Results on Relatively Prime Domination Number of Vertex Switching of Some Graphs

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Abstract

If a set $S \subseteq V$ has at least two members and every pair of vertices u and v is such that $(d(u), d(v)) = 1$, then it is said to be a relatively prime dominating set. The relatively prime domination number, represented by $\gamma_{rpd}(G)$, is the lowest cardinality of a relatively prime dominating set. The switching of a finite undirected graph by a subset is defined as the graph $G^{\sigma}(V, E')$, which is obtained from G by removing all edges between σ and its complement $V - \sigma$ and adding as edges all non-edges between σ and $V - \sigma$. In this paper, we compute the relatively prime domination number of vertex switching of cycle type graphs like David Star Graph, Helm Graph, Friendship Graph and Book Graph .

Keywords: Dominating set, relatively prime dominating set, vertex switching.
2020 AMS subject classifications:05C69.[1]

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1 Introduction

In the realm of graph theory, a finite undirected graph without loops and multiple edges is typically denoted as $G = (V, E)$, where G's order and size are represented by the symbols $|V|(p)$ and $|E|(q)$, respectively. This foundational terminology is commonly cited in literature, with references to Harary[2] for graph theoretical concepts and Haynes [3] for domination-related terminology.

One fundamental concept in graph theory is the notion of domination sets. In a subset $V - S$ of vertices V, each vertex is considered adjacent to at least one vertex in set S if S functions as a dominating set in graph G. The smallest cardinality of such a dominating set in G is referred to as the domination number, denoted as $\gamma(G)$. This concept traces its origins to the works of Berge [2] and Ore [9], paving the way for a plethora of other domination-related graph metrics.

In our study, we consider nontrivial graphs and introduce the concept of relatively prime dominating sets. Specifically, if a set S is dominant, and every pair of its vertices u and v share a greatest common divisor of 1 (i.e) $((d(u), d(v)) = 1)$, then the set is termed relatively prime. The corresponding parameter, denoted as $\gamma_{rpd}(G)$ [5], represents the minimum cardinality of a relatively prime dominating set. Furthermore, our previous research [6, 7] introduced the notion of a substantially prime dominating polynomial.

Building on this foundation, we delve into the concept of switching in graphs, originally introduced by Lint and Sidel [8]. By switching vertices within the graph, we obtain a transformed graph $G^{\sigma}(V, E^{'})$, where edges between σ and its complement $V - \sigma$ are removed, and all non-edges between σ and $V - \sigma$ are added. This transformation, often referred to as vertex switching when $\sigma = v$ and G^{v} is used in place of G[4], plays a crucial role in our investigation.

In this paper, we venture into a novel direction by introducing the concept of relatively prime dominating sets (RPDS) in nontrivial graphs. The idea of RPDS emerges from the observation that in some scenarios, it is desirable for the vertices in a dominating set to be relatively prime with respect to their degrees. This concept not only enriches our understanding of domination in graphs but also has intriguing implications in various practical scenarios, such as resource allocation in networks.

Furthermore, our prior research introduced the substantially prime dominating polynomial, which offers an interesting connection between algebraic and combinatorial aspects of domination. This polynomial provides a powerful tool for analyzing the structure of dominating sets in graphs.

In the following sections of this paper, we narrow our focus to a specific class of graphs, the cycle-type graphs, which include well-known examples such as the David Star Graph, Helm Graph, Friendship Graph, and Book Graph. By studying these cycle-type graphs, we aim to uncover patterns and properties related to relatively prime domination numbers resulting from vertex switching. Our primary objective is to provide a comprehensive understanding of how the interplay of vertices and the unique characteristics of these graphs affect domination, shedding light on the broader implications of our findings.

2 Definition and examples

Definition 2.1. *For a finite undirected graph G(V, E) and $v \in V$, the vertex switching of G by v is the graph G^v which is obtained from G by removing all edges incident to v and adding edges which are not adjacent to v.*

Example 2.1. *The graphs C_5 and C_5^v are given in figures 2. 1 and 2. 2, respectively. Clearly, $\{u, x\}$ is a minimal relatively prime dominating set of C_5^v and hence $\gamma_{rpd}(C_5^v)$ = 2.*

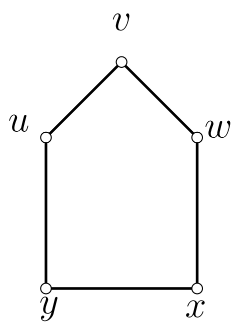


Fig.2.1.C_5

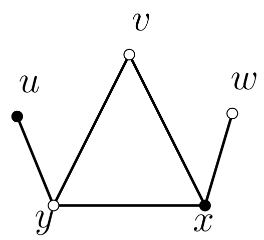


Fig.2.2.C_5^v

Example 2.2. *Consider the graph G given in figure 2. 3. The graph G^{v_1} is given in figure 2. 4. Clearly, $\{v_1, v_2, v_4\}$ is a dominating set of G^{v_1}. Also $(d(v_1), d(v_2))$ = (3, 1) = 1; $(d(v_1), d(v_4))$ = (3, 2) = 1 and $(d(v_2), d(v_4))$ = (1, 2) = 1. By definition, $\{v_1, v_2, v_4\}$ is a relatively prime dominating set of G^{v_1}. Also $\{v_1, v_2, v_4\}$ is a minimal dominating set with this property and hence $\gamma_{rpd}(G^{v_1})$ = 3. But $\gamma(G^{v_1})$ = 2, since $\{v_3, v_6\}$ is the minimal dominating set.*

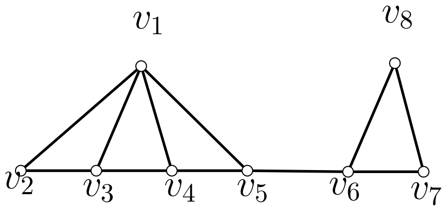


Fig.2.3.G

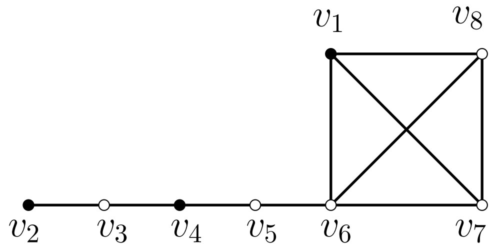


Fig.2.4.G^{v_1}

Definition 2.2. [10] Let $G = (V, E)$ be a graph and let x, y, z be three variables taking values + or -. The transformation graph G^{xyz} is the graph having $V(G) \cup E(G)$ as the vertex set, and for $\alpha, \beta \in V(G) \cup E(G)$, α and β are adjacent in G^{xyz} if and only if one of the following holds:

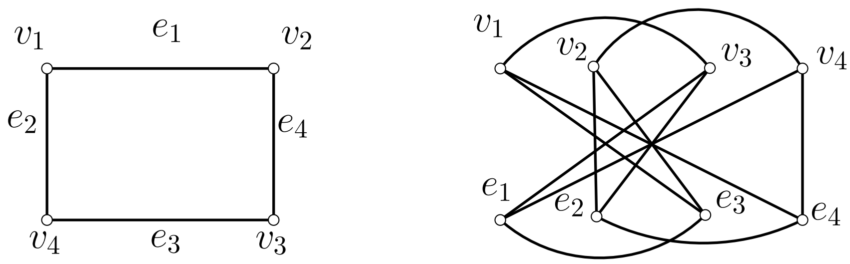
(i) For $\alpha, \beta \in V(G)$, α and β are adjacent in G if x = + ; α and β are not adjacent in G if $x = -$.

(ii) For $\alpha, \beta \in E(G)$, α and β are adjacent in G if y = + ; α and β are not adjacent in G if $y = -$.

(iii) For $\alpha \in V(G), \beta \in E(G)$, α and β are incident in G if z = + ; α and β are not incident in G if $z = -$.

Thus, we may obtain eight kinds of transformation graphs, in which G^{+++} is the total graph of G, and G^{---} is its complement. Also, G^{--+}, G^{-+-} and G^{-++} are the complements of G^{++-}, G^{+-+} and G^{+--} respectively.

Example 2.3. *The graph $G = C_4$ and G^{---} are given in figure 2. 5.*



$G = C_4$ $\quad$ G^{---}

$Fig.2.5$

Definition 2.3. *By adding a pendent edge at each node of the cycle, the wheel graph can be converted into the Helm graph H_n.*

Definition 2.4. *By combining n duplicates of the cycle graph C_3 with a common vertex, the friendship graph F_n can be created.*

Definition 2.5. *The Cartesian product of a star and a single edge is the book graph B_m. The definition of the $m-$book graph is the graph Cartesian product S_{m+1} times P_2, where $S_m + 1$ is a star graph and P_2 is a path graph on two vertices.*

Theorem 2.1. [5] $\gamma_{rpd}(G) = 0$ if a graph G has an isolated vertex and each dominating set contains at least one vertex with degree > 1.

Result 2.1. [5]For any graph G with more than one isolated vertex, $\gamma_{rpd}(G)$= 0.

Theorem 2.2. [5]For a complete bipartite graph $K_{m,n}$, $\gamma_{rpd}(K_{m,n})$= 2 if and only if (m, n) = 1.

Theorem 2.3. [5] $\gamma_{rpd}(K_m \cup K_n)$ = $\begin{cases} 2\ if\ and\ only\ if\ (m-1, n-1) = 1 \\ 0\ otherwise \end{cases}$.

Theorem 2.4. [5]If $G = nK_2 \cup K_1$, then $\gamma_{rpd}(G) = n + 1$.

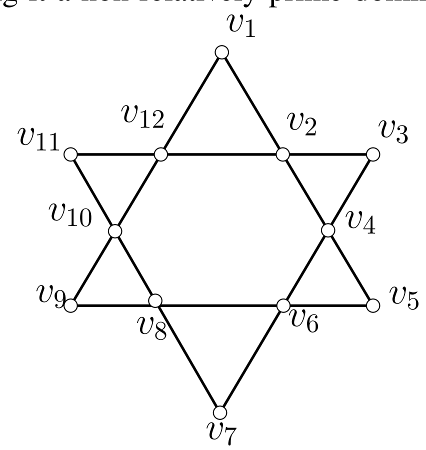
Notation : We use the symbols $d(u)$ and $d'(u)$ to denote the degree of a vertex u in G and G^v, respectively.

3 Main Results

Theorem 3.1. *Let G be the David Star graph and let v be any vertex of G. Then,* $\gamma_{rpd}(G^v) = \begin{cases} 3\ if\ d(v) = 4 \\ 0\ if\ d(v) = 2 \end{cases}$.

Proof. Label the vertices of G having degree 2 as $v_1, v_3, ..., v_{11}$ and degree 4 as $v_2, v_4, ..., v_{12}$ such that $v_{2j}v_{2j+2}, v_2v_{12}, v_iv_{i+1}$ and v_1v_{12} are edges in $G, 1 \leq j \leq 5, 1 \leq i \leq 11$. The graph G is given in figure 3.1.
Case 1. $d_G(v) = 2$

In this case v is $v_i, i = 2n + 1, 0 \leq n \leq 5$. Clearly, $G^{v_1} \cong G^{v_3} \cong ... \cong G^{v_{11}}$. Let v be v_1. The graph G^{v_1} is given in figure 3.2. In G, v_1 is adjacent to v_2 and v_{12}. This implies v_1 is adjacent to all the vertices except v_2 and v_{12} in G^{v_1}. Also v_2 and v_{12} are adjacent. Hence either $\{v_1, v_2\}$ or $\{v_1, v_{12}\}$ is a minimal dominating set of G^{v_1} and $d'(v_1)$ = 9, $d'(v_2)$ = 3 and $d'(v_{12})$ = 3. Now, $(d'(v_1), d'(v_2))$ = (9, 3) = 3 and $(d'(v_1), d'(v_{12}))$ = (9, 3) = 3. This implies that, neither $\{v_1, v_2\}$ nor $\{v_1, v_{12}\}$ is a minimal relativly prime dominating set of G^{v_1} and hence $\gamma_{rpd}(G^{v_1})$ is either 0 or greater than 2. Any dominating set that has more than two vertices either has two vertices with degrees 3 and 9 or at least a pair of vertices of the same degree, making it a non-relatively prime dominating set. Hence, $\gamma_{rpd}(G^{v_1})$ = 0.



$Fig.3.1.\ G$

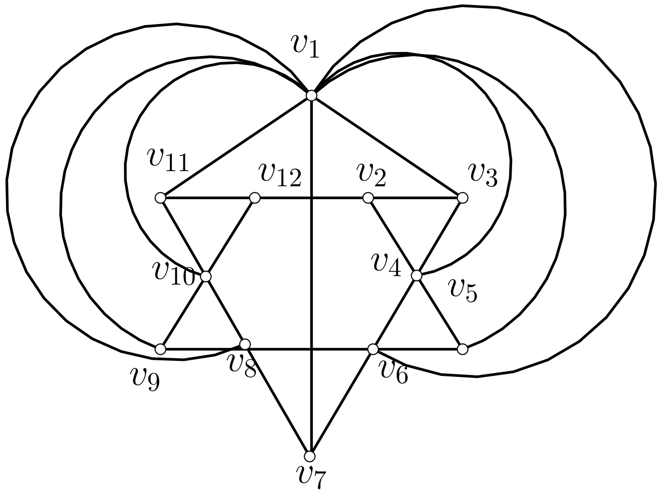


Fig.3.2. G^{v_1}

Case 2. $d_G(v) = 4$

Here v is $v_i, i = 2n+2, 0 \leq n \leq 5$. Clearly, $G^{v_2} \cong G^{v_4} \cong ... \cong G^{v_{12}}$. Let v be v_2. The graph G^{v_2} is given in figure 3.3. In G, v_2 is adjacent to v_1, v_3, v_4 and v_{12} only and hence v_2 is adjacent to all vertices except v_1, v_3, v_4 and v_{12} in G^{v_2}. Also v_1 is adjacent to v_{12} and v_3 is adjacent to v_4. Therefore, $\{v_1, v_2, v_3\}$ is a minimal dominating set of G^{v_2}. Clearly, $d'(v_1) = d'(v_3)$ = 1, $d'(v_2)$ = 7 and $(d'(v_1), d'(v_2)) =$ $(d'(v_2), d'(v_3))$ = $(d'(v_1), d'(v_3))$ = 1. This implies that, $\{v_1, v_2, v_3\}$ is a minimal relatively prime dominating set of G^{v_2} and thereby $\gamma_{rpd}(G^{v_2})$ = 3.

The theorem follows from cases 1 and 2. □

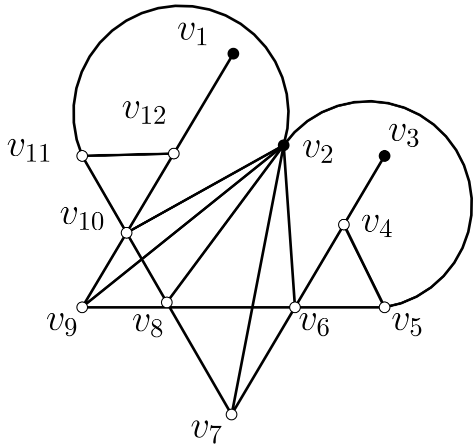


Fig.3.3. G^{v_2}

Theorem 3.2. *For the helm graph H_n,*

$$\gamma_{rpd}(H_n^v) = \begin{cases} 2 \; if \; v \; is \; an \; end \; vertex \; of \; H_n \; and \; 2n \neq 3r+1, 5r+1 \\ \quad where \; r \geq 1 \; is \; odd \; and \; n \neq 3r-1, r \geq 1 \\ 0 \; otherwise \end{cases}.$$

Proof. Let u be centre and $v_1, v_2, ..., v_n$ be vertices of the outer cycle of H_n. Let u_i be the vertex attached with $v_i, 1 \leq i \leq n$. The resultant graph G is H_n with $V(G) = \{u, v_i, u_j/1 \leq i, j \leq n\}$ and $E(G) = \{uv_i, uv_n, v_iv_{i+1}, v_1v_n, v_iu_i, v_nu_n/ 1 \leq i \leq n-1\}$. We consider three cases.

Case 1. v is the central vertex of H_n

Here v is u. The graph H_4 ios given in figure3 3.4 and the graph H_4^u.

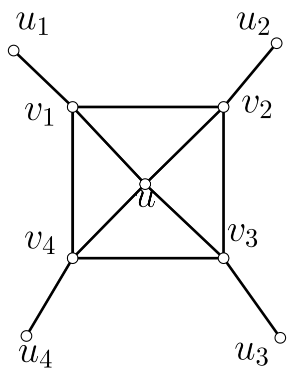


Fig.3.4. H_4

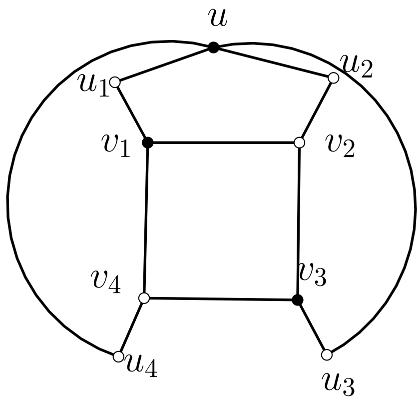


Fig.3.5. H_4^u

In H_n, u is adjacent to $v_i, 1 \leq i \leq n$ and non-adjacent to $u_i, 1 \leq i \leq n$. Hence u is adjacent to u_i and non-adjacent to v_i in H_n^u, $1 \leq i \leq n$. Clearly, $d'(u) = n$, $d'(u_i) = 2$, $d'(v_i) = 3$, $1 \leq i \leq n$. Now, if n is odd, then $\{u, v_1, v_3, ..., v_{n-2}\}$

is a minima1 dominating set of H_n^u and if n is even, then $\{u, v_1, v_3, ..., v_{n-1}\}$ is a minima1 dominating set of H_n^u. Clearly, the minimal dominating sets contain at least two vertices having equal degrees. Hence neither $\{u, v_1, v_4, ..., v_{n-2}\}$ nor $\{u, v_1, v_4, ..., v_{n-1}\}$ is a minimal relatively prime dominating set of H_n^u. Hence $\gamma_{rpd}(H_n^u)$ = 0.

Case 2. v is a vertex of the outer cycle of H_n

In this case v is v_i, $1 \leq i \leq n$. Now $H_n^{v_i}$ is a disconnected graph with the isolated vertex u_i and all other vertices have degree > 1. By Theorem 2.9, $\gamma_{rpd}(H_n^{v_i})$ = 0.

Case 3. v is an end vertex of H_n

Here v is u_i, $1 \leq i \leq n$. Let v be u_1. The graph $H_4^{u_1}$ is given in figure 3.6.

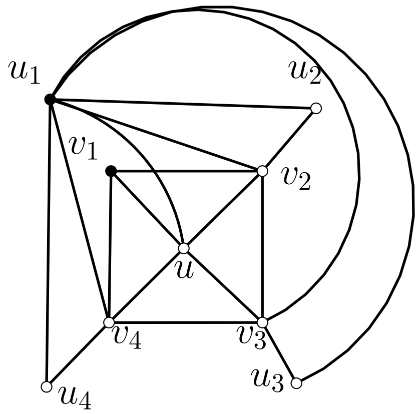


Fig.3.6.$H_4^{u_1}$

In H_n, u_1 is adjacent to v_1 and hence u_1 is adjacent to all vertices except v_1 in $H_n^{u_1}$. Therefore, $\{u_1, v_1\}$ is a minimal dominating set of $H_n^{u_1}$. Also v_1 is adjacent to u, v_2 and v_n in $H_n^{u_1}$. Hence $\{u_1, u\}, \{u_1, v_2\}$ and $\{u_1, v_n\}$ are also a minimal dominating sets of $H_n^{u_1}$. Clearly, $d'(u_1) = 2n-1$, $d'(v_1)$ = 3, $d'(u) = n+1$, $d'(v_i)$ = 5, $2 \leq i \leq n$, $d'(u_i) = 2, 2 \leq i \leq n$. Therefore, $(d'(u_1), d'(v_1)) = (2n-1, 3)$ = 1 if $2n - 1 \neq 3r$. This implies that $2n \neq 3r + 1$. Since $2n$ is even, r must be odd. Therefore, $(2n - 1, 3) = 1$ if $2n \neq 3r + 1$ where $r \geq 1$ is odd. This implies that $\{u_1, v_1\}$ is a minimal relatively prime dominating set of $H_n^{u_1}$ and hence $\gamma_{rpd}(H_n^{u_1})$ = 2 when $2n \neq 3r + 1$ and $r \geq 1$ is odd. Also, $(d'(u_1), d'(v_2))$ = $(d'(u_1), d'(v_n))$ = $(2n - 1, 5)$ = 1 if $2n - 1 \neq 5r$. This implies that $2n \neq 5r + 1$. Since $2n$ is even, r must be odd. Therefore, $(2n - 1, 5)$ = 1 if $2n \neq 5r + 1$ where $r \geq 1$ is odd. This implies that $\{u_1, v_2\}$ and $\{u_1, v_n\}$ are minimal relatively prime dominating sets of $H_n^{u_1}$ and hence $\gamma_{rpd}(H_n^{u_1})$ = 2 when $2n \neq 5r + 1$ and $r \geq 1$ is odd. Now $(d'(u_1), d'(u)) = (2n - 1, n + 1)$ = 1 if $n \neq 3r - 1, r \geq 1$. Therefore, $\{u_1, u\}$ is a minimal relatively prime dominating set of $H_n^{u_1}$ if $n \neq 3r - 1$ and so $\gamma_{rpd}(H_n^{u_1})$

= 2.

The theorem follows from cases 1, 2 and 3. □

Theorem 3.3. *Let G = P_m^{+++} and v be an end vertex of P_m, then $\gamma_{rpd}(G^v)$ =2.*

Proof. Let $v_1v_2...v_m$ be the path $P_m, m \geq 2$ and let $e_i = v_ivi + 1$ be the edges of $P_m, 1 \leq i \leq m-1$. Then $V(P_m^{+++})$ = $\{v_1, v_2, ..., v_m, e_1, e_2, ..., e_{m-1}\}$ and hence G has $2m - 1$ vertices. We consider two cases.

Case 1. $m = 2$

In this case v is either v_1 or v_2 and G is K_3. Clearly, $G^v = K_1 \cup K_2$ where K_1 is v. Clearly $\{v, e_1\}$ is a minimal relatively prime dominating set of G^v and hence by Theorem 2. 12, $\gamma_{rpd}(G^v)$= 2.

Case 2. $m \geq 3$

Here v is either v_1 or v_n. Without loss of generality, let v be v_1. In G, v_1 is adjacent to v_2 and e_1 and hence v_1 is non-adjacent to v_2 and e_1 in G^v. Also v_2 is adjacent to e_1 in G^v. Therefore, $\{v_1, v_2\}$ is a minimal dominating set of G^v. In G^v, $d'(v_1) = 2m - 4 = 2(m - 2)$ is even and $d'(v_2)$ = 3 is odd and hence $(d'(v_1), d'(v_2))$ = 1. Thus $\{v_1, v_2\}$ is a minimal relatively prime dominating set of G^v and hence $\gamma_{rpd}(G^v)$ = 2. □

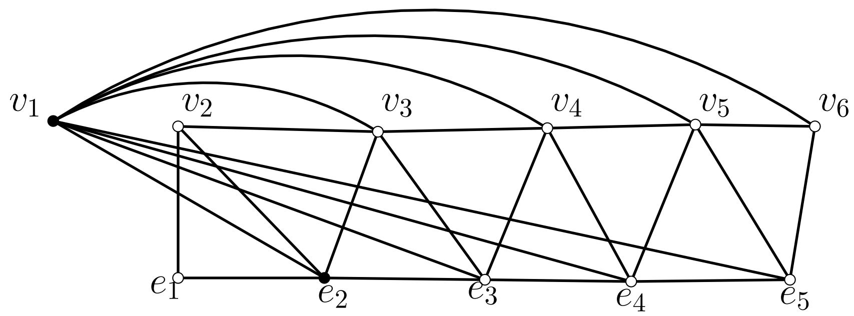


Fig.3.7.G^v where $G = P_6^{+++}$

Theorem 3.4. *Let G be a friendship graph F_n and let v be any vertex of G. Then,*

$$\gamma_{rpd}(F_n^v) = \begin{cases} n+1 \; if \; v \; is \; the \; central \; vertex \; of \; F_n \\ 2 \; otherwise \end{cases}.$$

Proof. Let v_{i1}, v_{i2}, v_{i3} be the vertices of i^{th} copy of cycle C_3. Identify the vertices $v_{i3}, 1 \leq i \leq n$ and denote it by u. The resultant graph G is the friendship graph F_n with vertex set $V(F_n)$ = $\{v_{i1}, v_{i2}, u/1 \leq i \leq n\}$ and edge set $E(G) = \{uv_{i1}, uv_{i2}, v_{i1}v_{i2}/1 \leq i \leq n\}$. Clearly, $d(v_{i1}) = d(v_{i2}) = 2, 1 \leq i \leq n$ and $d(u) = 2n$. The graphs $F_4, F_4^{v_{11}}$ and F_4^u are given in figures 3.8, 3.9 and 3.10 respectively.

Case 1. v is the central vertex of F_n

Here v is u and $F_n^u = nK_2 \cup K_1$. By Theorem 2. 13, $\gamma_{rpd}(F_n^u)$= $n + 1$.

Case 2. v is not the central vertex of F_n

Here v is $v_{ij}, 1 \leq i \leq n, j = 1, 2$. Let v be v_{11}. In F_n, v_{11} is adjacent to both v_{12} and u and hence v_{11} is adjacent to every vertices except v_{12} and u in $F_n^{v_{11}}$. Since u and v_{12} are adjacent in $F_n^{v_{11}}$, $\{u, v_{11}\}$ is a minimal dominating set of $F_n^{v_{11}}$. Clearly, $d'(u) = 2n-1, d'(v_{11}) = 2n-2$ and $(d'(u), d'(v_{11})) = (2n-1, 2n-2) = 1$. Thus $\{u, v_{11}\}$ is a minimal relatively prime dominating set of $F_n^{v_{11}}$ and hence $\gamma_{rpd}(F_n^v)$= 2.

The theorem follows from cases 1 and 2. □

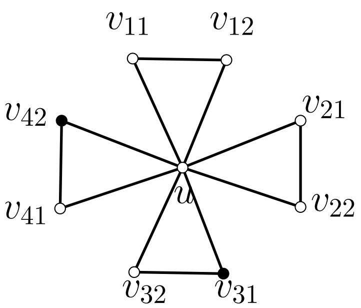


Fig.3.8.F_4

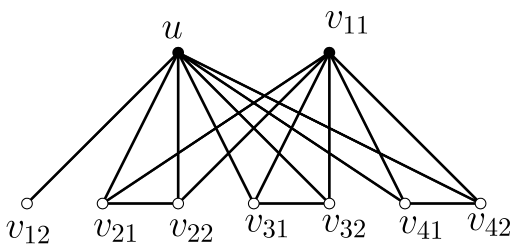


Fig.3.9.$F_4^{v_{11}}$

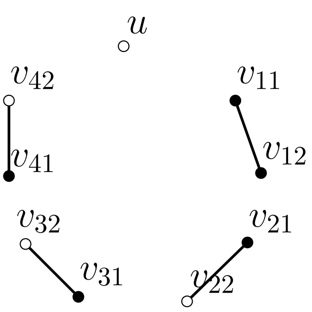


Fig.3.10.F_4^u

Theorem 3.5. *Let G be the book graph $B_m, m \geq 2$ and v be any vertex of G.*

(i) If $d(v)$ = 2 and $m \equiv 3(mod\ 5)$, then $\gamma_{rpd}(G^v)$= 3.

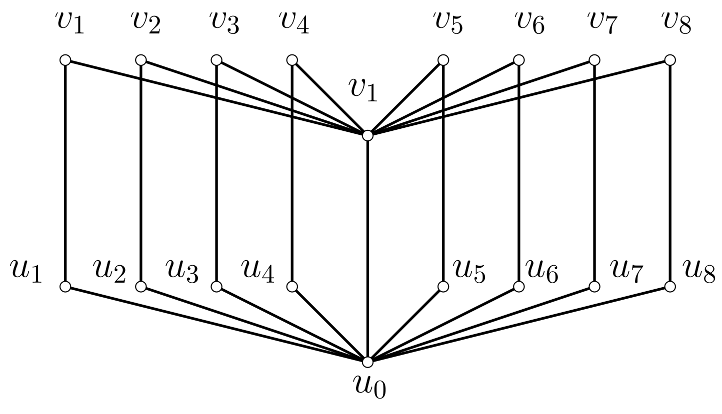
(ii) If $d(v)$ = 2 and $m \not\equiv 3(mod\ 5)$, then $\gamma_{rpd}(G^v)$= 2.

(iii) If $d(v) = m$, then $\gamma_{rpd}(G^v)$ = m.

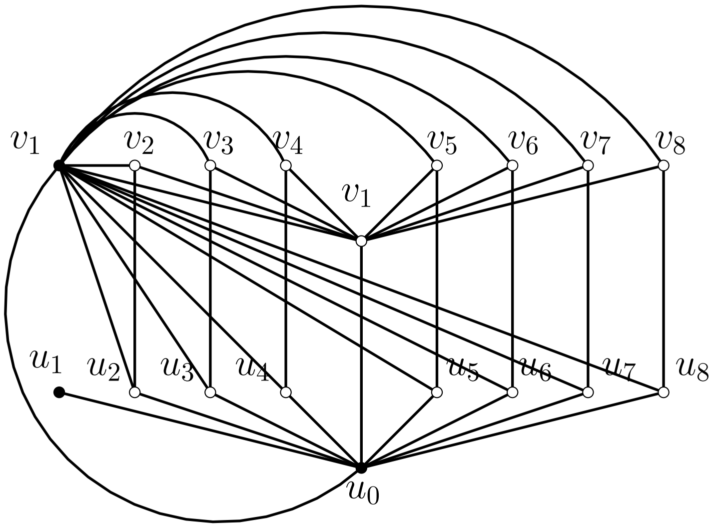
Proof. Let $v_0, v_1, ..., v_m$ and $u_0, u_1, ..., u_m$ be the two copies of star $K_{1,n}$ with central vertices v_0 and u_0 respectively. Join u_i with v_i for all $i, 1 \leq i \leq m$. The resultant graph G is B_m with vertex set $V(G) = \{v_0, u_0, v_i, u_i/1 \leq i \leq m\}$ and edge set $E(G) = \{u_0v_0, u_iv_i, v_0v_i, u_0u_i/1 \leq i \leq m\}$. Then G has $2m+2$ vertices, $3m+1$ edges, $d(v) = 2$ if $v \in \{u_i, v_i/1 \leq i \leq m\}$ and $d(v) = m$ if $v \in \{u_0, v_0\}$. The graphs $G = B_8, G^{v_1}$ and G^{v_0} are given in figures 3.11, 3.12 and 3.13 respectively.

Case 1. $d(v) = 2$

In this case v is either v_i or $u_i, 1 \leq i \leq m$. Clearly, $G^{v_i} \cong G^{u_i}$. Let v be v_1. In G, v_1 is adjacent to only v_0 and u_1 and hence v_1 is adjacent to all vertices except u_1 and v_0 in G^{v_1}. But u_0 is adjacent to both u_1 and v_0 in G^{v_1}. Clearly, $\{v_1, u_0\}$ is the minimal dominating set of G^{v_1} and hence $\gamma(G^{v_1})$= 2. Now, $d'(v_1) = 2m-1, d'(u_0) = m+2$ and $(d'(v_1), d'(u_0)) = (2n-1, n+2)$. If n is even, then $2m-1$ is odd and so $(2m-1, m+2)$ = 1. This implies that $\gamma_{rpd}(G^{v_1})$= 2. If n is odd, then both $2n-1$ and $n+2$ are odd. Also $2m-1 > m+2$ for $m > 3$ and $2m-1 = m+2$ for $m = 3$. If $m \not\equiv 3(mod\, 5)$ which implies that $m = 3+5k+c$ where $1 \leq c \leq 4$. Then $n+2 = 5(k+1)+c$ and $2n-1 = 5(2k+1)+2c$. Since $c \not\equiv 0(mod\, 5), 2c \not\equiv 0(mod\, 5)$ and so $(2m-1, m+2)$ = 1 and hence $\gamma_{rpd}(G^{v_1})$= 2. Suppose $n \equiv 3(mod\, 5)$, then $(d'(v_1), d'(u_0)) = (2m-1, m+2) = (2(5r+3)-1, (5r+3)+2) = (10r+5, 5r+5)$ = 5$\neq$ 1. This implies that $\{v_1, u_0\}$ is not a relatively prime dominating set of G^{v_1}. Consider the dominating set $\{v_1, u_1, v_0\}$ of G^{v_1} in which $d'(v_1) = 2m-1, d'(u_1) = 1, d'(v_0) = m$. Clearly, $(d'(u_1), d'(v_1)) = (d'(u_1), d'(v_0)) = (d'(v_1), d'(v_0))$ = 1 if $m \equiv 3(mod\, 5)$. This implies that $\{v_1, u_1, v_0\}$ is a minimal relatively prime dominating set of G^{v_1} and hence $\gamma_{rpd}(G^{v_1})$= 3.



$Fig.3.11.G = B_8$

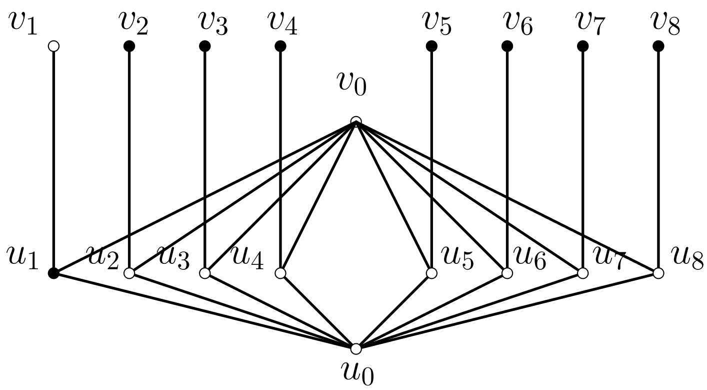


$Fig.3.12.G^{v_1}$

Case 2. $d_G(v) = m$

Here v is either u_0 or v_0. Clearly, $G^{v_0} \cong G^{u_0}$. A minimal dominating set of G^{v_0} is $\{u_1, v_2, ..., v_m\}$. Clearly, $d'(u_1)$= 3 and $d'(v_i)$ = 1, $1 \leq i \leq m$. This implies that $(d'(u_1), d'(v_i))$ = 1 and so $\{u_1, v_2, ..., v_m\}$ is a minimal relatively prime dominating set of G^{v_0}. Hence $\gamma_{rpd}(G^{v_0}) = m$.

The theorem follows from cases 1 and 2. □



$Fig.3.13.\ G^{v_0}$

Conclusion

As a result, we have demonstrated how to identify the relatively prime domination number of vertex switching graphs in this study. Additionally, we determine the relatively prime domination number for vertex switching in cycle type graphs such the David Star, Helm, Friendship, and Book graphs.

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