# Algorithm approaches for shortest path problem in an interval - valued triangular Pythagorean fuzzy network

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**Abstract**

In this paper, it has been proposed to use Pythagorean fuzzy sets (PFS), is an extensionof fuzzy sets (FSs), to address uncertainty in practical decision– making problems. Then the Shortest Path Problem (SPP) is a well– known network improvement issue with numerous practical applications.Then the shortest Path (SP) and the shortest distance (SD)in an Interval – valued Pythagorean fuzzy graph (I-VPFG) are foundusing a method in the current communication. Nodes and connectionsare crisp, while the edge weights are I-V triangular Pythagoreanfuzzy numbers (I-VTPFN). Additionally, a numerical example hasbeen used to demonstrate the suggested strategy.

**Keywords**: I - V Pythagorean fuzzy sets (I-VPFS), I – V Pythagorean fuzzy graph (I-VPFG), I – V triangular Pythagorean fuzzy number (I-VTPFN), ranking function (RF), I – V Pythagorean fuzzy shortest path problem (I-VPFSPP)
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# 1 Introduction

The pair of sets (V, E) that make up a graph are V, which represents the set of vertices, and E, which represents the set of edges that connect the pair of vertices. Numerous domains, including computer, social and natural research, among others, use graph theory. Any mathematical, scientific, or engineering problem can be represented as a graph. Leonard Euler first introduced the idea of graph theory in the year 1736. In order to address the issue of the seven Konigsberge bridges constructed across the Pregel River of Prussia, he produced the first graph. The key factor contributing to graph theory's explosive growth is its application across a variety of disciplines. Even though graphs can be used to simulate events that happen in real-world problems, graph theory became interesting. Graphs are useful for studying these issues.

In the graphs are a very crucial model for networks. The SPPs are very useful and are used in various areas, applications for the road network, transportation, communication channel routing, and scheduling issues are a few examples. The weights of the edges in the basic network issue are taken to be actual numbers. After all, in most constructive applications, the bounds are generally not precise. Therefore, they can be considered fuzzy numbers (FN) in the real world.

Zadeh, 1965 extracted the idea of a Fuzzy Set (FS) as a way to identify uncertainty in 1965. As a generalisation of FSs, Zadeh introduced the idea of I – V fuzzy sets (I-VFS). Intuitionistic fuzzy sets (IFS), which Atanassov [1986] suggested in 1986, look more exactly at uncertainty dimension and give the chance to precisely model the issue based on the available data and consideration I-VFS was introduced three years later by Atanassov and Gargov [1989]. Then the definition of an I-V fuzzy graph (FG) was given by Kumar et al. [2009]. On Intuitionistic fuzzy graph (IFGs), Mohamed and Ali [2018] defined several new applications. Dubois and Prade [1980] were the first to investigate the SPP in an ambiguous and unreliable surroundings. They claim that while the length of the shortest way may be determined, the network may not contain a path that corresponds to it. Numerous academics have looked into the SPP in literature, each in a different way. Lin and Chen proposed the Fuzzy SP length in a network created using fuzzy linear programming. Furthermore, Yao and Lin [2003] created two distinct fuzzy SP network challenges, the first of which makes use of triangular fuzzy integers. Okada [2004] proposed an algorithm to complete the degree of incident for each arc in the network and introduce the similarity index among the sums of Fuzzy numbers (FN) by taking into consideration interactivity with FNs. Kumar and Kaur [2011] suggested one such approach for locating the SP in a network flow with imprecise arc lengths. A based on model the idea of dynamic scripting was presented by Karunambigai et al. [2007]et al. to locate the SP in IFGs. Additionally, Gani and Jabarulla [2010] created a technique for finding the IFSP in a network.

To handle the complex, imprecise, and ambiguity in challenges with effective decision-making, Yager [2014] introduced the concept of the PFSs, as a generalisation of the IFS. The key feature of the PF model is that it relaxes the action that the sum of its mem-ship degree and non-mem-ship degree i.e., $(0.5+0.4 \leq 1)$ with the squar sum of its mem-ship and non-mem-ship degree i.e., $(0.6^2 + 0.7^2 \leq 1)$. Following Yager [2013] invention of the PFS, Xu and Zhang [2014] granted the mathematical form of the PFS as well as the concept of the PFN. Then the idea of a PFGs was proposed by Naz et al. [2013]. As a generalisation of PFSs, presented the idea of I-VPF sets Xindong and Young [2016]. The idea of an I-VPFG was suggested by Mohamed and Ali [2018]. In order to accomplish this, we first offer a mathematical formulation for SP issues where contradiction costs of arcs are expressed in terms of I-VPF numbers. Then, in order to construct a solution algorithm, we present the optimality in I-VPF networks. To compare the costs of several pathways, whose arc are expressed by I-VPF numbers, an improved score function (SF) is employed.

In this study, we present a novel method for finding the SP using I-VPFSs. Using the suggested approach, a decision maker can determine the SP and the SD between each node and the SN. The following is how the study is structured. Sec 2 goes over some fundamental ideas such as I-VTPFNs, arithmetic operations, and RFs. In sec 3, a algorithm for determining the SP and the SD in an I-VPFG is proposed. Sec 4 contains an example of determining the SP and SD between the SN and the DNs. The final section includes some closing remarks.

# 2 Preface

This chapter discusses some fundamental ideas, operations in mathematics, and rank order functions for I-VTPFNs are covered.

**Definition 2.1.** (I-VPFS)Xindong and Young [2016]

An I-VPFS $\widetilde{A}_{\tilde{P}}$, in the global set $Z'$ is defined by

$$\widetilde{A}_{\tilde{P}} = \langle x, \check{\mu}_{\widetilde{A}_{\tilde{P}}}(x), \check{\lambda}_{\widetilde{A}_{\tilde{P}}}(x) \rangle : x \in Z^{'}$$

where $\check{\mu}_{\widetilde{A}_{\tilde{P}}} = [\check{\mu}_{\widetilde{A}_{\tilde{P}_L}}(x), \check{\mu}_{\widetilde{A}_{\tilde{P}U}}(x)] \subseteq [0,1]$ and $\check{\lambda}_{\widetilde{A}_{\tilde{P}}} = \left[\check{\lambda}_{\widetilde{A}_{\tilde{P}_L}}(x), \check{\lambda}_{\widetilde{A}_{\tilde{P}U}}(x)\right] \subseteq [0,1]$ are interval numbers satisfied as $0 \leq \left[\check{\mu}_{\widetilde{A}_{\tilde{P}U}}(x)\right]^2 + \left[\check{\lambda}_{\widetilde{A}_{\tilde{P}U}}(x)\right]^2 \leq 1$ for separate element $x \in Z^{'}$. For each I-VPFS $\widetilde{A}_{\tilde{P}}$ and $x \in X'$,

$$\check{\pi}_{\widetilde{A}_{\tilde{P}}}(x) = \left[\pi^{'}_{\widetilde{A}_{\tilde{P}_L}}(x), \pi^{'}_{\widetilde{A}_{\tilde{P}U}}(x)\right]$$

$$= \left[\sqrt{1 - \left[\check{\mu}_{\widetilde{A}_{\tilde{P}U}}(x)\right]^2 - \left[\check{\lambda}_{\widetilde{A}_{\tilde{P}U}}(x)\right]^2}, \sqrt{1 - \left[\check{\mu}_{\widetilde{A}_{\tilde{P}_L}}(x)\right]^2 - \left[\check{\lambda}_{\widetilde{A}_{\tilde{P}_L}}(x)\right]^2}\right]$$

is called the hesitancy interval of $x$ to $\widetilde{A}_{\widetilde{P}}$. For an I-VPFS $\widetilde{A}_{\widetilde{P}}$ is defined the pair $\left\langle \left[\breve{\mu}_{\widetilde{A}_{\widetilde{P}_L}}(x), \breve{\mu}_{\widetilde{A}_{\widetilde{P}U}}(x)\right]\right\rangle, \left\langle \left[\breve{\lambda}_{\widetilde{A}_{\widetilde{P}_L}}(x), \breve{\lambda}_{\widetilde{A}_{\widetilde{P}U}}(x)\right]\right\rangle$ is called an I-VPFN is denoted by $\widetilde{A}_{\widetilde{P}} = \left\langle \left[\breve{\mu}_{\widetilde{A}_{\widetilde{P}_L}}, \breve{\mu}_{\widetilde{A}_{\widetilde{P}U}}\right]\right\rangle, \left\langle \left[\breve{\lambda}_{\widetilde{A}_{\widetilde{P}_L}}, \breve{\lambda}_{\widetilde{A}_{\widetilde{P}U}}\right]\right\rangle$.

**Definition 2.2:-** (Interval - valued Pythagorean fuzzy graph)Mohamed and Ali [2018]

An I-VPFG with the underneath set $v''$ is defined as a pair of $G' = \left(\breve{P}, \breve{Q}\right)$ where,

(i) In the functions $\breve{\mu}_{\widetilde{P}} : v'' \to [0,1]$ and $\breve{\lambda}_{\widetilde{P}} : v'' \to [0,1]$ express the degree of membership and nonmembership element of the $x' \in v''$, respectively, such that $0 \le \breve{\mu}_{\widetilde{P}}(x') + \breve{\lambda}_{\widetilde{P}}(x') \le 1,\ \forall\, x' \in v''$.

(ii) In the functions $\breve{\mu}_{\widetilde{Q}} : E' \subseteq v'' \times v'' \to [0,1]$, $\breve{\lambda}_{\widetilde{Q}} : E' \subseteq v'' \times v'' \to [0,1]$ is,

$$\breve{\mu}_{\widetilde{Q}_L}((x', y')) \le \min\left(\breve{\mu}_{\widetilde{P}_L}(x'), \breve{\mu}_{\widetilde{P}_L}(y')\right) and$$
$$\breve{\mu}_{\widetilde{Q}_L}((x', y')) \ge \max\left(\breve{\lambda}_{\widetilde{P}_L}(x'), \breve{\lambda}_{\widetilde{P}_L}(y')\right)$$
$$\breve{\mu}_{\widetilde{Q}^U}((x', y')) \le \min\left(\breve{\mu}_{\widetilde{P}^U}(x'), \breve{\mu}_{\widetilde{P}^U}(y')\right) and$$
$$\breve{\mu}_{\widetilde{Q}^U}((x', y')) \ge \max\left(\breve{\lambda}_{\widetilde{P}^U}(x'), \breve{\lambda}_{\widetilde{P}^U}(y')\right)$$

such that,

$0 \le \breve{\mu}^2_{\widetilde{Q}^U}((x', y')) + \breve{\lambda}^2_{\widetilde{P}^U}((x', y')) \le 1,\ \forall\, (x', y') \in E'$.

**Definition 2.3** Kumar et al. [2015]

Let $\widetilde{A}_{\widetilde{P}} = (\widetilde{a_1}, \widetilde{a_2}, \widetilde{a_3}) \left\langle \left[\breve{\mu}_{\widetilde{A}_{\widetilde{P}_L}}, \breve{\mu}_{\widetilde{A}_{\widetilde{P}U}}\right]\right\rangle, \left\langle \left[\breve{\lambda}_{\widetilde{A}_{\widetilde{P}_L}}, \breve{\lambda}_{\widetilde{A}_{\widetilde{P}U}}\right]\right\rangle$ and
$\widetilde{B}_{\widetilde{P}} = \left(\widetilde{b_1}, \widetilde{b_2}, \widetilde{b_3}\right) \left\langle \left[\breve{\mu}_{\widetilde{B}_{\widetilde{P}_L}}, \breve{\mu}_{\widetilde{B}_{\widetilde{P}U}}\right]\right\rangle, \left\langle \left[\breve{\lambda}_{\widetilde{B}_{\widetilde{P}_L}}, \breve{\lambda}_{\widetilde{B}_{\widetilde{P}U}}\right]\right\rangle$ be two I-VPTFNs,
Then defined as,

(i)

$$\begin{aligned}\widetilde{A}_{\widetilde{P}} \oplus \widetilde{B}_{\widetilde{P}} = \langle &\left(\sqrt{\widetilde{a_1}^2 + \widetilde{b_1}^2}, \sqrt{\widetilde{a_2}^2 + \widetilde{b_2}^2}, \sqrt{\widetilde{a_3}^2 + \widetilde{b_3}^2}\right) \\ &\sqrt{\left(\breve{\mu}_{\widetilde{A}_{\widetilde{P}_L}}\right)^2 + \left(\breve{\mu}_{\widetilde{B}_{\widetilde{P}_L}}\right)^2 - \left(\breve{\mu}_{\widetilde{A}_{\widetilde{P}_L}}\right)^2 \cdot \left(\breve{\mu}_{\widetilde{B}_{\widetilde{P}_L}}\right)^2} \\ &\times \sqrt{\left(\breve{\mu}_{\widetilde{A}_{\widetilde{P}U}}\right)^2 + \left(\breve{\mu}_{\widetilde{B}_{\widetilde{P}U}}\right)^2 - \left(\breve{\mu}_{\widetilde{A}_{\widetilde{P}U}}\right)^2 \cdot \left(\breve{\mu}_{\widetilde{B}_{\widetilde{P}U}}\right)^2} \\ &\times \left[\left(\breve{\lambda}_{\widetilde{A}_{\widetilde{P}_L}}\right) \cdot \left(\breve{\lambda}_{\widetilde{B}_{\widetilde{P}_L}}\right), \left(\breve{\lambda}_{\widetilde{A}_{\widetilde{P}U}}\right)\left(\breve{\lambda}_{\widetilde{B}_{\widetilde{P}U}}\right)\right]\rangle .\end{aligned}$$

**Definition 2.4** (Score and accuracy functions) Garg [2016]

The SF and the AF of any I-VPTFN

$$\widetilde{A}_{\widetilde{P}} = \left\langle \left(\widetilde{a}, \widetilde{b}, \widetilde{c}\right) \left[\check{\mu}_{\widetilde{A}_{\widetilde{P}_L}}, \check{\mu}_{\widetilde{A}_{\widetilde{P}U}}\right] \cdot \left[\check{\lambda}_{\widetilde{A}_{\widetilde{P}_L}}, \check{\lambda}_{\widetilde{A}_{\widetilde{P}U}}\right] \right\rangle$$

are defined as,

$S\left(\widetilde{A_{\widetilde{P}}}\right) = \frac{1}{2}\left[\left(\check{\mu}_{\widetilde{A}_{\widetilde{P}_L}}\right)^2 + \left(\check{\mu}_{\widetilde{A}_{\widetilde{P}U}}\right)^2 - \left(\check{\mu}_{\widetilde{A}_{\widetilde{P}_L}}\right)^2 - \left(\check{\mu}_{\widetilde{A}_{\widetilde{P}U}}\right)^2\right],\ S\left(\widetilde{A_{\widetilde{P}}}\right) \in [-1, 1]$ and

$H\left(\widetilde{A_{\widetilde{P}}}\right) = \frac{1}{2}\left[\left(\check{\mu}_{\widetilde{A}_{\widetilde{P}_L}}\right)^2 + \left(\check{\mu}_{\widetilde{A}_{\widetilde{P}U}}\right)^2 + \left(\check{\mu}_{\widetilde{A}_{\widetilde{P}_L}}\right)^2 + \left(\check{\mu}_{\widetilde{A}_{\widetilde{P}U}}\right)^2\right],\ H\left(\widetilde{A_{\widetilde{P}}}\right) \in [0, 1]$.

**Remark 2.5**Kumar et al. [2015]

Let $\widetilde{A_{\widetilde{P}}}$ and $\widetilde{B_{\widetilde{P}}}$ be two I-VTPFVs the collection of real numbers. Then we define a method of ranking is,

(i) If $S\left(\widetilde{A_{\widetilde{P}}}\right) > S\left(\widetilde{B_{\widetilde{P}}}\right)$, then $\left(\widetilde{A_{\widetilde{P}}}\right) > \left(\widetilde{B_{\widetilde{P}}}\right)$, that is $\left(\widetilde{A_{\widetilde{P}}}\right)$ is sup to $\left(\widetilde{B_{\widetilde{P}}}\right)$, denoted by $\left(\widetilde{A_{\widetilde{P}}}\right) > \left(\widetilde{B_{\widetilde{P}}}\right)$.

(ii) If $S\left(\widetilde{A_{\widetilde{P}}}\right) = S\left(\widetilde{B_{\widetilde{P}}}\right)$ and $H\left(\widetilde{A_{\widetilde{P}}}\right) > H\left(\widetilde{B_{\widetilde{P}}}\right)$ then $\left(\widetilde{A_{\widetilde{P}}}\right) > \left(\widetilde{B_{\widetilde{P}}}\right)$, that is $\left(\widetilde{A_{\widetilde{P}}}\right)$ is sup to $\left(\widetilde{B_{\widetilde{P}}}\right)$ denoted by $\left(\widetilde{A_{\widetilde{P}}}\right) > \left(\widetilde{B_{\widetilde{P}}}\right)$.

**I-VTPF distance 2.6**Mohamed and Ali [2018]

Let F be any connected I-VPFG. For any path $\left\{\widetilde{P} : \widetilde{u_1}, \widetilde{u_2}, \ldots, \widetilde{u_n}\right\}$ length of $\widetilde{P}$ is described as the weights added together. $\left(\widetilde{W_i}\right)$ those arcs in $\widetilde{P}$, in which the weight among two adjacent I-VTriPFNs vertices are $J\left(\widetilde{P'}\right) = \sum_{i=1}^{n} (\widetilde{W'_{i-1}},\ \widetilde{W'_i})$. any two nodes $u, v \in F$, let $\widetilde{P} = \{\widetilde{P_i}$ is a $u - v$ path, $i = 1, 2, 3, \ldots\}$ The definition of the I-VTriPF distance is, $I_{\widetilde{P}}\{u, v\} = \min\left\{J\left(\widetilde{P'_i}\right);\ \widetilde{P}, i = 1, 2, \ldots, n\right\}$.

# 3 Algorithm for SP

An algorithm is suggested in this section for the SP and SD of each node from the source node (SN) Okada [2004].

The following steps for algorithm are defined by

Step – 1: Surmise $I_{\widetilde{P_i}} = [0, 0, 0]; [0, 0]\,[1, 1]$ and label the SN is,

$[0, 0, 0]; [0, 0]\,[1, 1] \quad J\left(\widetilde{G}\right) = --$.

Step – 2: Calculate $I_{\widetilde{P'_j}} = min\{I_{\widetilde{P'_i}} \oplus I_{\widetilde{P'_j}}\ /\ i \in \widetilde{G}_{\widetilde{P}}(j)\}; j = 2, 3, \ldots, n.$

Step – 3: A minimum value appearing from step 2 and matching the unique value of $i$, the label node next $j$ as $I_{\widetilde{P_i}}(x)$. If the minimum value appears to be equal to one of the values of $i$, then it indicates that there is just one remaining path between the SN as well as the node $j$because the separation between all pathways is $I_{\widetilde{P_j}}$, choose any value for $i$.

Step – 4: The DN is denoted by $I_{\widetilde{P_n}}\gamma$, then the SD between the SN and the DN is $I_{\widetilde{P_n}}$.

Step – 5: The DN is now marked $I_{\widetilde{P_n}}\gamma$. Check the marking of node $\gamma$ to find the SP between the SN and the DN. Let it be $I_{\widetilde{P_n}}\eta$, now check the marking of node $q$, and so on. Repeat the process until node 1. The SP can be found by combining all nodes.

# 4 Numerical explanation

Consider an I-VPF – scaled graph, in Fig. 1, where the I-VTPFNs represents the distance between any two vertices. The problem is find to the SD and the path enclosed by SN and the DN in network.

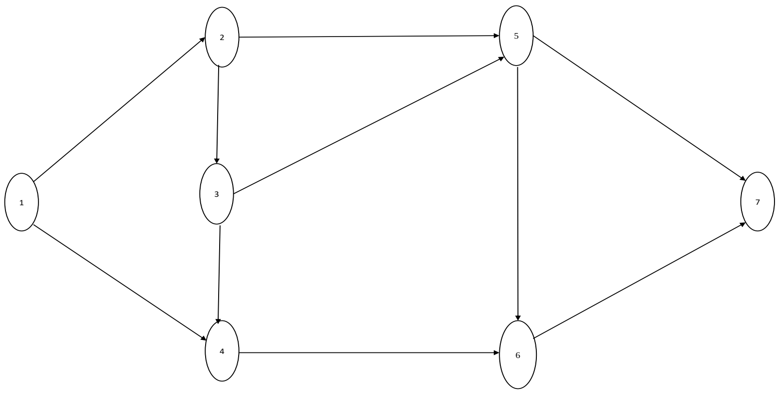


Figure 1: I-VTPF directed graph..

The below (fig 1) network shall be formed in each edges to evaluate the I-VTPFN as follows:

**Explanation**

Let node 7 is the DN is defined as, $n = \mathbf{7}$.

Let us deal with the initial distance is $\widetilde{d_1''} = [0,0,0]\,;[0,0]\,[1,1]$ and the SN [declared nod 1] as $\left[[0,0,0]\,;[0,0]\,[1,1]\,, J\left(\widetilde{G}\right) = --\right]$, the values of $\widetilde{d_{j'}''}; j = 2,3,4,5,6,7$ can be assigned the network terminology:

**Iteration – I** Let taken away from fig.1 PN 2 is node 1, so place the values from $i = 1$ & $j = 2$ appropriately in the St – 2 being the above algorithm.

| Edges | I-VTPF distance |
|---|---|
| $1-2$ | $[0.3, 0.5, 0.5]\,;[0.4, 0.6]\,[0.6, 0.8]$ |
| $1-4$ | $[0.4, 0.5, 0.6]\,;[0.5, 0.6]\,[0.7, 0.8]$ |
| $2-3$ | $[0.3, 0.5, 0.7]\,;[0.6, 0.7]\,[0.6, 0.7]$ |
| $2-5$ | $[0.4, 0.5, 0.7]\,;[0.4, 0.6]\,[0.7, 0.8]$ |
| $3-4$ | $[0.3, 0.5, 0.6]\,;[0.6, 0.7]\,[0.7, 0.7]$ |
| $3-5$ | $[0.5, 0.5, 0.5]\,;[0.5, 0.6]\,[0.6, 0.8]$ |
| $4-6$ | $[0.4, 0.5, 0.6]\,;[0.6, 0.7]\,[0.6, 0.8]$ |
| $5-6$ | $[0.5, 0.6, 0.5]\,;[0.5, 0.6]\,[0.6, 0.8]$ |
| $5-7$ | $[0.4, 0.5, 0.7]\,;[0.4, 0.7]\,[0.7, 0.7]$ |
| $6-7$ | $[0.5, 0.6, 0.7]\,;[0.5, 0.7]\,[0.6, 0.7]$ |

Table 1: I-VTPFNs edge weights

From the path for $\widetilde{d''_2}$ is, $\widetilde{d''_2} = min\{\widetilde{d''_1} \oplus \widetilde{d''_{12}}\}$
$= \min\{[0, 0, 0]\,;[0, 0]\,[1, 1] \oplus [0.3, 0.5, 0.5]\,;[0.4, 0.6]\,[0.6, 0.8]\}$

$$\widetilde{d'_2} = \min\{[0.3, 0.5, 0.5]\,;[0.4, 0.6]\,[0.6, 0.8]\}$$

$$\widetilde{d'_2} = \{[0.3, 0.5, 0.5]\,;[0.4, 0.6]\,[0.6, 0.8]\}$$

From the above calculation, there is a min that corresponds to node 1, and then node 2 is considered to be $\left\{[0.3, 0.5, 0.5]\,;[0.4, 0.6]\,[0.6, 0.8]\,, J\left(\widetilde{G}\right) = 1\right\}$

**Iteration – II** The PN for node 3 is node 2, then the distance between the SN and node 3 according to the above procedure is
$\widetilde{d''_3} = [0.424264, 0.707107, 0.860233]\,;[0.68, 0.820731]\,[0.36, 0.56]$
and the node 3 is

$$\left\{[0.424264, 0.707107, 0.860233]\,;[0.68, 0.820731]\,[0.36, 0.56]\,, J\left(\widetilde{G}\right) = 2\right\}$$

**Iteration – III** The PN nodes for node 4 are nodes 1 and 3, then the value of $\widetilde{d''_4}$ is given,

$$\widetilde{d''_4} = \min\{\widetilde{d''_1} \oplus \widetilde{d''_{14}}, \widetilde{d''_3} \oplus \widetilde{d''_{34}}\}$$

$$= \min\left\{\begin{array}{c} [0, 0, 0]\,;[0, 0]\,[1, 1] \oplus [0.4, 0.5, 0.6]\,;[0.5, 0.6]\,[0.7, 0.8]\,, \\ [0.424264, 0.707107, 0.860233]\,;[0.68, 0.820731]\,[0.36, 0.56] \\ \oplus [0.3, 0.5, 0.6]\,;[0.6, 0.7]\,[0.7, 0.7] \end{array}\right\}$$

$$= \min \left\{ \begin{array}{c} [0.5, 0.707107, 0.781025]\,; [0.60828, 0.768375]\,[0.42, 0.64]\,, \\ [0.519645, 0.86602, 1.048809]\,; [0.809899, 0.912968]\,[0.252, 0.396] \end{array} \right\}$$

Applying the ranking method (based on the SF) results in the following value for $\widetilde{d''}_4$.

$$[0.5, 0.707107, 0.781025]\,; [0.60828, 0.768375]\,[0.42, 0.64]$$

and the term for $\widetilde{d''}_4$ is
$\left\{[0.5, 0.707107, 0.781025]\,; [0.60828, 0.768375]\,[0.42, 0.64]\,, J\left(\widetilde{G}\right) = 1\right\}$.

**Iteration – IV** The PNs for node 5 are nodes 2 & 3, then the value of $\widetilde{d''}_5$ is

$$\widetilde{d''}_5 = \min\left\{\widetilde{d''}_2 \oplus \widetilde{d''}_{25}, \widetilde{d''}_3 \oplus \widetilde{d''}_{35}\right\}$$

Using the rank order method, the value of $\widetilde{d''}_5$ is

$$\widetilde{d''}_5 = [0.5, 0.707107, 0.860233]\,; [0.542586, 0.768375]\,[0.42, 0.64]$$

$\widetilde{d''}_5$ and the name for $\widetilde{d''}_5$ is,
$\left\{[0.5, 0.707107, 0.860233]\,; [0.542586, 0.768375]\,[0.42, 0.64]\,,\ J\left(\widetilde{G}\right) = 2\right\}$.

**Iteration – V** The PN nodes for node 6 are nodes 4 and 5, then the value of $\widetilde{d''}_6$ is
$\widetilde{d''}_6 = \min\left\{\widetilde{d''}_4 \oplus \widetilde{d''}_{46}, \widetilde{d''}_5 \oplus \widetilde{d''}_{56}\right\}$
Using the ranking approach, $\widetilde{d''}_6$ value is
$\widetilde{d''}_6 = [0.707107, 0.92279, 0.994988]\,; [0.686148, 0.858986]\,[0.252, 0.512]$ and its label is
$\{[0.707107, 0.92279, 0.994988]; [0.686148, 0.858986][0.252, 0.512],\ J(\widetilde{G}) = 5\}$.

**Iteration – VI** The PN nodes for node 7 are nodes 5 and 6, then the value of $\widetilde{d''}_7$ is
$\widetilde{d''}_6 = \min\left\{\widetilde{d''}_5 \oplus \widetilde{d''}_{57}, \widetilde{d''}_6 \oplus \widetilde{d''}_{67}\right\}$
Using the ranking approach, $\widetilde{d''}_7$ value is
$\widetilde{d''}_7 = [0.640312, 0.86602, 1.109028]\,; [0.809919, 0.931328]\,[0.252, 0.448]$ and its label is
$\left\{[0.640312, 0.86602, 1.109028]\,; [0.809919, 0.931328]\,[0.252, 0.448]\,,\ J\left(\widetilde{G}\right) = 5\right\}$.

Path $\mathbf{1} \to \mathbf{2} \to \mathbf{5} \to \mathbf{7}$ is determined to be the SP from the computations above, and the distance between the SN and DNs is

$[0.640312, 0.86602, 1.109028]$ ; $[0.809919, 0.931328]$ $[0.252, 0.448]$.

The following table lists the SP taken by all nodes from the SN using the suggested algorithm:

| Node | $\mathbf{I}_{\widetilde{\mathbf{P_j}}}$ | $\mathbf{W}$ |
|---|---|---|
| 2 | $[0.3, 0.5, 0.5]$ ; $[0.4, 0.6]$ $[0.6, 0.8]$ | $1 \to 2$ |
| 3 | $[0.424, 0.707, 0.860]$ ; $[0.68, 0.821]$ $[0.36, 0.56]$ | $1 \to 2 \to 3$ |
| 4 | $[0.3, 0.5, 0.7]$ ; $[0.6, 0.7]$ $[0.6, 0.7]$ | $1 \to 4$ |
| 5 | $[0.5, 0.707, 0.860]$ ; $[0.543, 0.768]$ $[0.42, 0.64]$ | $1 \to 2 \to 5$ |
| 6 | $[0.707, 0.923, 0.995]$ ; $[0.686, 0.859]$ $[0.252, 0.512]$ | $1 \to 5 \to 6$ |
| 7 | $[0.640, 0.866, 1.109]$ ; $[0.809, 0.931]$ $[0.252, 0.448]$ | $\mathbf{1 \to 2 \to 5 \to 7}$ |

Table 2: SP and SD

where $\mathbf{W} = SP$ between the SN and the $\mathbf{j}^{\text{th}}$ node

# 5 Conclusions

In a SPP, the SD and I-VPFSP provide useful information for making decisions. The distance function, which aids in determining the SP, has been defined for I-VTPFNs in this study. On a network with I-VTPF length, a new approach for resolving the I-VPFSP issue has been suggested. The ranking of the paths is a very helpful decision-making tool for selecting the best alternative path. The process of determining the SP has been adequately explained. Furthermore, a used to explain how the suggested algorithm can be implemented.

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