

Domination in m – polar soft fuzzy graphs

S. Ramkumar *

R. Sridevi †

Abstract

In this article, we have introduced dominating set, minimal dominating set, independent dominating set, maximal independent dominating set in m – polar soft fuzzy graphs. We proved theorems and also some properties of dominating set in m – polar soft fuzzy graphs.

Keywords: Dominating set, Independent dominating set, Maximal independent dominating set in m – polar soft fuzzy graphs.

2020 AMS subject classifications: 03E72, 05C72. ¹

*Research Scholar , PG and Research Department of Mathematics, Sri S. Ramasamy Naidu Memorial College, Sattur-626 203, Tamil Nadu, India. Affiliated to Madurai Kamaraj University, Madurai-625 021, Tamil Nadu, India; ramkumarmat2015@gmail.com.

†Assistant Professor, PG and Research Department of Mathematics, Sri S. Ramasamy Naidu Memorial College, Sattur-626 203, Tamil Nadu, India. Affiliated to Madurai Kamaraj University, Madurai-625 021, Tamil Nadu, India; danushsairam@gmail.com.

¹Received on September 15, 2022. Accepted on December 15, 2022. Published on March 20, 2023. DOI: 10.23755/rm.v46i0.1070. ISSN: 1592-7415. eISSN: 2282-8214. ©The Authors. This paper is published under the CC-BY licence agreement.

[illegible]

$\forall v \in V \setminus \{u\}$ in $\tilde{H}_{P,V}(e) \forall e \in P$. so that $N^{hd}(u) = \phi$. That means an isolated vertex does not dominate any other vertex in $\tilde{G}_{P,V}$.

Example 2.1. Consider a 3-psf-graph $\tilde{G}_{P,V}$.

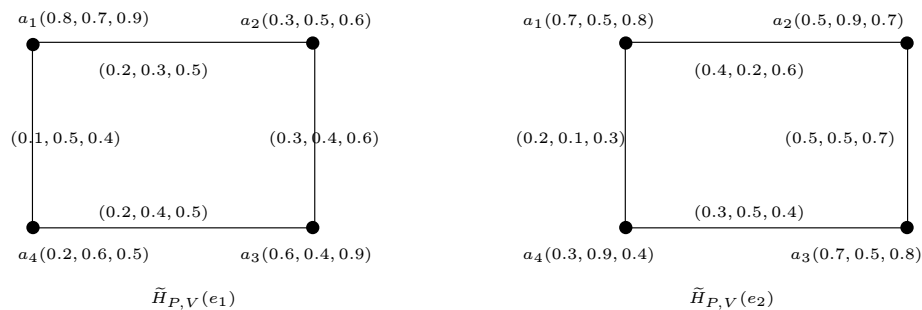


Fig.1. Effective edges of 3-psf-graph $\tilde{G}_{P,V} = \{\tilde{H}_{P,V}(e_1), \tilde{H}_{P,V}(e_2)\}$.

In this example, a_2a_3 and a_3a_4 are effective edges. Also $N^{hd}(a_1) = \{\phi\}$, $N^{hd}(a_2) = \{a_3\}$, $N^{hd}(a_3) = \{a_4a_2\}$, $N^{hd}(a_4) = \{a_3\}$. Here a_1 is an isolated vertex.

Definition 2.3. Let $\tilde{G}_{P,V} = (G^*, \tilde{\rho}, \tilde{\mu}, P)$ be an m -psf-graph. For any two vertices $u, v \in V$, we call u dominates v in $\tilde{G}_{P,V}$ if

[illegible]

in $\tilde{H}_{P,V}(e) \forall e \in P$ and $\forall u, v \in V$.

Definition 2.4. Consider a subset $\tilde{\mathcal{S}}$ of V in an m -psf-graph $\tilde{G}_{P,V}$. The cardinality of $\tilde{\mathcal{S}}$ is defined as,

$$|\tilde{\mathcal{S}}| = \sum_{e \in P} (\sum_{u \in \tilde{\mathcal{S}}} \tilde{\rho}_e(u)) \text{ in } \tilde{H}_{P,V}(e) \forall e \in P.$$

Definition 2.5. A subset $\tilde{\mathcal{S}}$ of V is called a dominating set of an m -psf-graph $\tilde{G}_{P,V}$ if for every vertex $u \in V \setminus \tilde{\mathcal{S}}$ then $\exists v \in \tilde{\mathcal{S}}$ such that u dominates v in $\tilde{H}_{P,V}(e) \forall e \in P$. The domination number $\gamma(\tilde{G}_{P,V})$ means the infimum cardinality of all dominating set in $\tilde{G}_{P,V}$ and $\gamma(\tilde{G}_{P,V}) = \min_{\tilde{\mathcal{S}} \in V} \sum_{e \in P} (\sum_{u \in \tilde{\mathcal{S}}} \tilde{\rho}_e(u))$.

Definition 2.6. A dominating set $\tilde{\mathcal{S}}$ is called a minimal dominating set of m -psf-graph $\tilde{G}_{P,V} = (G^*, \tilde{\rho}, \tilde{\mu}, P)$ if for any $a \in \tilde{\mathcal{S}}, \tilde{\mathcal{S}} \setminus \{a\}$ is not a dominating set in $\tilde{H}_{P,V}(e) \forall e \in P$.

Definition 2.7. Lower and upper domination number of an m -psf-graph $\tilde{G}_{P,V} = (G^*, \tilde{\rho}, \tilde{\mu}, P)$ is denoted by $\gamma(\tilde{G}_{P,V})$ and $\Gamma(\tilde{G}_{P,V})$, respectively, and defined by infimum cardinality and supremum cardinality of all minimal dominating set of that m -psf-graph, respectively.

Example 2.2. Consider a 3-psf-graph $\tilde{G}_{P,V}$.

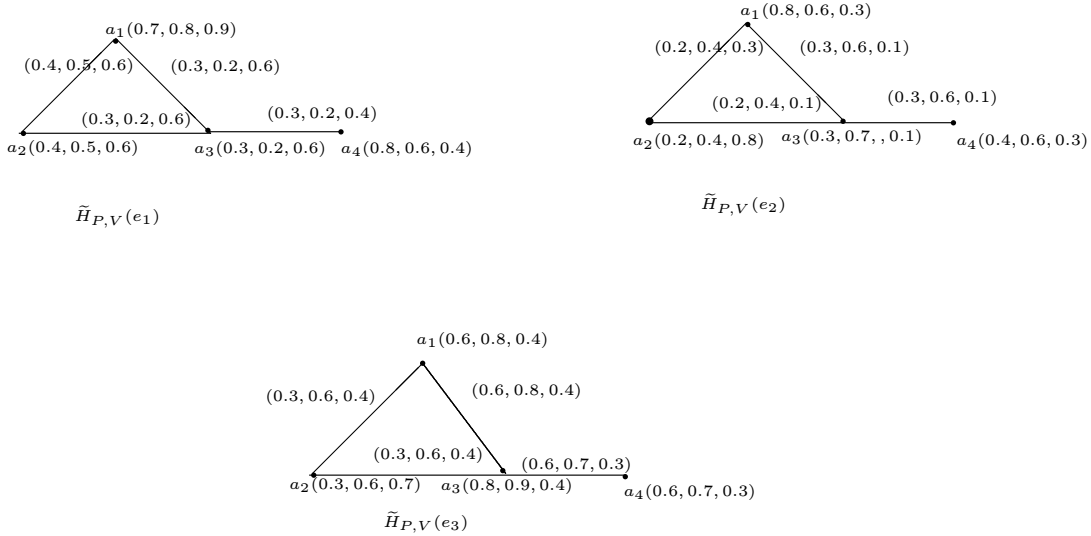


Fig.2. Minimal dominating set of 3-psf-graph

In Fig.2. Here, the minimal dominating sets in each parameterized graph is $\tilde{\mathcal{S}}_1 = \{a_1, a_4\}, \tilde{\mathcal{S}}_2 = \{a_2, a_4\}, \tilde{\mathcal{S}}_3 = \{a_3\}$.

Theorem 2.1. A dominating set $\tilde{\mathcal{S}}$ is minimal if and only if one of the below mentioned criteria holds.

- (1) $N^{hd}(a) \cap \tilde{\mathcal{S}} = \phi$.
- (2) There is a vertex $b \in V \setminus \tilde{\mathcal{S}}$ such that $N^{hd}(b) \cap \tilde{\mathcal{S}} = \{a\}$, for each $a \in \tilde{\mathcal{S}}$.

Proof. For a minimal dominating set \tilde{S} of a 3-psf-graph $\tilde{G}_{P,V}$, for every vertex $a \in \tilde{S}$, $\tilde{S} \setminus \{a\}$ is not dominating set and so then $\exists b \in V \setminus (\tilde{S} \setminus \{a\})$ which is not dominated by any vertex in $\tilde{S} \setminus \{a\}$ of $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. If $a = b$ then $N^{hd}(a) \subseteq V \setminus \tilde{S} \Rightarrow N^{hd}(a) \cap \tilde{S} = \phi$ in $\tilde{H}_{P,V}(e_i)$ for $i = 1, 2, \dots, n$. If $a \neq b$, then b is not dominated by $\tilde{S} \setminus \{a\}$ but is dominated by \tilde{S} , i.e., b is dominated only by a in \tilde{S} in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. $\therefore N^{hd}(b) \cap \tilde{S} = \{a\}$ in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Hence $N^{hd}(b) \cap \tilde{S} = \{a\}$ in 3-psf-graph $\tilde{G}_{P,V}$.

Conversely, let \tilde{S} holds one of the two given criterias. If \tilde{S} is not minimal dominating set in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Then \exists a vertex $a \in \tilde{S}$ such that $\tilde{S} \setminus \{a\}$ is a dominating set in $\tilde{H}_{P,V}(e_i)$ for $i = 1, 2, \dots, n$. Thus a is dominated by atleast one vertex in $\tilde{S} \setminus \{a\}$ in $\tilde{H}_{P,V}(e_i)$ for $i = 1, 2, \dots, n$. Then $N^{hd}(a) \not\subseteq \tilde{S} \setminus \{a\}$ in $\tilde{H}_{P,V}(e_i)$ for $i = 1, 2, \dots, n$. Hence condition (1) does not hold. Also, If $\tilde{S} \setminus \{a\}$ is a dominating set in $\tilde{H}_{P,V}(e_i)$ for $i = 1, 2, \dots, n$. Then every vertex b in $V \setminus (\tilde{S} \setminus \{a\})$ is dominated by at least one vertex in $\tilde{S} \setminus \{a\}$ in $\tilde{H}_{P,V}(e_i)$ for $i = 1, 2, \dots, n$. So $N^{hd}(b) \cap \tilde{S} \neq \{a\}$ in $\tilde{H}_{P,V}(e_i)$ for $i = 1, 2, \dots, n$. hence condition (2) does not hold. This leads to a $\Rightarrow \Leftarrow$. $\therefore \tilde{S}$ must be a minimal dominating set in $\tilde{H}_{P,V}(e_i)$ for $i = 1, 2, \dots, n$. Hence \tilde{S} must be a minimal dominating set in 3-psf-graph $\tilde{G}_{P,V}$. \square

Theorem 2.2. Let $\tilde{G}_{P,V} = ((P, \tilde{\rho}), (P, \tilde{\mu}))$ be a 3-psf-graph without isolated vertices. If \tilde{S} is a minimal dominating set of $\tilde{G}_{P,V}$ then $V \setminus \tilde{S}$ is a dominating set of $\tilde{G}_{P,V}$.

Proof. Let \tilde{S} be a minimal dominating set and $a \in \tilde{S}$. Since $\tilde{G}_{P,V}$ has no isolated vertices \exists a vertex $b \in N^{hd}(a)$ in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Utilization of the same content similar to the proof given for Theorem 2.1, we get that b in $V \setminus \tilde{S}$. Thus every element of \tilde{S} is dominated by some element of $V \setminus \tilde{S}$ in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Consequently $V \setminus \tilde{S}$ is a dominating set of $\tilde{G}_{P,V}$. \square

Theorem 2.3. Superset of a dominating set in $\tilde{G}_{P,V}$ is a dominating set.

Proof. Assume that \tilde{S} is a dominating set in $\tilde{G}_{P,V}$. Then all the vertices in $V \setminus \tilde{S}$ are adjacent to some element of \tilde{S} in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Let $b \in V \setminus \tilde{S}$. Then $\tilde{S} \cup \{b\}$ is a dominating set, Since all the vertices in $(V \setminus \tilde{S}) \setminus \{b\}$ adjacent to some elements of $\tilde{S} \cup \{b\}$. Since b is arbitrary, any superset of a dominating set is a dominating set. \square

Remark 2.1. Subset of a dominating set in $\tilde{G}_{P,V}$ need not to be dominating set.

Example 2.3. Consider a 3-psf-graph.

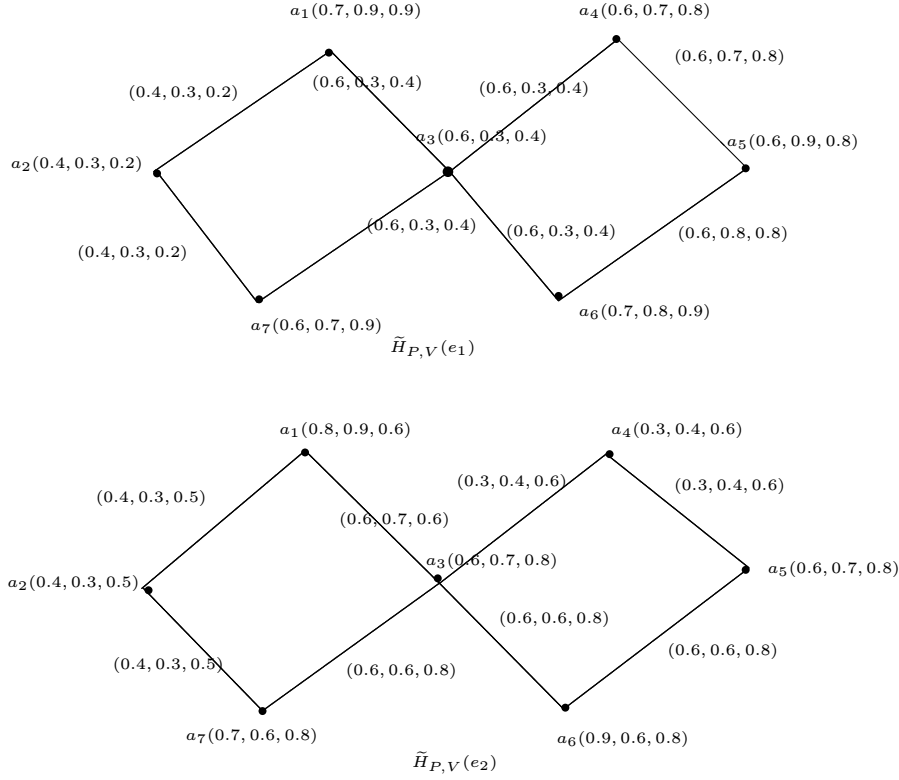


Fig.3. 3-psf-graphs

Here each parameterized graph is $\tilde{\mathcal{S}} = \{a_2, a_3, a_5\}$ and $\tilde{\mathcal{S}} \cup \{b\} = \{a_1, a_2, a_3, a_5\}$ $\tilde{\mathcal{S}} \setminus \{b\} = \{a_2, a_5\}$. Here $\{a_1, a_2, a_3, a_5\}$ is a dominating set. But $\{a_2, a_5\}$ is not a dominating set.

Definition 2.8. A set $\tilde{\mathcal{S}} \subseteq V$ of an m -psf-graph $\tilde{G}_{P,V}$ is called an independent set if

$$\begin{aligned} \tilde{\mu}_e x_1(uv) &< (\tilde{\rho}_e x_1(u) \wedge \tilde{\rho}_e x_1(v)) \\ \tilde{\mu}_e x_2(uv) &< (\tilde{\rho}_e x_2(u) \wedge \tilde{\rho}_e x_2(v)) \\ &\dots\dots\dots \\ \tilde{\mu}_e x_m(uv) &< (\tilde{\rho}_e x_m(u) \wedge \tilde{\rho}_e x_m(v)) \end{aligned}$$

in $\tilde{H}_{P,V}(e) \forall e \in P$ and for all $u, v \in \tilde{\mathcal{S}}$.

Definition 2.9. An independent set $\tilde{\mathcal{S}}$ of an m -psf-graph $\tilde{G}_{P,V} = (G^*, \tilde{\rho}, \tilde{\mu}, P)$ is said to be maximal independent set if for every vertex $u \in V \setminus \tilde{\mathcal{S}}$, the set $\tilde{\mathcal{S}} \cup \{u\}$ is not independent in $\tilde{H}_{P,V}(e) \forall e \in P$.

Definition 2.10. Lower and upper independence number of an m -psf-graph $\tilde{G}_{P,V} = (G^*, \tilde{\rho}, \tilde{\mu}, P)$ is denoted by $i(\tilde{G}_{P,V})$ and $I(\tilde{G}_{P,V})$, respectively, and defined by infimum cardinality and supremum cardinality among all the maximum independent set of that m -psf-graph, respectively.

Example 2.4. Consider a 3-psf-graph $\tilde{G}_{P,V}$.

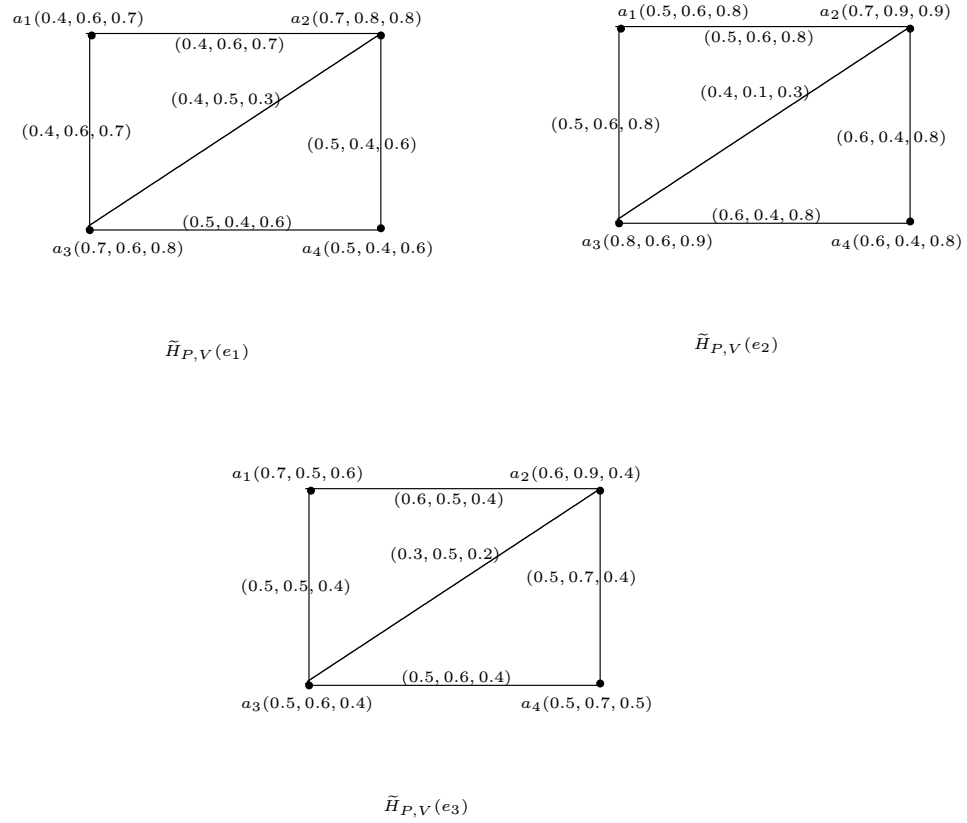


Fig.4. Independent set of a 3-psf-graphs

In this example, Here $\{a_2, a_3\}$ is an independent set in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, 3$. Similarly, other independent sets in each parameterized graph is $\{\{a_3, a_4\}, \{a_2, a_3, a_4\}, \{a_2, a_4\}, \{a_1, a_4\}\}$. The maximal independent set in each parameterized graph is $\{\{a_1, a_4\}, \{a_2, a_3, a_4\}\}$. Here $\{a_1, a_4\}$ is a maximal independent set of $\tilde{G}_{P,V}$ with infimum cardinality and $i(\tilde{G}_{P,V}) = (3.2, 3.2, 4.0)$. Also here $\{a_2, a_3, a_4\}$ is maximal independent set of $\tilde{G}_{P,V}$ with supremum cardinality and $I(\tilde{G}_{P,V}) = (5.6, 5.9, 6.1)$.

Proposition 2.1. For any 3-psf-graph $\tilde{G}_{P,V} = (G^*, \tilde{\rho}, \tilde{\mu}, P)$, $\gamma(\tilde{G}_{P,V}) \leq i(\tilde{G}_{P,V})$.

Example 2.5. Consider a 3-psf-graph.

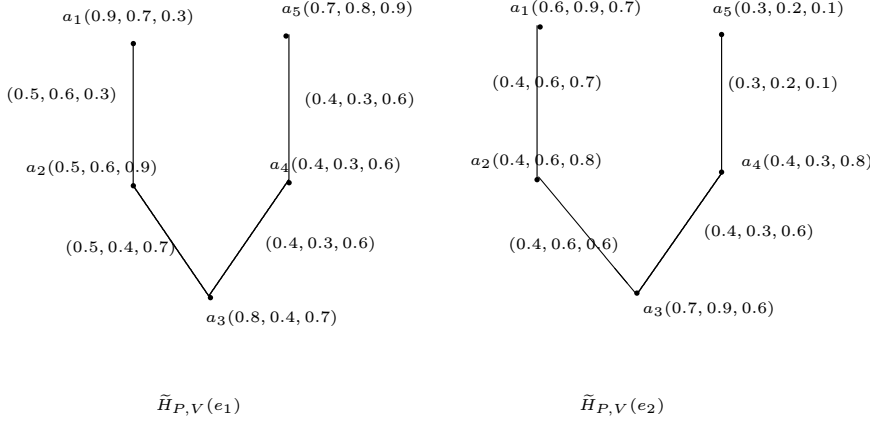


Fig.5. Independent set of a 3-psf-graphs

In Fig.5. that minimum dominating set of a 3-psf-graph $\tilde{G}_{P,V}$ is $\{a_2, a_4\}$ and the maximal independent set of $\tilde{G}_{P,V}$ is $\{a_1, a_3, a_5\}$ in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2$. Also here $\gamma(\tilde{G}_{P,V}) = (1.7, 1.8, 3.1)$ and $i(\tilde{G}_{P,V}) = (4.0, 3.9, 3.3)$ in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2$. Clearly, $\gamma(\tilde{G}_{P,V}) \leq i(\tilde{G}_{P,V})$.

Theorem 2.4. A set $\tilde{\mathcal{S}} \subseteq V$ is a maximal independent set of a 3-psf-graph $\tilde{G}_{P,V}$ if and only if it is independent and dominating set.

Proof. Assume that $\tilde{\mathcal{S}}$ is a maximal independent set of $\tilde{G}_{P,V}$. Then for each vertex $a \in V \setminus \tilde{\mathcal{S}}$, the set $\tilde{\mathcal{S}} \cup \{a\}$ is not independent set in $\tilde{H}_{P,V}(e_i)$ for $i = 1, 2, \dots, n$. In this manner for every vertex $a \in V \setminus \tilde{\mathcal{S}}$, then \exists vertex $b \in \tilde{\mathcal{S}}$ such that b dominates a in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Hence $\tilde{\mathcal{S}}$ is a dominating set in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Therefore $\tilde{\mathcal{S}}$ is both independent and dominating set in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Hence $\tilde{\mathcal{S}}$ is both independent and dominating set in $\tilde{G}_{P,V}$.

Conversely if we suppose that $\tilde{\mathcal{S}}$ is not maximal independent set in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Then \exists a vertex $a \in V \setminus \tilde{\mathcal{S}}$, such that $\tilde{\mathcal{S}} \cup \{a\}$ is independent set. Thus there \nexists any vertex b in $\tilde{\mathcal{S}}$ which dominates a in $\tilde{H}_{P,V}(e_i)$ for $i = 1, 2, \dots, n$. So $\tilde{\mathcal{S}}$ is not a dominating set in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Which $\Rightarrow \Leftarrow$ to the choice of $\tilde{\mathcal{S}}$. Accordingly $\tilde{\mathcal{S}}$ is a maximal independent set in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Hence $\tilde{\mathcal{S}}$ is a maximal independent set of $\tilde{G}_{P,V}$. \square

Theorem 2.5. In a 3-psf-graph $\tilde{G}_{P,V} = ((P, \tilde{\mu}), (P, \tilde{\rho}))$, every maximal independent set is a minimal dominating set

Proof. For a maximal independent \tilde{S} of $\tilde{G}_{P,V}$. By Theorem 2.4, \tilde{S} is dominating set in $\tilde{G}_{P,V}$. If we consider \tilde{S} being not minimal dominating set in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Then \exists atleast one vertex $a \in \tilde{S}$ so that $\tilde{S} \setminus \{a\}$ is a dominating set that means $\tilde{S} \setminus \{a\}$ dominates $V \setminus (\tilde{S} \setminus \{a\})$. Thus, there exists atleast one vertex in \tilde{S} which dominates a . This contradict our assumption. Therefore \tilde{S} is a minimal dominating set in $\tilde{H}_{P,V}(e_i) \forall e_i \in P$ for $i = 1, 2, \dots, n$. Hence \tilde{S} is a minimal dominating set in $G_{P,V}$. \square

3 Conclusions

The multiplicity of applications and vast range of domination parameters that can be defined make domination theory research attractive. The terms dominating set, independent set, dominance number etc. are used in this study that proposed the m -polar soft fuzzy graphs, and several intriguing findings have been demonstrated. In a similar situation, future studies can define and examine additional domination parameters.

References

- [1] M. I. Ali. *A note on soft sets, rough sets and fuzzy soft sets*, Applied soft computing ,11(2011), 3329-3332.
- [2] A. Somasundaram and S. Somasundaram. *Domination in fuzzy graph-I*. Pattern Recognition Letter, 19 (9):77-95, 1998.
- [3] Mohinto Sumit, T. K. Samanta. *An Introduction to Fuzzy Soft Graph*, Mathematica Moravica, 2015, vol. 19, br. 3, str 35-48.
- [4] M. Akram and B. Davvaz. *Certain types of domination in m -polar fuzzy graphs*. Journal of Multiple-Valued Logic and Soft Computing, 29 (6),2017.
- [5] A. Rosenfeld, L. A. Zadeh, K. S. Fu, and M. Shimura. [Eds] *Fuzzy graphs in Fuzzy sets and their Applications*, Academic Press, New York 1975, 77-95.
- [6] S. Ramkumar, R. Sridevi. *Proper m -polar Soft fuzzy Graphs*, Adv.Math.,Sci.J. 10(2021), No.4, 1845-1856.