1)$ (3.2)

[From the proposition 3.5]

Using (3.2) in (3.1), we have $T_R((j_1 \to (k_1 \to l_1)) \to l_1)$

$\geq \min\{T_R(j_1), T_R(k_1)\}$

Similarly, we get $I_R((j_1 \to (k_1 \to l_1)) \to l_1) \leq \min\{I_R(j_1), I_R(k_1)\}$ and

$F_R((j_1 \to (k_1 \to l_1)) \to l_1) \leq \min\{F_R(j_1), F_R(k_1)\}$.

Conversely, assume (i) and (ii) hold.

Since $T_R(k_1) = T_R(1 \to k_1)$

$= T_R((j_1 \to k_1) \to (j_1 \to k_1) \to k_1)$

$\geq \min\{T_R(j_1 \to k_1), T_R(k_1)\}$.

Similarly, $I_R(k_1) \leq \min\{I_R(j_1 \to k_1), I_R(k_1)\}$and

$F_R(k_1) \leq \min\{F_R(j_1 \to k_1), F_R(k_1)\}$.

From (i), R is a neutrosophic filter of $\mathcal{B}$. ■

Proposition 3.11 The intersection of two neutrosophic filters of $\mathcal{B}$ is also a neutrosophic filter of $\mathcal{B}$.

Proof: Let R and S be two neutrosophic filters of $\mathcal{B}$,

To Prove: $R \cap S$ is a neutrosophic filter of $\mathcal{B}$.

We have $T_R(k_1) \geq \min\{T_R(l_1), T_R(j_1)\}$ and $T_S(k_1) \geq \min\{T_S(l_1), T_S(j_1)\}$ for all $j_1, k_1, l_1 \in \mathcal{B}$.

Since $T_{R\cap S}(k_1) = \min\{T_R(k_1), T_S(k_1)\}$

$\geq \min\{\min\{T_R(l_1), T_R(j_1)\}, \min\{T_S(l_1), T_S(j_1)\}\}$

$= \min\{\min\{T_R(l_1), T_S(l_1)\}, \min\{T_R(j_1), T_S(j_1)\}\}$

$= \min\{\min\{T_{R\cap S}(l_1), T_{R\cap S}(j_1)\}\}$.

Similarly,$I_{R\cap S}(k_1) = \min\{\min\{I_{R\cap S}(l_1), I_{R\cap S}(j_1)\}\}$ and

$F_{R\cap S}(k_1) = \min\{\min\{F_{R\cap S}(l_1), F_{R\cap S}(j_1)\}\}$.

Hence, $T_{R\cap S}(k_1) \geq \min\{T_{R\cap S}(l_1), T_{R\cap S}(j_1)\}$,

$I_{R\cap S}(k_1) \leq \min\{I_{R\cap S}(l_1), I_{R\cap S}(j_1)\}$

and $F_{R\cap S}(k_1) \leq \min\{F_{R\cap S}(l_1), F_{R\cap S}(j_1)\}$.

Thus $R \cap S$ is a neutrosophic filter of $\mathcal{B}$. ■

Corollary 3.12 Let $P_i$be a family of neutrosophic filters of $\mathcal{B}$, where $i \in I$, I is a index set, then $\cap_{i\in I} P_i$ is a neutrosophic filter of $\mathcal{B}$. ■

4. Discussion and Conclusions

In this paper, we have introduced the notion of a neutrosophic filter in BL-algebras with illustrations. Moreover, several features of the neutrosophic filters are conferred. Also, we have derived a few equivalent conditions for a neutrosophic set of a BL-algebra to be a neutrosophic filter. In BL-algebras, fuzzy, vague, and many additional filters have already been defined. The main focus of this paper is to establish the neutrosophical nature of BL-algebras. Further, research on BL-algebras structure and the creation of the associated many-valued logical system will benefit from the aforementioned study. In the future, these neutrosophic filters can be extended to include fantastic filters, implicative filters, normal filters, and so on.

References

[1] H. H. Abbass,Q. M. Luhaib. On Smarandache Filter of a Smarandache BH-Algebra. Journal of Physics, 12-34, 2019.

[2] K. T. Atanassov. Intuitionistic fuzzy sets. ITKR'S session: sofia, 1983.

[3] K. T. Atanassov. Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20, 87-86, 1986.

[4] Biao Long Meng. On filters in BE- Algebras. Scientiae MathematicaeJaponicae,71(2), 201-207, 2010.

[5] R. A. Borzooei. Neutrosophic deductive filters on BL-Algebras. Intelligent and Fuzzy Systems, 26, 2993- 3004,2014.

[6] P. Hajek. Metamathematics of fuzzy logic. Klower Academic Publishers: Dordrecht, 1999.

[7] L.Z. Liu, K.T. Li. Fuzzy filters of BL-algebras. Information Sciences, 173, 141-154,2005.

[8] A. A. Salama, H. Alagamy. Neutrosophic Filters. Journal of Computer Science Engineering, 307-312, 2013.

[9] F. Smarandache.A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, 1999.

[10] C. A .C. Sweety, I. Arockiarani. Annals of Fuzzy Mathematics and Informatics. Rough Sets in Neutrosophic Approximation Space,13(4), 449-463,2017.

[11] E. Turunen. BL-algebras of Basic Fuzzy Logic. Mathware and Soft Computing, 6, 49-61, 1999.

[12] Y. Xu and K.Y.Qin. On filters of lattice implication algebra. Journal of Fuzzy Maths, 2, 251-260,1993.

[13] S. Yahya Mohamed, P.Umamaheshwari. Vague filter of BL- algebras. Computer and MathematicalSciences,9(8), 914-920,2018.

[14] L. A. Zadeh.Fuzzy sets. Information and Control. 8, 338-353,1965.

[15] X. Zhang, X. Mao, Y. Wu, X. Zhai.Neutrosophic Filters in Pseudo-BCI algebras. International Journal for Uncertainty Quantification, 8(6),511-526, 2018.